

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/
TECHNOLOGY—MARCH, 2012

TECHNICAL MATHEMATICS-II

[Time : 3 hours

(Maximum marks : 100)

Marks

PART—A

I Answer all questions. Each question carries 2 marks.

1. Evaluate $\lim_{x \rightarrow \alpha} \frac{7-x}{3x+1}$

2. Find $\frac{dy}{dx}$ if $y = \frac{1}{\sec \sqrt{x}}$

3. Find the slope of the curve $y = \frac{3}{x^2}$ at the point (1, 3)

4. Evaluate $\int_0^3 x(x^2+1) dx$

5. Solve $\frac{d^2y}{dx^2} = \sin x$.

(5×2=10)

PART—B

II Answer any five questions. Each question carries 6 marks.

1. (a) Evaluate $\lim_{\theta \rightarrow 0} \frac{\sin 3\theta \cos \theta}{\theta}$

(b) Find 'a' if $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

2. If $y = x^2 \sin x$, prove that $x^2 y'' - 4xy' + (x^2 + 6)y = 0$.

3. A balloon is spherical in shape. Gas is escaping from it at the rate of 10 c.c/sec. How fast is the surface area shrinking when the radius is 15 cms?

4. Find the maximum value of y , if $y = 2x^3 - 9x^2 + 12x$.

5. Evaluate :

(a) $\int \sqrt{1 + \sin 2x} dx$

(b) $\int \frac{\sin^{-1} 2x}{\sqrt{1-4x^2}} dx$

6. Find $\int (\log x)^2 dx$.

7. Solve $\frac{dy}{dx} + \frac{x\sqrt{1+y^2}}{y\sqrt{1+x^2}} = 0$

(5×6=30)

PART—C

(Answer *one* full question from each unit. Each question carries 15 marks.)

UNIT—I

- III 1. Evaluate $\lim_{x \rightarrow 4} \frac{x-4}{x^2-16}$ 2
2. Find $\frac{dy}{dx}$ if:
- (a) $y = \frac{e^{2x} \log x}{x^2}$ (3+3=6)
- (b) $y = \frac{\tan^{-1} x}{(1+x^2)^2}$
3. Find $\frac{dy}{dx}$ if x and y are connected by :
- $x = 3 \sin \theta - \sin^3 \theta$
- $y = 3 \cos \theta - \cos^3 \theta$ 4
4. If $2x^2 + 3xy + 5y^2 = 0$, find $\frac{dy}{dx}$. 3

OR

- IV 1. Using 1st principles, find the derivative of $\sin x$. 5
2. If $y = a \cos(\log x) + b \sin(\log x)$ prove that $x^2 y'' + xy' + y = 0$. 4
3. If $p = \frac{x}{x+R}$, find $\frac{d^2p}{dx^2}$ 3
4. If x and y are connected by $y \log x = x - y$, prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$. 3

UNIT—II

- V 1. Find the equation of the tangent and normal to the rectangular hyperbola $x = ct$, $y = \frac{c}{t}$ at $(ct, \frac{c}{t})$. 6
2. Find the rate of change of the volume of a cone with respect to radius when radius is equal to height. 3
3. An open tank is to be constructed with a square base and vertical sides to hold a given quantity of water, show that the expense of lining it with lead will be the least if the depth is half the width. 6

OR

- VI 1. Prove that the function $x^3 + 6x^2 + 12x - 9$ is an increasing function for all real values of x . 5
2. A circular plate contracts when cooled. Find the rate of decrease in area if the radius decreases at the rate of 0.1 cm/min, when the radius is 4 cm. 5
3. A particle is projected vertically upwards and its height 'h' feet at time 't' is given by $h = 60t - 16t^2$. Find the greatest height attained. 5

UNIT—III

VII Find :

1. $\int \frac{2x^4}{1+x^{10}} dx.$

2. $\int_0^{\sqrt{3}} x\sqrt{1+x^2} dx.$

3. $\int (\tan x + \cot x)^2 dx.$

4. $\int \sqrt{1+\sin 2x} dx.$

5. $\int xe^{-x} dx.$

(5×3=15)

OR

VIII Evaluate :

1. $\int \frac{x^2}{(8+x^3)^4} dx.$

3

2. $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx.$

3

3. $\int_0^{\pi} \frac{1}{1+\sin x} dx.$

4

4. $\int_0^2 x^2 \log x.$

5

UNIT—IV

- IX 1. Find the area enclosed between the curve $y = x^2 - x - 2$ and the x - axis. 5
2. Find the volume generated when the portion of the parabola $y^2 = 4x$ between $x = 0$ and $x = 4$ revolves about the x — axis. 5
3. Solve $\frac{dy}{dx} = e^{x+y} + x^2 e^y.$ 5

OR

- X 1. Find the area enclosed between the curves $y = x^2$ and $2x + y - 3 = 0.$ 5
2. Find the volume of the solid obtained by rotating one arc of the curve $y = \sin x$ about the x — axis. 5
3. Solve $x(1+y^2) dx + y(1+x^2) dy = 0.$ 5