

**APPLIED SCIENCE –I (PHYSICS)
MARCH 2012**

PART A

(Answer the following questions in one or two sentence. Each question carries 2 marks)

- I. 1. Define a Newton. Write dimensional formula for force. (2)

Ans: Newton is the SI unit of force. One Newton is that force will cause a mass of 1kg to have an acceleration 1m/s^2 . Force= Mass * Acceleration. Therefore dimensional formula for force is MLT^{-2} .

2. How do you account for the lack of atmosphere on the moon? (2)

Ans: Moon has no atmosphere because the value of acceleration due to gravity 'g' on the surface of moon is small. The value of escape velocity is small so that all molecules of gases escapes and there is no atmosphere on moon.

PART B

(Answer any two full question. Each carries 8 marks)

- II. 1. Derive an expression for the period of a simple pendulum using dimensional analysis. (4)

Ans: Let the period 'T' of the simple pendulum depend on the length 'l' of the pendulum, mass 'm' of the bob and acceleration due to gravity 'g'.

Therefore $T \propto l^x m^y g^z$(1)

Or, $T = K \cdot l^x m^y g^z$

Taking dimensions on both sides ,

$$L^0 M^0 T^1 = (L^1 M^0 T^0)^x (L^0 M^1 T^0)^y (L^1 M^0 T^{-2})^z$$

$$L^0 M^0 T^1 = L^{x+z} M^y T^{-2z}$$

Equating the powers of M,L and T on both sides .

$$0=y$$

$$0= x+z$$

$$1= -2z$$

Solving, we get, $x=1/2$, $y=0$, $z= -1/2$.

Substituting in (1),

$$T = K l^{1/2} g^{-1/2}$$

I.e. $T = K \sqrt{\frac{l}{g}}$

2. b) State Newton's universal law of gravitation. Distinguish between G and g. (4)

Ans: Newton's universal law of gravitation states that every body in universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

'g' is the acceleration due to gravity. It is the acceleration experienced by the body falling freely. Its value is 9.8 m/s^2 .

'G' is the universal gravitational constant. $G = Fr^2/m_1 m_2$. Its value is $6.67 \times 10^{-11} \text{ m/s}^2$.

The universal gravitational constant is equal to the force of attraction acting between two bodies each of unit mass, whose centers are placed unit distance apart.

- III. 1. State the law of conservation of momentum. Prove it in the case of collision of two bodies moving in the same direction. (4)

Ans: Law of conservation of momentum states that when two or more bodies collide, the sum of their momenta before impact is equal to the sum of momenta after impact.

Consider two bodies of masses m_1 and m_2 moving along a line with velocities u_1 & u_2 respectively. After colliding for a time t , their velocities are v_1 and v_2 .

Momentum of m_2 before Collision = $m_2 u_2$.

Momentum of m_2 after Collision = $m_2 v_2$.

Changes of momentum in t seconds = $m_2 v_2 - m_2 u_2$.

Rate of change of momentum $m_2 = m_2 v_2 - m_2 u_2 / t$.

A change of momentum will occur only by a force. In this case the force causing the change in momentum is action of the body m_1 on m_2 .

Therefore

$$\text{Action} = \frac{m_2 v_2 - m_2 u_2}{t}$$

Change of momentum of first body in t seconds = $m_1 v_1 - m_1 u_1$.

Rate of change of momentum of the first body = $m_1 v_1 - m_1 u_1 / t$.

This rate of change of first body is the reaction. Since action and reaction are equal and opposite.

$$m_2 v_2 - m_2 u_2 / t = - (m_1 v_1 - m_1 u_1 / t)$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

i.e., total momentum before collision is equal to the total momentum after collision.

2. Explain Hooke's law. Formulate the three elastic moduli. (4)

Ans: Hooke's law states that the strain produced in a body is directly proportional to the stress provided the stress is not very large. i.e., Stress/Strain=Constant.

This Constant is called Modulus of elasticity.

Theorem of Parallel axes: -

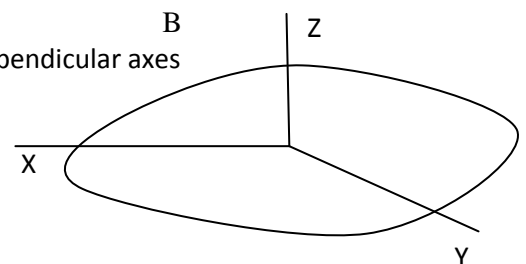
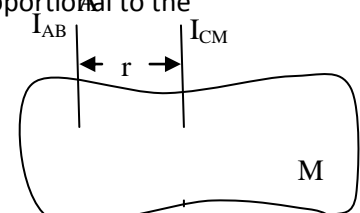
The moment of inertia I_{AB} of any rigid body about a given axis is equal to the sum of its moment of inertia I_{CM} about a parallel axis passing through the centre of gravity and product of the mass of the body and the square of the distance between the axes

$$I_{AB} = I_{CM} + Mr^2$$

Theory of perpendicular axes:-

The sum of moments of inertia of a plane about two mutually perpendicular axes lying in its plane is equal to the moment of inertia about an axis perpendicular to the plane passing through the point of intersection of the first two axes

$$I_x + I_y = I_z$$



IV. 1. Define angular displacement and angular velocity. Derive the relation between linear velocity and angular velocity. (4)

Ans: consider a particle moving in a circular path from P to Q. The radius vector substance an angle θ at the centre when we covers a displacement 's'. This angle ' θ ' is called the angular displacement. $\theta = S/r$. Angular velocity ' ω ' of a particle is the rate at which the angular displacement is taking place $\omega = \theta/t$.

Relation between ω and v :

$$\omega = \theta/t.$$

$$v = S/t.$$

$$\text{Since } S/r = \theta, S = r \theta.$$

$$\text{Therefore } v = r \theta/t = r \omega$$

$$\text{Therefore } v = r \omega$$

2. Derive an expression for the kinetic energy of a rolling disc.

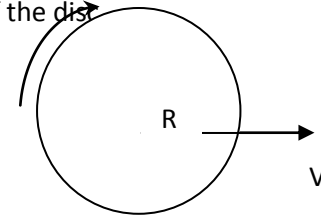
Ans: Consider a disc of radius R and mass M. When it rolls, it has both linear velocity (v) and angular velocity (ω). Consequently, it has two types of kinetic energies – translational and rotational. If ω is the angular velocity and I is the moment of inertia about the axis of the disc

Rotational kinetic energy = $\frac{1}{2} I \omega^2$.

For a disc rolling about its own axis, moment of inertia is $\frac{1}{2} MR^2$

If v is the linear velocity, $\omega = v/R$

Hence rotational kinetic energy = $\frac{1}{2} (\frac{1}{2} MR^2) (v/R)^2$
 $= \frac{1}{4} Mv^2$



In addition to the rotational kinetic energy, the disc has a translational kinetic energy $\frac{1}{2} Mv^2$.

Therefore the total kinetic energy of the rolling disc is $\frac{3}{4} Mv^2$.

PART C

(Answer one full question from each unit. Each question carries 15 marks.)

UNIT 1

V. 1. State Newton's second law of motion and derive the relation for force. (3)

Ans: Newton's second law states that the rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Consider a body of mass moving with a velocity u. When an unbalanced force F acts on it for a time t, its velocity changes to v. So, its initial momentum = mu

Final momentum = mv

Therefore changes in momentum = mv - mu

$$\begin{aligned} \text{Rate of change of momentum} &= \frac{mv - mu}{t} \\ &= \frac{m(v - u)}{t} = m \left(\frac{v - u}{t} \right) = ma \end{aligned}$$

According to the second law, the rate of change of momentum is proportional to force F. Thus,

$$F \propto ma \quad \rightarrow \quad f = kma \quad \text{Here } k=1$$

Therefore $F = ma$

2. The ceiling of a long hall is 25m high. What is the maximum horizontal distance that a ball thrown with a speed of 40m/s can go without hitting of the hall. (6)

Ans: H = 25m

U = 40m/s

H = $u^2 \sin^2 \theta / 2g$

$\sin^2 \theta = H * 2g / u^2 = 25 * 2 * 9.8 / (40)^2 = 0.306$

Therefore $\sin \theta = 0.5534$

i.e. $\theta = 33.36^\circ$

R = $u^2 \sin 2\theta / g$

R = $(40)^2 * \sin(2 * 33.36^\circ) / 9.8 = 150.5\text{m}$

3. A gun weighting 10 kg fires a bullet of 30g with a velocity of 330m/s. With what velocity does the gun recoil?

Ans: Mass of gun, M=10kg

Mass of bullet, m=30kg

Velocity of bullet, v=330m/s

Recoil velocity of gun, V = $-mv/M = -30 * 330 / 10 = -9.90\text{m/s}$

VI. 1. Explain friction. Why is it called a necessary evil? (3)

Ans: friction is defined as the force which opposes the relative motion of two surfaces in contact with one another.

It is necessary evil because it has its own necessities and evil effects. Walking on board or paper, brakes etc work only due to friction.

Friction causes wear and tear of machines, reduces its efficiency and causes unnecessary expense of energy.

2. One end of a string of length 74.2cm is attached to a bucket contain water and the bucket is rotated in a vertical circle. Find the maximum speed with which it can be rotated without spilling water at the highest point. How many revolutions/min does it make? (6)

Ans: Velocity at the highest point, $V = \sqrt{5rg}$

$$V = \sqrt{5 * 0.742 * 9.8} = 6.0298m/s$$

Angular velocity, $\omega = v/r = 6.0298/ 0.748 = 8.1264 \text{ rad/s}$.

1 revolution = $2\pi \text{ rad}$

1 rad = $1/ 2\pi$ revolution. Also $1s = 1/60 \text{ min}$.

Therefore $\omega = 8.1264 * 60 / 2\pi = 77.6 \text{ revolutions/min}$.

3. A body moving with uniform acceleration describes 10m in the 2nd second and 20m in the 4th second of its motion. Calculate the distance moved by it in the 5th second of its motion. (6)

Ans: $S_n = u + a(n-1/2)$

$$10 = u + a(2-1/2)$$

$$10 = u + a3/2$$

$$20 = 2u + 3a \dots \dots \dots (1)$$

$$20 = u + a(4-1/2)$$

$$20 = u + a7/2$$

$$40 = 2u + 7a \dots \dots \dots (2)$$

Solving, $a = 5m/s^2$, $u = 2.5m/s$

$$S_5 = 2.5 + 5(5-1/2) = 25m.$$

VII. 1. Distinguish between torque and angular momentum.

Ans: Torque is the rotating or turning effect produced by a force. It is measured by the product of the force and the perpendicular distance between the point of application of the force and the axes of rotation.

$T = r * F$. Unit is Nm.

Angular momentum is the moment of linear momentum about an axis.

$$L = Pr = m v r = mr^2\omega.$$

Relation between T and L : $T = dL/dt$

2. A uniform circular disc of mass 2kg and radius 0.5m is rotated about an axis passing through its center and perpendicular to its plane. Find its moment of inertia about this axis and about one of its diameters.

Ans: $M = 2kg$

$r = 0.5m$

$$I = 1/2 Mr^2 = 1/2 * 2 * (0.5)^2 = 0.25kgm^2$$

(About the perpendicular axis).

$$\text{Moment of inertia about one of its diameter} = I = 1/4 Mr^2 = 1/4 * 2 * (0.5)^2 = 0.125kgm^2$$

3. What is geostationary orbit? Determine height from the earth. (6)

Ans: An artificial satellite whose orbital period is the same as the rotational period of the earth is called a geostationary satellite. The period of a satellite is,

$$T = \frac{2\pi\sqrt{(R + H)^3}}{gR^2}$$

For geostationary satellite $T = 24\text{hrs} = 864400 \text{ seconds}$. Therefore, $864400 = \frac{2\pi\sqrt{(6400000+h)^3}}{9.8 * 6400000^2}$

$$\frac{9.8 * 86400^2 * 6400000^2}{4 * 3.14 * 3.14} = (6400000 + h)^3$$

$$h = 35954 * 10^3 m = 35954 km$$

VIII. 1. If the ice on the polar caps of the earth melts, how will affect the duration of the day?
(3)

Ans: When the ice on the polar caps melts, the radius increases. Increase in radius result in the increase of moment of inertia since moment of inertia increases, the angular velocity decreases ($L=I, \omega$). I.e. earth will take more time to complete a revolution. I.e. the duration of day increases.

2. On taking a solid ball of rubber from the surface to the bottom of a lake 200m deep, the reduction in volume of the ball is 0.1%. The density of water is $1 * 10^3 kg/m^3$. Determine the value of bulk modulus of rubber ($g=10m/s^2$).
(6)

Ans: $v/V = 0.1\%$

Density, $\rho = 1 * 10^3 kg/m^3$.

Pressure, $P = h \rho g = 200 * 10^3 * 9.8 = 19.6 * 10^5 Pa$

Therefore bulk modulus, $K = PV/v$

Therefore $K = 19.6 * 10^5 / 0.1$

$K = 19.6 * 10^8 Pa$

3. If a satellite is moving around the earth in a circular orbit, derive the formula for the orbital velocity and period of revolution of satellite.
(6)

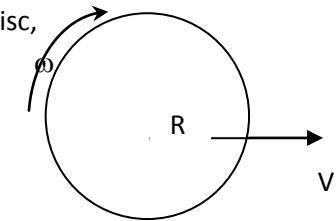
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: $M=300 kg, R=2m, \omega_1=90 rpm, \omega_2=60rpm, t=30s$.

Torque, $T = I\alpha$

Moment of Inertia, $I = \frac{1}{2} MR^2$

$I = 1/2 * 300 * 2^2 = 600 kg m^2$

Angular acceleration, $\alpha = \frac{\omega_2 - \omega_1}{t}$

Initial angular velocity $\omega_1 = 90 rpm = 2\pi * 90 / 60 = 3\pi rad/s$.

Angular velocity after 30s $\omega_2 = 60 rpm = 2\pi * 60 / 60 = 2\pi rad/s$.

Therefore angular acceleration $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{3\pi - 2\pi}{30} = 0.033\pi = 0.1037 rad/s^2$

Therefore torque, $T = I\alpha = 600 * 0.1037 = 62.22 Nm$.