TED (10)-1015 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2011

TECHNICAL MATHEMATICS- II

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) $\lim_{x\to a} \frac{x^{\frac{3}{5}} - a^{\frac{3}{5}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}}$

$$\lim_{x\to a} \frac{x^{\frac{3}{5}} - a^{\frac{3}{5}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}} = \frac{na^{n-1}}{ma^{m-1}} = \frac{\frac{3}{5} \text{x} a^{\frac{3}{5}-1}}{\frac{1}{3} \text{x } a^{\frac{1}{3}-1}}$$

$$\left\{\lim_{x\to a} \frac{x^n - a^n}{x^m - a^m} = \frac{na^{n-1}}{ma^{m-1}}\right.$$

$$= \frac{\frac{3}{5} a^{\frac{-2}{5}}}{\frac{1}{3} a^{\frac{-2}{3}}} = \frac{3}{5} \text{x} \frac{3}{1} \text{x} \frac{a^{\frac{2}{3}}}{a^{\frac{2}{5}}}$$

$$= \frac{9}{5} \text{x} a^{\frac{2}{3}-\frac{2}{5}}$$

$$= \frac{9}{5} a^{\frac{4}{15}}$$

(b) State the product rule and quotient rule of differentiation

<u>Product rule</u>

$$\frac{d}{dx}(u.v) = \text{u}\frac{dv}{dx} + \text{v}\frac{du}{dx}$$

Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$$

(c) The displacement of a particle moving on a straight line is given by $s = ae^{nt} + be^{-nt}$. Find the velocity and acceleration.

$$s = ae^{nt} + be^{-nt}$$

Velocity V = $\frac{ds}{dt}$

V = $ane^{nt} + nbe^{-nt}$

Acceleration, a = $\frac{d^2s}{dt^2}$

a = $an^2e^{nt} + n^2be^{-nt}$

(d) Evaluate $\int cosec(9x+7)\cot(9x+7)dx$

$\int cosec(9x+7)\cot(9x+7)dx = \frac{-cosec(9x+7)}{9} + C$

(e) Solve $\frac{dy}{dx}$ + ytanx = $\cos^2$x

$\frac{dy}{dx}$ + ytanx = $\cos^2$x

$\frac{dy}{dx}$ + Py = Q

P = tanx, Q = $\cos^2$x

∴ I.F = $e^{\int p\,dx} = e^{\int tanx\,dx} = e^{\log secx}$ = secx

∴Solution is y x IF = ∫ Q IF dx

ysecx = $\int \cos^2 x$ x $secx$ dx

= $\int \cos x$ x $\cos x$ x $\frac{1}{secx}$ dx

= sinx + c

Ysecx = sinx + c

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a)

i. Evaluate $\lim_{x\to\infty} \frac{x^2-4x+8}{4x^3+8}$

$$\lim_{x\to\infty} \frac{x^2-4x+8}{4x^3+8} = \lim_{x\to\infty} \frac{x^2\left(1-\frac{4}{x}+\frac{8}{x}\right)}{x^3\left(4+\frac{8}{x^3}\right)}$$

$$= \lim_{x\to\infty} \frac{\left(1-\frac{4}{x}+\frac{8}{x}\right)}{x\left(4+\frac{8}{x^3}\right)}$$

$$= \frac{\left(1-\frac{4}{\infty}+\frac{8}{\infty}\right)}{\infty\left(4+\frac{8}{\infty^3}\right)}$$

$$= \frac{1}{\infty} = 0$$

ii. Y = acosmx

$$\frac{dy}{dx} = \text{-aSin(mx).x}$$

$$\frac{d^2y}{dx^2} = -am^2\text{Cosmx}$$

$$\frac{d^2y}{dx^2} = -m^2(a\ \text{cosmx})$$

$$= -m^2y$$

$$\frac{d^2y}{dx^2} + m^2y = 0$$

(b)

i. If y = log(sin$\sqrt{x}$) find $\frac{dy}{dx}$

y = log(sin$\sqrt{x}$)

$$\frac{dy}{dx} = \frac{1}{sin\sqrt{x}} \text{ x } \cos\sqrt{x} \text{ x } \frac{1}{2\sqrt{x}}$$

ii. Prove that $f(x) = \begin{cases} 3x - 2, & x \leq 0 \\ x + 1, & x > 0 \end{cases}$ is discontinuous at x= 0

f(0) = -2

l.h.l = $\lim_{x \to 0} 3x - 2$

= -2

r.h.l = $\lim_{x \to 0} x + 1$

= 1

∴ l.h.l ≠ r.h.l

∴ So it is discontinuous at x = 0

(c)

i. Find the slop of the curve y = $x^2 - 4x$ at the point(2, -4)

Y = $x^2 - 4x$ at(2, -4)

$\frac{dy}{dx}$ = 2x – 4

∴Slop m = 2 x 2 – 4

= 4 – 4

= 0

ii. A particle is projected vertically upward.Its height 'h' and time't' are connected by $h = 60t - 16t^2$. Find the greatest height attained.

$$h = 60t - 16t^2$$

$\frac{dh}{dt}$ = 60 – 32t

$\frac{d^2h}{dt^2}$ = -32 >0

∴h is maximum

∴ $\frac{dh}{dt} = 0$ ===> 60 – 32t = 0

===> 60 = 32t

∴ t = $\frac{60}{32} = \frac{15}{8}$

∴ Maximum height h =$60t - 16t^2$

$= 60 \text{ x } \frac{15}{8} - 16\frac{225}{64}$

$= \frac{225}{2} - \frac{225}{4} = \frac{225}{4}$

(d)

i. A particles moves such that the displacement from a point 'o' is always given by $s = 5cosnt + 4sinnt$, where n is a constant. Prove that the acceleration varies as the displacement

$$s = 5cosnt + 4sinnt$$

$\frac{ds}{dt} = -5nsinnt + 4ncosnt$

$\frac{d^2s}{dt^2} = -5n^2cosnt - 4n^2sinnt$

$\frac{d^2s}{dt^2} = -n^2(5cosnt - 4sinnt)$

$\frac{d^2s}{dt^2} = -n^2s$

$\frac{d^2s}{dt^2}$=ks, where k = $-n^2$

∴Acceleration varies as the distances.

ii. Find the range of values of 'x' for which $(x^2 - 2x + 3)$ is

a. Increasing

b. Decreasing

Y = $x^2 - 2x + 3$

$\frac{dy}{dx}$=2x – 2

a. Increasing ,$\frac{dy}{dx}$>0

2x – 2 = > 0

2x > 2

X > 1

b. Decreasing , $\frac{dy}{dx} < 0$

$2x - 2 < 0$

$2x < 2$

$X < 1$

(e)

i. Evaluate $\int \frac{3x^2}{\sqrt{1-x^6}}dx$

Let $u = x^3$

$\frac{du}{dx} = 3x^2$

$3x^2\, dx = du$

$$\int \frac{3x^2}{\sqrt{1-x^6}}dx = \int \frac{3x^2}{\sqrt{1-(x^3)^2}}dx$$

$$= \int \frac{du}{\sqrt{1-u^2}}$$

$$= \int \frac{1}{\sqrt{1-u^2}}du$$

$$= \sin^{-1} u$$

$$= \sin^{-1} x^3 + C$$

ii. Evaluate $\int xe^{-x}dx$

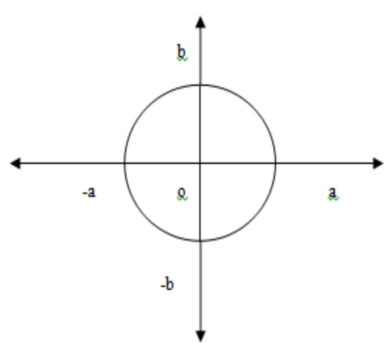
$$\int xe^{-x}dx = x\frac{e^{-x}}{-1} - \int \frac{e^{-x}}{-1} \times 1\, dx$$

$$= -xe^{-x} + \int e^{-x}dx$$

$$= -xe^{-x} + e^{-x} + c$$

(f)

i. Calculate the entire area of the eclipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}= 1$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}= 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$x = 0, x = a$

$$= \frac{a^2-x^2}{a^2}$$

$Y^2 = \frac{b^2}{a^2}\ (a^2 - x^2)$

$Y = \frac{b}{a}\sqrt{a^2 - x^2}$

Area in 1st quadrant = $\int_a^b y\,dx$

$= \int_a^b \frac{b}{a}\sqrt{a^2 - x^2}$ dx

$= \frac{b}{a}\int_a^b \sqrt{a^2 - x^2}$ dx

$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$= \frac{b}{a}\left[\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right) - \frac{0}{2}\sqrt{a^2 - 0^2} - \frac{a^2}{2}\sin^{-1}\left(\frac{0}{a}\right)\right]$

$= \frac{b}{a}\left[0 + \frac{a^2}{2}\sin^{-1}(1) - 0 - 0\right]$

$= \frac{ba^2}{a} \times \frac{\pi}{4}$

$= \frac{\pi a\ \ b}{4}$ square units

Total area = 4 x $\frac{\pi a\ \ b}{4}$

$= \pi ab$ square units

ii. Solve $\frac{dy}{dx} - 10y = 0$

$\frac{dy}{dx} - 10y = 0$

$\frac{dy}{dx} = 10y$

$\frac{dy}{y} = 10dx$

$\int \frac{1}{y}dy = \int 10x\ \ dx$

Logy = 10x + C

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Evaluate $\lim_{\theta \to 0} \frac{tan3\theta + sin5\theta}{8\theta}$

$$\lim_{\theta \to 0} \frac{tan3\theta + sin5\theta}{8\theta} = \lim_{\theta \to 0} \frac{tan3\theta}{8\theta} + \lim_{\theta \to 0} \frac{tan5\theta}{8\theta}$$

$$= \frac{3}{8} + \frac{5}{8}$$

$$= \frac{8}{8}$$

$$=1$$

(b) Find $\frac{dy}{dx}$, if x = 3cosθ – cos$^3\theta$, y = 3sinθ –sin$^3\theta$

If x = 3cosθ – cos$^3\theta$

$\frac{dx}{d\theta}$= -3sinθ – 3cos$^2\theta$x -sinθ

= -3sinθ + 3sinθ cos$^2\theta$

= 3sinθ (cos$^2\theta$– 1)

= 3sinθ x –sin$^2\theta$

= -3sin$^3\theta$

y = 3sinθ –sin$^3\theta$

$\frac{dy}{d\theta}$= 3cosθ – 3sin$^2\theta$cosθ

= 3cos θ(1 – sin$^2\theta$)

= 3cosθ x cos$^2\theta$

= 3cos$^3\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \frac{3\cos^3\theta}{-3\sin^3\theta} = -\cot^3\theta$$

$$\frac{dy}{dx} = -\cot^3\theta$$

IV.

(a) By the method of 1st principle, find the derivatives of cosx

Y = cosx

f(x) = cosx

f(x + Δx) = cos(x + Δx)

f(x + Δx) – f(x) = cos(x + Δx) – cosx

$$= -2\sin\frac{(x+\Delta x+x)}{2} x \sin\frac{(x+\Delta x-x)}{2}$$

$$= -2\sin\frac{(2x+\Delta x)}{2} x \sin\frac{(\Delta x)}{2}$$

$$= -2\sin\left(x + \frac{(\Delta x)}{2}\right) x \sin\frac{(\Delta x)}{2}$$

$$\therefore \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-2\sin\left(x+\frac{(\Delta x)}{2}\right) x \sin\frac{(\Delta x)}{2}}{\Delta x}$$

$$= \frac{-2\sin\left(x+\frac{(\Delta x)}{2}\right) x \sin\frac{(\Delta x)}{2}}{\frac{\Delta x}{2} x\ 2}$$

$$\therefore \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin\left(x+\frac{(\Delta x)}{2}\right)}{\frac{\Delta x}{2}} x \lim_{\Delta x \to 0} \sin\left(x + \frac{(\Delta x)}{2}\right)$$

$$= 1\ x -\sin(x+\frac{0}{2})$$

$$\therefore \frac{dy}{dx} = -\sin x$$

(b) Find $\frac{dy}{dx}$, if y = log(2x + 3)e^{2x}

$$y = \log(2x + 3)e^{2x}$$

$$\frac{dy}{dx} = \log(2x + 3)e^{2x} x\ 2 + e^{2x}\ x \frac{1}{2x+3} x\ 2$$

(c) If y = sin(m $\sin^{-1}$x). prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

y = sin (m $\sin^{-1}$x)

$\sin^{-1}y = m \sin^{-1}x$

Differentiating on both sides

$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = m\frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dx} = m\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\,y' = m\sqrt{1-y^2}$

Differentiating again w.r.to x

$\sqrt{1-x^2}\,y'' + y'\frac{1}{2\sqrt{1-x^2}} \times -2x = m\frac{1}{2\sqrt{1-y^2}} \times -2y \times \frac{dy}{dx}$

$\sqrt{1-x^2}\,y'' - \frac{xy'}{\sqrt{1-x^2}} = \frac{-my}{\sqrt{1-y^2}}\frac{dy}{dx}$

$\frac{(1-x^2)y'' - xy'}{\sqrt{1-x^2}} = \frac{-my}{\sqrt{1-y^2}}y'$

$Ie\frac{(1-x^2)y'' - xy'}{\sqrt{1-x^2}} + \frac{my}{\sqrt{1-y^2}}y' = 0$

Solving these we get,

$\frac{(1-x^2)y'' - xy'}{\sqrt{1-x^2}} + \frac{my \times m\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}}{\sqrt{1-y^2}}y' = 0$

$\frac{(1-x^2)y'' - xy' + m^2y}{\sqrt{1-x^2}} = 0$

$ie(1-x^2)y'' - xy' + m^2y = 0$

V.

(a) The radius of a circular plate is decreasing in length at a rate of 0.2cm/s. what is the rate at which the area decreasing when the radius is 6cm

Given $\frac{dr}{dt} = -0.2$

A= πr^2

$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$, given r = 6

$\frac{dA}{dt} = \pi$ 2 x 6 x $-$ 0.2

$\frac{dA}{dt} = -2.4\pi cm^2/sec$

∴Area is decreasing at a rate of 2.4$\pi cm^2/sec$

(b) Find the minimum value of $2x^3 - 3x^2 - 36x + 10$

Y = $2x^3 - 3x^2 - 36x + 10$

$\frac{dy}{dx} = 6x^2 - 6x - 36$

= $6(x^2 - x - 6)$

∴ $\frac{dy}{dx}$= 0 ===> $6(x^2 - x - 6)$ = 0

===>$(x^2 - x - 6)$ = 0

===> (x – 3) (x + 3) = 0

===> x = 3, -2

$\frac{d^2y}{dx^2} =$ 12x – 6

At x = 3 $\frac{d^2y}{dx^2} =$ 12 x 3 – 6 = 36 – 6 = 30>0

∴at x = 3 value is minimum

At x = -2 $\frac{d^2y}{dx^2} =$ 12 x -2 – 6 = -24 – 6 = -30 < 0

∴at x = -2 value is maximum

∴ Maximum value y = $2x^3 - 3x^2 - 36x + 10$

= $2(-2)^3 - 3(-2)^2 - 36$x$(-2) + 10$

= 2x – 8 – 12 + 72 + 10

= -16 – 12 +82

= -28 + 82

= 54

Minimum value y = $2x^3 - 3x^2 - 36x + 10$

$= 2(3)^3 - 3(3)^2 - 36x(3) + 10$

= 54 – 27 – 108 + 10

= 64 – 135

= -71

(c) A spherical rubber bladder of radius increases at a uniform rate of 1 inch/minute. Find the rate at which the volume is increasing at the end of 2 minutes

Given radius r = 3 inches

$\frac{dr}{dt}$= 1 inch/minute

V =$\frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi r^2$x 1

$\frac{dv}{dt} = 4\pi r^2$

∴ Required radius r = given radius + t x$\frac{dr}{dt}$

r = 3 + 2 x 1

= 3 + 2

r = 5

∴ $\frac{dv}{dt}$= 4π x 5^2

= 4π x 25

$\frac{dv}{dt}$= 100π inch^3/minute

VI.

(a) Find the equation of the tangent and normal to the curve $x^2 + y^2 = 25$

$x^2 + y^2 = 25$ (3, -4)

2x + 2y $\frac{dy}{dx}$= 0

$2y\frac{dx}{dx}$= -2x

$\frac{dy}{dx}$= $\frac{-2x}{2y}$

$\frac{dy}{dx}$= $\frac{-x}{y}$

m = $\frac{-3}{-4}$

m = $\frac{3}{4}$

Equation of the tangent is $y - y_1 = m(x - x_1)$

$y - -4 = \frac{3}{4}(x-3)$

$y + 4 = \frac{3}{4}(x-3)$

4y + 16 = 3x – 9

3x – 4y – 25 = 0

Equation of normal is $y - y_1 = -\frac{1}{m}(x - x_1)$

$y - -4 = -\frac{4}{3}(x-3)$

$y + 4 = -\frac{4}{3}(x-3)$

3y + 12 = -4x + 12

4x + 3y = 0

(b) If S denotes the displacement of a particle at the time 't' seconds and S = $t^3 - 6t^2 + 8t - 4$.Find the time when the acceleration is 12cm/sec^2 and the velocity at the time.

S = $t^3 - 6t^2 + 8t - 4$

$\frac{ds}{dt} = 3t^2 - 12t + 8$

$a = \frac{d^2s}{dt^2} =$ 6t – 12

But given a = 12cm/sec^2

12 = 6t – 12

12 + 12 = 6t

6t = 24

t = 4 sec

∴ Velocity V = $\frac{ds}{dt} = 3t^2 - 12t + 8$

$= 3 \times 4^2 - 12 \times 4 + 8$

$= 3 \times 16 - 48 + 8$

$= 48 - 48 + 8$

$= 8$

(c) Find the slop of the normal to the curve y = $x^2 + x - 1$ at (2, 7)

y = $x^2 + x - 1$ at (2, 7)

Slop m = $\frac{dy}{dx}$ = 2x + 1

m = 2 x 2 + 1

m = 4 + 1 = 5

VII.

(a) Evaluate $\int (tanx + cotx)^2\, dx$

$$\int (tanx + cotx)^2\, dx$$

$= \int tan^2 x\, dx + \int \cot^2 x\; dx + 2\int tanxcotx\, dx$

$= \int (sec^2 x - 1)dx + \int (cose^2 x - 1)\, dx + 2\int 1\, dx$

= tanx – x + -cotx – x + 2x

= tanx – cotx – 2x + 2x

= tanx – cotx + C

(b) Evaluate $\int \sin^2 5x\, dx$

$\int \sin^2 5x\, dx = \int \frac{1 - cose10x}{2}$dx

Sin^{2}x = $\frac{1-cos2x}{2}$

$= \frac{1}{2}\left[x - \frac{sin10x}{10}\right]$

$$= \frac{x}{2} - \frac{sin10x}{20} + \text{c}$$

(c) Evaluate $\int \tan^3 xsec^2x\,dx$

$\int \tan^3 xsec^2x\,dx = \int u^5\,du$

$= \frac{u^6}{6}$

$= \frac{\tan^6 x}{6} + \text{c}$

U = tanx

$\frac{du}{dx} = \sec^2 x$

Du = $\sec^2$x dx

(d) Evaluate $\int logx\,dx$

$\int logx\,dx = logx$ x $x - \int x\frac{1}{x}$dx

= x logx - $\int$ 1dx

= x logx – x + C

(e) Evaluate $\int_0^{\frac{\pi}{2}} sinx(1 - cosx)^5 dx$

$\int_0^{\frac{\pi}{2}} sinx(1 - cosx)^5 dx$

$= \int_0^1 u^5\,du$

$= \left[\frac{u^6}{6}\right]\begin{matrix}1\\0\end{matrix}$

$= \frac{1}{6}$

U = $1 - cosx$

$\frac{du}{dx} = sinx$

du = sinx dx

x = 0, u = 1 – cos0

= 1 – 1 = 0

x = $\frac{\pi}{2}$, u = 1 - cos$\frac{\pi}{2}$

1 – 0 = 1

VIII.

(a) Evaluate $\int \frac{x^2+2x+1}{x^2} dx$

$$\int \frac{x^2 + 2x + 1}{x^2} dx$$

$= \int \left(1 + \left(\frac{2}{x}\right) + \frac{1}{x^2}\right) dx$

$= \int 1 \ dx + 2\int \left(\frac{1}{x}\right) dx + \int x^{-2}\, dx$

$= x + 2logx - \frac{1}{x} + c$

(b) Evaluate $\int \frac{1}{\sqrt{3x+4}}\, dx$

$\int \frac{1}{\sqrt{3x+4}}\, dx = \int \frac{1}{(3x+4)^{\frac{1}{2}}}\, dx$

$= \int (3x+4)^{-\frac{1}{2}}\, dx$

$= \frac{(3x+4)^{-\frac{1}{2}}}{\frac{1}{2} \text{ x } 3}$

$= \frac{\sqrt{3x+4}}{\frac{3}{2}}$

$= \frac{2}{3}\sqrt{3x+4} + C$

(c) Evaluate $\int e^x cosec^2(e^x)\, dx$

$\int e^x cosec^2(e^x)\, dx$

Let u = e^x

$\frac{du}{dx} = e^x$

$= \int cosec^2 u\, du$

= -cot u

du = e^x du

= -cot e^x + C

(d) Evaluate $\int sin^{-1}x\, dx$

$\int sin^{-1}x\, dx = \int sin^{-1}x \text{ x } 1\, dx$

$= sin^{-1}x \text{ x } x - \int x \frac{1}{\sqrt{1-x^2}} dx$

Let u = 1 – x^2

$\frac{du}{dx} = -2x$

$= x\, sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}} dx$

$\frac{-du}{2} = xdx$

$= x\, sin^{-1}x - \int \frac{1}{\sqrt{u}} \text{ x } \frac{-du}{2}$

$= x\ sin^{-1}x + \frac{1}{2}\int u^{-\frac{1}{2}}$ du

$= x\ sin^{-1}x + \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}$

$= x\ sin^{-1}x + \sqrt{u}$

$= x\ sin^{-1}x + \sqrt{1-x^2}$ + C

(e) Evaluate $\int_0^{\pi} \frac{1-sinx}{x+cosx}\,dx$

Let u = x + cosx

$\frac{du}{dx} = 1 - sinx$

du = $(1 - sinx)dx$

when x = 0,

u = 0 + cos0 = 1

when x = π,

u = $\pi + cos\pi$

u = π + -1 = $\pi - 1$

$\int_0^{\pi} \frac{1-sinx}{x+cosx} = \int_0^{\pi} \frac{1}{u}$du

= $[\log u]_1^{\pi-1}$

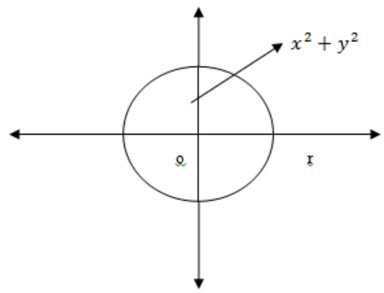
= $\log(\pi - 1)$ – log 1

= $\log(\pi - 1)$

IX.

(a) Find the area of circle of radius r using integration

We know the equation of a circle is



$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$y = \sqrt{r^2 - x^2}$ x = 0, x = r

Area in the 1st quadrant, A = $\int_a^b y\, dx$

$$= \int_0^r \sqrt{r^2 - x^2}\, dx$$

$$= \left[\frac{x^2}{2}\sqrt{r^2 - x^2} + \frac{r^2}{2}\sin^{-1}\left(\frac{x}{r}\right)\right]_0^r$$

$$= \frac{r^2}{2}\sqrt{r^2 - r^2} + \frac{r^2}{2}\sin^{-1}\left(\frac{r}{r}\right)$$

$$- \frac{0^2}{2}\sqrt{r^2 - 0^2} + \frac{r^2}{2}\sin^{-1}\left(\frac{0}{r}\right)$$

$$= \frac{r^2}{2} \times 0 + \frac{r^2}{2}\sin^{-1}(1) - 0 - \frac{r^2}{2} \times 0$$

$= \frac{r^2}{4}\frac{\pi}{}$ total area= 4xarea in first quadrant=

$$A = \pi r^2$$

(b) Find the volume generated by the rotation of area bounded by the curve $y = 3$ and y = 9 about the y – axis

Y = $2x^2 + 1$

$\therefore x^2 = \frac{y-1}{2}$ y = 3, y = 9

$\therefore$volume V = $\pi \int_a^b x^2 dy$

$= \pi \int_a^b \frac{y-1}{2} dy$

$= \frac{\pi}{2}\left[\frac{y^2}{2} - y\right]_3^9$

$= \frac{\pi}{2}\left[\left(\frac{81}{2} - \frac{9}{2}\right) - (9-3)\right]$

$= \frac{\pi}{2}\left[\left(\frac{72}{2}\right) - 6\right]$

$= \frac{\pi}{2}[36 - 6]$

$= \frac{\pi}{2} \times 30$

$= 15\pi \ cm^3$

(c) Solve $\frac{dy}{dx} = e^{x+y} + x^2e^y$

$\frac{dy}{dx} = e^{x+y} + x^2e^y$

$\frac{dy}{dx} = e^x e^y + x^2e^y$

$\frac{dy}{dx} = e^y(e^x + x^2)$

$\frac{dy}{e^y} = (e^x + x^2)\ dx$

$e^y dy = (e^x + x^2)\ dx$

$\therefore \int e^y dy = \int (e^x + x^2)\ dx$

$ie - e^{-y} = e^x + \frac{x^3}{3} + C$

X.

(a) Find the area bounded by the curve $y = x^2 + x$ and the x - axis

$y = x^2 + x$

$\therefore x^2 + x = 0$

x (x + 1) = 0

x = 0 or x = -1

∴ Area A = $\int_a^b y\,dx$

$$= \int_{-1}^{0} x^2 + x\,dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{0}$$

$$= \frac{0^3}{3} + \frac{0^2}{2} - \frac{-1^3}{3} - \frac{-1^2}{2} + \frac{1}{3} - \frac{1}{1}$$

$$= \frac{-1}{6}$$

Area cannot be –ve, so Area = $\frac{1}{6}m^2$

(b) Find the volume of solid obtained by rotatory one arch of the curve y = sin3x about the x - axis

Given that y = sin3x x = 0, x = $\frac{\pi}{3}$

∴Volume V = $\pi \int_a^b y^2$ dx

$$= \pi \int_0^{\frac{\pi}{3}} \sin^2 3x \, dx$$

$$= \pi \int_0^{\frac{\pi}{3}} \frac{1-\cos 6x}{2} \, dx$$

$$= \frac{\pi}{2}\left[x - \frac{\sin 6x}{6}\right]_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{2}\left[\frac{\pi}{3} - \frac{\sin 6\frac{\pi}{3}}{6} - 0 + \frac{\sin 0}{6}\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{3} - \frac{\sin 2\pi}{6} - 0 + 0\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{3} - 0\right]$$

$= \frac{\pi^2}{6}$cubic units

(c) Solve $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

Dividing $(1 + x^2)$ on both sides, we get

$\frac{dy}{dx} + \frac{1}{(1 + x2)}y = \frac{e^{\tan^{-1} x}}{(1 + x2)}$

$\frac{dy}{dx} + Py = Q$

$\therefore P = \frac{1}{(1 + x2)}$

$Q = \frac{e^{\tan^{-1} x}}{(1 + x2)}$

$\therefore$ I.F $= e^{\int p\, dx} = e^{\int \frac{1}{(1 + x2)}\, dx}$

$= e^{\tan^{-1} x}$

$\therefore$Solution is Y x IF $= \int Q$ x IF dx

$$Y \times e^{\tan^{-1} x} = \int \frac{e^{\tan^{-1} x}}{(1 + x2)} \times e^{\tan^{-1} x}\, dx$$

Let u = $e^{\tan^{-1} x}$

$\frac{du}{dx} = e^{\tan^{-1} x} \frac{1}{(1 + x2)}$

du $= e^{\tan^{-1} x} \frac{1}{(1 + x2)} dx$

$\therefore$ Solution is $Y \times e^{\tan^{-1} x} = \int u\, du$

$= \frac{u^2}{2}$

$$Y \times e^{\tan^{-1} x} = \frac{e^{2tan^{-1}x}}{2} + c$$