

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/
TECHNOLIGY- MARCH, 2011

TECHNICAL MATHEMATICS- II
(Common – Except DCP and CABM)

[Time: 3 hours

(Maximum marks: 100)

Marks

PART –A
(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

$$(a) \lim_{x \rightarrow a} \frac{x^{\frac{3}{5}} - a^{\frac{3}{5}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}}$$

$$\lim_{x \rightarrow a} \frac{x^{\frac{3}{5}} - a^{\frac{3}{5}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}} = \frac{na^{n-1}}{ma^{m-1}} = \frac{\frac{3}{5}xa^{\frac{3}{5}-1}}{\frac{1}{3}a^{\frac{1}{3}-1}}$$

$$\left\{ \lim_{x \rightarrow a} \frac{x^n - a^n}{x^m - a^m} = \frac{na^{n-1}}{ma^{m-1}} \right.$$

$$= \frac{\frac{3}{5}a^{\frac{-2}{5}}}{\frac{1}{3}a^{\frac{-2}{3}}} = \frac{3}{5} \times \frac{3}{1} \times \frac{a^{\frac{2}{3}}}{a^{\frac{2}{5}}}$$

$$= \frac{9}{5} \times a^{\frac{2}{3} - \frac{2}{5}}$$

$$= \frac{9}{5} a^{\frac{4}{15}}$$

(b) State the product rule and quotient rule of differentiation

Product rule

$$\frac{d}{dx} (u \cdot v) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient rule

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

- (c) The displacement of a particle moving on a straight line is given by $s = ae^{nt} + be^{-nt}$. Find the velocity and acceleration.

$$s = ae^{nt} + be^{-nt}$$

$$\text{Velocity } V = \frac{ds}{dt}$$

$$V = ane^{nt} + nbe^{-nt}$$

$$\text{Acceleration, } a = \frac{d^2s}{dt^2}$$

$$a = an^2e^{nt} + n^2be^{-nt}$$

- (d) Evaluate $\int \operatorname{cosec}(9x + 7) \cot(9x + 7) dx$

$$\int \operatorname{cosec}(9x + 7) \cot(9x + 7) dx = \frac{-\operatorname{cosec}(9x+7)}{9} + C$$

- (e) Solve $\frac{dy}{dx} + y \tan x = \cos^2 x$

$$\frac{dy}{dx} + y \tan x = \cos^2 x$$

$$\frac{dy}{dx} + Py = Q$$

$$P = \tan x, Q = \cos^2 x$$

$$\therefore \text{IF} = e^{\int p dx} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$$

$$\therefore \text{Solution is } y \times \text{IF} = \int Q \text{IF} dx$$

$$y \sec x = \int \cos^2 x \times \sec x dx$$

$$= \int \cos x \times \cos x \times \frac{1}{\sec x} dx$$

$$= \sin x + c$$

$$Y \sec x = \sin x + c$$

PART -B

Answer any five questions. Each question carries 6 marks

II.

(a)

i. Evaluate $\lim_{x \rightarrow \infty} \frac{x^2 - 4x + 8}{4x^3 + 8}$

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x^2 - 4x + 8}{4x^3 + 8} &= \lim_{x \rightarrow \infty} \frac{x^2 \left(1 - \frac{4}{x} + \frac{8}{x}\right)}{x^3 \left(4 + \frac{8}{x^3}\right)} \\ &= \lim_{x \rightarrow \infty} \frac{\left(1 - \frac{4}{x} + \frac{8}{x}\right)}{x \left(4 + \frac{8}{x^3}\right)} \\ &= \frac{\left(1 - \frac{4}{\infty} + \frac{8}{\infty}\right)}{\infty \left(4 + \frac{8}{\infty^3}\right)} \\ &= \frac{1}{\infty} = 0\end{aligned}$$

ii. $Y = a \cos mx$

$$\frac{dy}{dx} = -a \sin(mx) \cdot x$$

$$\frac{d^2y}{dx^2} = -am^2 \cos mx$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= -m^2(a \cos mx) \\ &= -m^2y\end{aligned}$$

$$\frac{d^2y}{dx^2} + m^2y = 0$$

(b)

i. If $y = \log(\sin \sqrt{x})$ find $\frac{dy}{dx}$

$$y = \log(\sin \sqrt{x})$$

$$\frac{dy}{dx} = \frac{1}{\sin \sqrt{x}} \times \cos \sqrt{x} \times \frac{1}{2\sqrt{x}}$$

ii. Prove that $f(x) = \begin{cases} 3x - 2, & x \leq 0 \\ x + 1, & x > 0 \end{cases}$ is discontinuous at $x = 0$

$$f(0) = -2$$

$$\text{l.h.l} = \lim_{x \rightarrow 0} 3x - 2$$

$$= -2$$

$$\text{r.h.l} = \lim_{x \rightarrow 0} x + 1$$

$$= 1$$

$$\therefore \text{l.h.l} \neq \text{r.h.l}$$

\therefore So it is discontinuous at $x = 0$

(c)

i. Find the slope of the curve $y = x^2 - 4x$ at the point $(2, -4)$

$$Y = x^2 - 4x \quad \text{at}(2, -4)$$

$$\frac{dy}{dx} = 2x - 4$$

$$\therefore \text{Slop } m = 2 \times 2 - 4$$

$$= 4 - 4$$

$$= 0$$

ii. A particle is projected vertically upward. Its height 'h' and time 't' are connected by $h = 60t - 16t^2$. Find the greatest height attained.

$$h = 60t - 16t^2$$

$$\frac{dh}{dt} = 60 - 32t$$

$$\frac{d^2h}{dt^2} = -32 < 0$$

\therefore h is maximum

$$\therefore \frac{dh}{dt} = 0 \implies 60 - 32t = 0$$

$$\implies 60 = 32t$$

$$\therefore t = \frac{60}{32} = \frac{15}{8}$$

\therefore Maximum height $h = 60t - 16t^2$

$$= 60 \times \frac{15}{8} - 16 \frac{225}{64}$$

$$= \frac{225}{2} - \frac{225}{4} = \frac{225}{4}$$

(d)

- i. A particles moves such that the displacement from a point 'o' is always given by $s = 5\cos nt + 4\sin nt$, where n is a constant. Prove that the acceleration varies as the displacement

$$s = 5\cos nt + 4\sin nt$$

$$\frac{ds}{dt} = -5n\sin nt + 4n\cos nt$$

$$\frac{d^2s}{dt^2} = -5n^2\cos nt - 4n^2\sin nt$$

$$\frac{d^2s}{dt^2} = -n^2(5\cos nt - 4\sin nt)$$

$$\frac{d^2s}{dt^2} = -n^2s$$

$$\frac{d^2s}{dt^2} = ks, \text{ where } k = -n^2$$

∴ Acceleration varies as the distances.

- ii. Find the range of values of 'x' for which $(x^2 - 2x + 3)$ is

- a. Increasing
- b. Decreasing

$$Y = x^2 - 2x + 3$$

$$\frac{dy}{dx} = 2x - 2$$

- a. Increasing, $\frac{dy}{dx} > 0$

$$2x - 2 > 0$$

$$2x > 2$$

$$X > 1$$

b. Decreasing, $\frac{dy}{dx} < 0$

$$2x - 2 < 0$$

$$2x < 2$$

$$x < 1$$

(e)

i. Evaluate $\int \frac{3x^2}{\sqrt{1-x^6}} dx$

Let $u = x^3$

$$\int \frac{3x^2}{\sqrt{1-x^6}} dx = \int \frac{3x^2}{\sqrt{1-(x^3)^2}} dx$$

$$\frac{du}{dx} = 3x^2$$

$$3x^2 dx = du$$

$$= \int \frac{du}{\sqrt{1-u^2}}$$

$$= \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \sin^{-1} u$$

$$= \sin^{-1} x^3 + C$$

ii. Evaluate $\int x e^{-x} dx$

$$\int x e^{-x} dx = x \frac{e^{-x}}{-1} - \int \frac{e^{-x}}{-1} x \cdot 1 dx$$

$$= -x e^{-x} + \int e^{-x} dx$$

$$= -x e^{-x} + e^{-x} + c$$

(f)

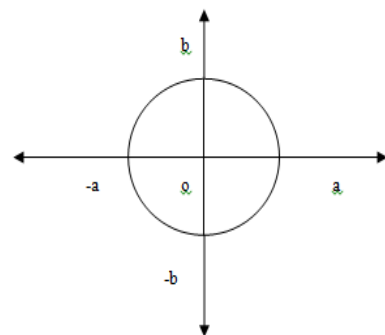
i. Calculate the entire area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$x = 0, x = a$$

$$= \frac{a^2 - x^2}{a^2}$$



$$Y^2 = \frac{b^2}{a^2} (a^2 - x^2)$$

$$Y = \frac{b}{a} \sqrt{a^2 - x^2}$$

$$\text{Area in 1}^{\text{st}} \text{ quadrant} = \int_a^b y \, dx$$

$$= \int_a^b \frac{b}{a} \sqrt{a^2 - x^2} \, dx$$

$$= \frac{b}{a} \int_a^b \sqrt{a^2 - x^2} \, dx$$

$$= \frac{b}{a} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a$$

$$= \frac{b}{a} \left[\frac{a}{2} \sqrt{a^2 - a^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{a}{a} \right) - \frac{0}{2} \sqrt{a^2 - 0^2} - \frac{a^2}{2} \sin^{-1} \left(\frac{0}{a} \right) \right]$$

$$= \frac{b}{a} \left[0 + \frac{a^2}{2} \sin^{-1}(1) - 0 - 0 \right]$$

$$= \frac{ba^2}{a} \times \frac{\pi}{4}$$

$$= \frac{\pi a}{4} b \text{ square units}$$

$$\text{Total area} = 4 \times \frac{\pi a}{4} b$$

$$= \pi ab \text{ square units}$$

ii. Solve $\frac{dy}{dx} - 10y = 0$

$$\frac{dy}{dx} - 10y = 0$$

$$\frac{dy}{dx} = 10y$$

$$\frac{dy}{y} = 10dx$$

$$\int \frac{1}{y} dy = \int 10x \, dx$$

$$\text{Log } y = 10x + C$$

PART –C
(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan 3\theta + \sin 5\theta}{8\theta}$

$$\begin{aligned}\lim_{\theta \rightarrow 0} \frac{\tan 3\theta + \sin 5\theta}{8\theta} &= \lim_{\theta \rightarrow 0} \frac{\tan 3\theta}{8\theta} + \lim_{\theta \rightarrow 0} \frac{\sin 5\theta}{8\theta} \\ &= \frac{3}{8} + \frac{5}{8} \\ &= \frac{8}{8} \\ &= 1\end{aligned}$$

(b) Find $\frac{dy}{dx}$, if $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$

$$\text{If } x = 3\cos\theta - \cos^3\theta$$

$$\frac{dx}{d\theta} = -3\sin\theta - 3\cos^2\theta \times -\sin\theta$$

$$= -3\sin\theta + 3\sin\theta \cos^2\theta$$

$$= 3\sin\theta (\cos^2\theta - 1)$$

$$= 3\sin\theta \times -\sin^2\theta$$

$$= -3\sin^3\theta$$

$$y = 3\sin\theta - \sin^3\theta$$

$$\frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta \cos\theta$$

$$= 3\cos\theta (1 - \sin^2\theta)$$

$$= 3\cos\theta \times \cos^2\theta$$

$$= 3\cos^3\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta} \cdot 3 \cos^3 \theta}{\frac{dx}{d\theta} \cdot -3 \sin^3 \theta} = -\cot^3 \theta$$

$$\frac{dy}{dx} = -\cot^3 \theta$$

IV.

(a) By the method of 1st principle, find the derivatives of $\cos x$

$$Y = \cos x$$

$$f(x) = \cos x$$

$$f(x + \Delta x) = \cos(x + \Delta x)$$

$$f(x + \Delta x) - f(x) = \cos(x + \Delta x) - \cos x$$

$$= -2 \sin \frac{(x + \Delta x + x)}{2} \times \sin \frac{(x + \Delta x - x)}{2}$$

$$= -2 \sin \frac{(2x + \Delta x)}{2} \times \sin \frac{(\Delta x)}{2}$$

$$= -2 \sin \left(x + \frac{(\Delta x)}{2} \right) \times \sin \frac{(\Delta x)}{2}$$

$$\therefore \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{-2 \sin \left(x + \frac{(\Delta x)}{2} \right) \times \sin \frac{(\Delta x)}{2}}{\Delta x}$$

$$= \frac{-2 \sin \left(x + \frac{(\Delta x)}{2} \right) \times \sin \frac{(\Delta x)}{2}}{\frac{\Delta x}{2} \times 2}$$

$$\therefore \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\sin \left(x + \frac{(\Delta x)}{2} \right)}{\frac{\Delta x}{2}} \times \lim_{\Delta x \rightarrow 0} \sin \left(x + \frac{(\Delta x)}{2} \right)$$

$$= 1 \times -\sin \left(x + \frac{0}{2} \right)$$

$$\therefore \frac{dy}{dx} = -\sin x$$

(b) Find $\frac{dy}{dx}$, if $y = \log(2x + 3)e^{2x}$

$$y = \log(2x + 3)e^{2x}$$

$$\frac{dy}{dx} = \log(2x + 3) \times 2 + e^{2x} \times \frac{1}{2x + 3} \times 2$$

(c) If $y = \sin(m \sin^{-1}x)$. prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

$$y = \sin(m \sin^{-1}x)$$

$$\sin^{-1}y = m \sin^{-1}x$$

Differentiating on both sides

$$\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = m \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = m \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}y' = m \sqrt{1-y^2}$$

Differentiating again w.r.to x

$$\sqrt{1-x^2}y'' + y' \frac{1}{2\sqrt{1-x^2}} x \cdot 2x = m \frac{1}{2\sqrt{1-y^2}} x \cdot 2y \times \frac{dy}{dx}$$

$$\sqrt{1-x^2}y'' - \frac{xy'}{\sqrt{1-x^2}} = \frac{-my}{\sqrt{1-y^2}} \frac{dy}{dx}$$

$$\frac{(1-x^2)y'' - xy'}{\sqrt{1-x^2}} = \frac{-my}{\sqrt{1-y^2}} y'$$

$$ie \frac{(1-x^2)y'' - xy'}{\sqrt{1-x^2}} + \frac{my}{\sqrt{1-y^2}} y' = 0$$

Solving these we get,

$$\frac{(1-x^2)y'' - xy'}{\sqrt{1-x^2}} + \frac{my \times m \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}}{\sqrt{1-y^2}} y' = 0$$

$$\frac{(1-x^2)y'' - xy' + m^2y}{\sqrt{1-x^2}} = 0$$

$$ie(1-x^2)y'' - xy' + m^2y = 0$$

V.

(a) The radius of a circular plate is decreasing in length at a rate of 0.2cm/s. what is the rate at which the area decreasing when the radius is 6cm

$$\text{Given } \frac{dr}{dt} = -0.2$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}, \text{ given } r = 6$$

$$\frac{dA}{dt} = \pi 2 \times 6 \times -0.2$$

$$\frac{dA}{dt} = -2.4\pi \text{ cm}^2/\text{sec}$$

∴ Area is decreasing at a rate of $2.4\pi \text{ cm}^2/\text{sec}$

(b) Find the minimum value of $2x^3 - 3x^2 - 36x + 10$

$$Y = 2x^3 - 3x^2 - 36x + 10$$

$$\frac{dy}{dx} = 6x^2 - 6x - 36$$

$$= 6(x^2 - x - 6)$$

$$\therefore \frac{dy}{dx} = 0 \implies 6(x^2 - x - 6) = 0$$

$$\implies (x^2 - x - 6) = 0$$

$$\implies (x - 3)(x + 3) = 0$$

$$\implies x = 3, -2$$

$$\frac{d^2y}{dx^2} = 12x - 6$$

$$\text{At } x = 3 \quad \frac{d^2y}{dx^2} = 12 \times 3 - 6 = 36 - 6 = 30 > 0$$

∴ at $x = 3$ value is minimum

$$\text{At } x = -2 \quad \frac{d^2y}{dx^2} = 12 \times -2 - 6 = -24 - 6 = -30 < 0$$

∴ at $x = -2$ value is maximum

$$\therefore \text{Maximum value } y = 2x^3 - 3x^2 - 36x + 10$$

$$= 2(-2)^3 - 3(-2)^2 - 36(-2) + 10$$

$$= 2(-8) - 3(4) + 72 + 10$$

$$= -16 - 12 + 82$$

$$= -28 + 82$$

$$= 54$$

$$\begin{aligned}
 \text{Minimum value } y &= 2x^3 - 3x^2 - 36x + 10 \\
 &= 2(3)^3 - 3(3)^2 - 36x(3) + 10 \\
 &= 54 - 27 - 108 + 10 \\
 &= 64 - 135 \\
 &= -71
 \end{aligned}$$

- (c) A spherical rubber bladder of radius increases at a uniform rate of 1 inch/minute. Find the rate at which the volume is increasing at the end of 2 minutes

Given radius $r = 3$ inches

$$\frac{dr}{dt} = 1 \text{ inch/minute}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi r^2 \times 1$$

$$\frac{dv}{dt} = 4\pi r^2$$

$$\therefore \text{Required radius } r = \text{given radius} + t \times \frac{dr}{dt}$$

$$r = 3 + 2 \times 1$$

$$= 3 + 2$$

$$r = 5$$

$$\therefore \frac{dv}{dt} = 4\pi \times 5^2$$

$$= 4\pi \times 25$$

$$\frac{dv}{dt} = 100\pi \text{ inch}^3/\text{minute}$$

VI.

- (a) Find the equation of the tangent and normal to the curve $x^2 + y^2 = 25$

$$x^2 + y^2 = 25 \quad (3, -4)$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dx}{dy} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$m = \frac{-3}{-4}$$

$$m = \frac{3}{4}$$

Equation of the tangent is $y - y_1 = m(x - x_1)$

$$y - -4 = \frac{3}{4}(x - 3)$$

$$y + 4 = \frac{3}{4}(x - 3)$$

$$4y + 16 = 3x - 9$$

$$3x - 4y - 25 = 0$$

Equation of normal is $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - -4 = -\frac{4}{3}(x - 3)$$

$$y + 4 = -\frac{4}{3}(x - 3)$$

$$3y + 12 = -4x + 12$$

$$4x + 3y = 0$$

- (b) If S denotes the displacement of a particle at the time 't' seconds and $S = t^3 - 6t^2 + 8t - 4$. Find the time when the acceleration is 12cm/sec^2 and the velocity at the time.

$$S = t^3 - 6t^2 + 8t - 4$$

$$\frac{ds}{dt} = 3t^2 - 12t + 8$$

$$a = \frac{d^2s}{dt^2} = 6t - 12$$

But given $a = 12\text{cm/sec}^2$

$$12 = 6t - 12$$

$$12 + 12 = 6t$$

$$6t = 24$$

$$t = 4 \text{ sec}$$

$$\begin{aligned}\therefore \text{Velocity } V &= \frac{ds}{dt} = 3t^2 - 12t + 8 \\ &= 3 \times 4^2 - 12 \times 4 + 8 \\ &= 3 \times 16 - 48 + 8 \\ &= 48 - 48 + 8 \\ &= 8\end{aligned}$$

(c) Find the slope of the normal to the curve $y = x^2 + x - 1$ at $(2, 7)$

$$y = x^2 + x - 1 \text{ at } (2, 7)$$

$$\text{Slop } m = \frac{dy}{dx} = 2x + 1$$

$$m = 2 \times 2 + 1$$

$$m = 4 + 1 = 5$$

VII.

(a) Evaluate $\int (\tan x + \cot x)^2 dx$

$$\begin{aligned}\int (\tan x + \cot x)^2 dx \\ &= \int \tan^2 x dx + \int \cot^2 x dx + 2 \int \tan x \cot x dx \\ &= \int (\sec^2 x - 1) dx + \int (\csc^2 x - 1) dx + 2 \int 1 dx \\ &= \tan x - x + -\cot x - x + 2x \\ &= \tan x - \cot x - 2x + 2x \\ &= \tan x - \cot x + C\end{aligned}$$

(b) Evaluate $\int \sin^2 5x dx$

$$\begin{aligned}\int \sin^2 5x dx &= \int \frac{1 - \cos 10x}{2} dx & \sin^2 x &= \frac{1 - \cos 2x}{2} \\ &= \frac{1}{2} \left[x - \frac{\sin 10x}{10} \right]\end{aligned}$$

$$= \frac{x}{2} - \frac{\sin 10x}{20} + C$$

(c) Evaluate $\int \tan^3 x \sec^2 x \, dx$

$$\begin{aligned} \int \tan^3 x \sec^2 x \, dx &= \int u^5 \, du \\ &= \frac{u^6}{6} \\ &= \frac{\tan^6 x}{6} + C \end{aligned}$$

$$U = \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$Du = \sec^2 x \, dx$$

(d) Evaluate $\int \log x \, dx$

$$\begin{aligned} \int \log x \, dx &= \log x \times x - \int x \frac{1}{x} \, dx \\ &= x \log x - \int 1 \, dx \\ &= x \log x - x + C \end{aligned}$$

(e) Evaluate $\int_0^{\frac{\pi}{2}} \sin x (1 - \cos x)^5 \, dx$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin x (1 - \cos x)^5 \, dx \\ &= \int_0^1 u^5 \, du \\ &= \left[\frac{u^6}{6} \right]_0^1 \\ &= \frac{1}{6} \end{aligned}$$

$$U = 1 - \cos x$$

$$\frac{du}{dx} = \sin x$$

$$du = \sin x \, dx$$

$$x = 0, u = 1 - \cos 0$$

$$= 1 - 1 = 0$$

$$x = \frac{\pi}{2}, u = 1 - \cos \frac{\pi}{2}$$

$$1 - 0 = 1$$

VIII.

(a) Evaluate $\int \frac{x^2 + 2x + 1}{x^2} \, dx$

$$\begin{aligned} &\int \frac{x^2 + 2x + 1}{x^2} \, dx \\ &= \int \left(1 + \left(\frac{2}{x}\right) + \frac{1}{x^2} \right) \, dx \end{aligned}$$

$$\begin{aligned}
 &= \int 1 \, dx + 2 \int \left(\frac{1}{x}\right) dx + \int x^{-2} \, dx \\
 &= x + 2 \log x - \frac{1}{x} + C
 \end{aligned}$$

(b) Evaluate $\int \frac{1}{\sqrt{3x+4}} \, dx$

$$\begin{aligned}
 \int \frac{1}{\sqrt{3x+4}} \, dx &= \int \frac{1}{(3x+4)^{\frac{1}{2}}} \, dx \\
 &= \int (3x+4)^{-\frac{1}{2}} \, dx \\
 &= \frac{(3x+4)^{-\frac{1}{2}+1}}{\frac{1}{2} \times 3} \\
 &= \frac{\sqrt{3x+4}}{\frac{3}{2}} \\
 &= \frac{2}{3} \sqrt{3x+4} + C
 \end{aligned}$$

(c) Evaluate $\int e^x \operatorname{cosec}^2(e^x) \, dx$

$$\begin{aligned}
 \int e^x \operatorname{cosec}^2(e^x) \, dx & \quad \text{Let } u = e^x \\
 &= \int \operatorname{cosec}^2 u \, du \quad \frac{du}{dx} = e^x \\
 &= -\cot u \quad du = e^x \, dx \\
 &= -\cot e^x + C
 \end{aligned}$$

(d) Evaluate $\int \sin^{-1} x \, dx$

$$\begin{aligned}
 \int \sin^{-1} x \, dx &= \int \sin^{-1} x \times 1 \, dx \\
 &= \sin^{-1} x \times x - \int x \frac{1}{\sqrt{1-x^2}} \, dx \quad \text{Let } u = 1-x^2 \\
 &= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx \quad \frac{du}{dx} = -2x \\
 &= x \sin^{-1} x - \int \frac{1}{\sqrt{u}} \times \frac{-du}{2} \quad \frac{-du}{2} = x \, dx
 \end{aligned}$$

$$\begin{aligned}
&= x \sin^{-1}x + \frac{1}{2} \int u^{-\frac{1}{2}} du \\
&= x \sin^{-1}x + \frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \\
&= x \sin^{-1}x + \sqrt{u} \\
&= x \sin^{-1}x + \sqrt{1-x^2} + C
\end{aligned}$$

(e) Evaluate $\int_0^{\pi} \frac{1-\sin x}{x+\cos x} dx$

Let $u = x + \cos x$

$$\frac{du}{dx} = 1 - \sin x$$

$$du = (1 - \sin x) dx$$

when $x = 0$,

$$u = 0 + \cos 0 = 1$$

when $x = \pi$,

$$u = \pi + \cos \pi$$

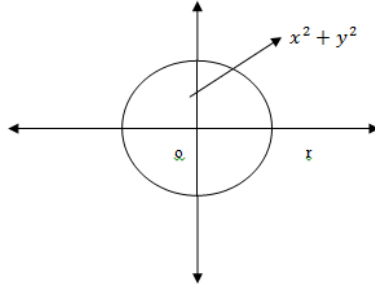
$$u = \pi - 1 = \pi - 1$$

$$\begin{aligned}
\int_0^{\pi} \frac{1-\sin x}{x+\cos x} dx &= \int_0^{\pi} \frac{1}{u} du \\
&= [\log u]_1^{\pi-1} \\
&= \log(\pi - 1) - \log 1 \\
&= \log(\pi - 1)
\end{aligned}$$

IX.

(a) Find the area of circle of radius r using integration

We know the equation of a circle is



$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$x = 0, x = r$$

Area in the 1st quadrant, $A = \int_a^b y \, dx$

$$= \int_0^r \sqrt{r^2 - x^2} \, dx$$

$$= \left[\frac{x^2}{2} \sqrt{r^2 - x^2} + \frac{r^2}{2} \sin^{-1}\left(\frac{x}{r}\right) \right]_0^r$$

$$= \frac{r^2}{2} \sqrt{r^2 - r^2} + \frac{r^2}{2} \sin^{-1}\left(\frac{r}{r}\right)$$

$$- \frac{0^2}{2} \sqrt{r^2 - 0^2} + \frac{r^2}{2} \sin^{-1}\left(\frac{0}{r}\right)$$

$$= \frac{r^2}{2} \times 0 + \frac{r^2}{2} \sin^{-1}(1) - 0 - \frac{r^2}{2} \times 0$$

$$= \frac{r^2}{4} \pi \quad \text{total area} = 4 \times \text{area in first quadrant} =$$

$$A = \pi r^2$$

- (b) Find the volume generated by the rotation of area bounded by the curve $y = 3$ and $y = 9$ about the y -axis

$$Y = 2x^2 + 1$$

$$\therefore x^2 = \frac{y-1}{2} \quad y = 3, y = 9$$

$$\therefore \text{volume } V = \pi \int_a^b x^2 \, dy$$

$$\begin{aligned}
&= \pi \int_a^b \frac{y-1}{2} dy \\
&= \frac{\pi}{2} \left[\frac{y^2}{2} - y \right]_3^9 \\
&= \frac{\pi}{2} \left[\left(\frac{81}{2} - \frac{9}{2} \right) - (9 - 3) \right] \\
&= \frac{\pi}{2} \left[\left(\frac{72}{2} \right) - 6 \right] \\
&= \frac{\pi}{2} [36 - 6] \\
&= \frac{\pi}{2} \times 30 \\
&= 15\pi \text{ cm}^3
\end{aligned}$$

(c) Solve $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

$$\frac{dy}{dx} = e^x e^y + x^2 e^y$$

$$\frac{dy}{dx} = e^y (e^x + x^2)$$

$$\frac{dy}{e^y} = (e^x + x^2) dx$$

$$e^y dy = (e^x + x^2) dx$$

$$\therefore \int e^y dy = \int (e^x + x^2) dx$$

$$ie - e^{-y} = e^x + \frac{x^3}{3} + C$$

X.

(a) Find the area bounded by the curve $y = x^2 + x$ and the x - axis

$$y = x^2 + x$$

$$\therefore x^2 + x = 0$$

$$x(x+1) = 0$$

$$x = 0 \text{ or } x = -1$$

$$\therefore \text{Area } A = \int_a^b y \, dx$$

$$= \int_{-1}^0 x^2 + x \, dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^0$$

$$= \frac{0^3}{3} + \frac{0^2}{2} - \frac{-1^3}{3} - \frac{-1^2}{2} + \frac{1}{3} - \frac{1}{2}$$

$$= \frac{-1}{6}$$

Area cannot be -ve, so Area = $\frac{1}{6} \text{m}^2$

(b) Find the volume of solid obtained by rotatory one arch of the curve $y = \sin 3x$ about the x-axis

$$\text{Given that } y = \sin 3x \quad x = 0, x = \frac{\pi}{3}$$

$$\therefore \text{Volume } V = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_0^{\frac{\pi}{3}} \sin^2 3x \, dx$$

$$= \pi \int_0^{\frac{\pi}{3}} \frac{1 - \cos 6x}{2} \, dx$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 6x}{6} \right]_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{2} \left[\frac{\pi}{3} - \frac{\sin 6 \cdot \frac{\pi}{3}}{6} - 0 + \frac{\sin 0}{6} \right]$$

$$= \frac{\pi}{2} \left[\frac{\pi}{3} - \frac{\sin 2\pi}{6} - 0 + 0 \right]$$

$$= \frac{\pi}{2} \left[\frac{\pi}{3} - 0 \right]$$

$$= \frac{\pi^2}{6} \text{cubic units}$$

(c) Solve $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

$$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$$

Dividing $(1 + x^2)$ on both sides, we get

$$\frac{dy}{dx} + \frac{1}{(1 + x^2)}y = \frac{e^{\tan^{-1} x}}{(1 + x^2)}$$

$$\frac{dy}{dx} + Py = Q$$

$$\therefore P = \frac{1}{(1 + x^2)}$$

$$Q = \frac{e^{\tan^{-1} x}}{(1 + x^2)}$$

$$\therefore \text{I.F} = e^{\int p dx} = e^{\int \frac{1}{(1 + x^2)} dx}$$

$$= e^{\tan^{-1} x}$$

$$\therefore \text{Solution is } Y \times \text{IF} = \int Q \times \text{IF} dx$$

$$Y \times e^{\tan^{-1} x} = \int \frac{e^{\tan^{-1} x}}{(1 + x^2)} \times e^{\tan^{-1} x} dx$$

$$\therefore \text{Solution is } Y \times e^{\tan^{-1} x} = \int u du$$

$$= \frac{u^2}{2}$$

$$Y \times e^{\tan^{-1} x} = \frac{e^{2\tan^{-1} x}}{2} + c$$

$$\text{Let } u = e^{\tan^{-1} x}$$

$$\frac{du}{dx} = e^{\tan^{-1} x} \frac{1}{(1 + x^2)}$$

$$du = e^{\tan^{-1} x} \frac{1}{(1 + x^2)} dx$$