TED (10)-1015 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2014

TECHNICAL MATHEMATICS- II

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) evaluate $\lim_{x\to 0}\frac{1-cos2x}{x^2}$

$$\lim_{x\to 0}\frac{1-cos2x}{x^2}$$

$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2}$

$= \lim_{x\to 0} 2\left(\frac{\sin^2 x}{x^2}\right)$

$= 2 \text{ x } 1^2$

$= 2$

(b) find k if $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at x = 2

f(2) = k

$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x^2-4}{x-2}$

$= \frac{2 \text{ x } 2^{2-1}}{1 \text{ x } 2^{1-1}}$

$$= \frac{2 \text{ x } 2}{1}$$

$$= 4$$

Since it is continuous, so f (2) $= \lim_{x \to 2} f(x)$

Ie, k = 4

(c) Find the velocity and acceleration at time t, of a particle moving according to the rate $s = 4t^2 + 3t$

$$s = 4t^2 + 3t$$

Velocity V = $\frac{ds}{dt}$

= 8t + 3

Acceleration, a = $\frac{d^2s}{dt^2}$

$= 8$

(d) Integrating with respect to x, $3\sec^2 x + 4sinx + e^x$

$$\int 3\sec^2 x + 4sinx + e^x \, dx$$

$$= 3tanx - 4cosx + e^x + C$$

(e) Find the integration factor(I.F) of $\frac{dy}{dx}$ + ytanx = $\cos^2$x

$\frac{dy}{dx}$ + ytanx = $\cos^2$x

$\frac{dy}{dx}$ + Py = Q

P = tanx, Q = $\cos^2$x

∴ I.F = $e^{\int p\, dx} = e^{\int tanx\, dx} = e^{\log secx}$ = secx

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Find the derivative of tanx with respect to x by 1st principle

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \_\_\_\_\_\_\_\_ (1)$$

$f(x)$ = tanx

$f(x + \Delta x) = \tan(x + \Delta x)$

$$f(x + \Delta x) - f(x) = \tan(x + \Delta x) - tanx$$

$$= \frac{sin(x+\Delta x)}{cos(x+\Delta x)} - \frac{sinx}{cosx}$$

$$= \frac{sin(x+\Delta x) \text{x} \cos x - sinx cos(x+\Delta x)}{cosx(cos(x+\Delta x))}$$

$$= \frac{sin(x+\Delta x - x)}{cosx(cos(x+\Delta x))}$$

{sinAcosB-cosAsinB = sin(A – B)}

$$\therefore (1) ===> \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{sin\Delta x}{cosxcos(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{sin\Delta x}{\Delta x} \text{x} \frac{1}{cosxcos(x+\Delta x)}$$

$$= 1\text{x} \frac{1}{cosxcos(x+0)}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

(b) If$x^3 + y^3 = 3x\,y$.Find $\frac{dy}{dx}$

$$x^3 + y^3 = 3x\,y$$

Differentiating with respect to x on both sides

$$3x^2 + 3y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$$

$$\frac{dy}{dx}(3y^2 - 3x) = 3y - 3x^2$$

$$\frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}$$

$$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$$

(c) The distance described by a particle in t seconds is given by $s = ae^t + be^{-t}$. Show that the acceleration is almost equal to the distance

$$s = ae^t + be^{-t}$$

$$\frac{ds}{dt} = ae^t - be^{-t}$$

$$\frac{d^2s}{dt^2} = ae^t + be^{-t}$$

$$\frac{d^2s}{dt^2} = s$$

(d) Evaluate $\int_0^{\frac{\pi}{2}} sin3xcosx\,dx$

$$\int_0^{\frac{\pi}{2}} sin3xcosx\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin(3x + x) + \sin(3x - x)\,dx$$

sinAcosB $=\frac{1}{2}$ [sin(A + B) + sin(A – B)]

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}(sin4x + sin2x)dx$$

$$= \frac{1}{2}\left[\frac{-cos4x}{4} - \frac{cos2x}{2}\right]_{0}^{\frac{\pi}{2}}$$

$$= \frac{-1}{4}\left[\frac{cos4x}{2} + cos2x\right]_{0}^{\frac{\pi}{2}}$$

$$= \frac{-1}{4}\left[\frac{cos4\frac{\pi}{2}}{2} + cos2\,\frac{\pi}{2} - \frac{cos4 \text{ x } 0}{2} + cos2 \text{ x } 0\right]$$

$$= \frac{-1}{4}\left[\frac{1}{2} - 1 - \frac{1}{2} - 1\right]$$

$$= \frac{-1}{4} \text{ x } - 2$$

$$= \frac{1}{2}$$

i. Find the area enclosed between one arch of the curve $y = sin3x$ and the x axis

$$y = sin3x$$

X = 0, $\frac{\pi}{3}$

Area A = $\int_{a}^{b} y\,dx$

A = $\int_{0}^{\frac{\pi}{3}} sin3x\,dx$

$$= \left[\frac{-cos3x}{3}\right]_{0}^{\frac{\pi}{3}}$$

$$= \left[\frac{-cos3 \text{ x } \frac{\pi}{3}}{3} - \frac{-cos3 \text{ x } 0}{3}\right]$$

$$= \left[\frac{-cos\pi}{3} + \frac{cos0}{3}\right]$$

$= \frac{--1}{3} + \frac{1}{3}$

$= \frac{1}{3} + \frac{1}{3}$

$=\frac{2}{3}$square units

(e) Evaluate $\int_0^2 x^2 logx$dx

$\int_0^2 x^2 logx$dx = $\int_0^2 logx$ x x^2dx

$=(\text{logx}\left(\frac{x^3}{3}\right)) \begin{matrix} 2 \\ 0 \end{matrix} - \int_0^2 \frac{x^3}{3} \frac{1}{x}\text{dx}$

$= \left(log2 \text{ x } \frac{2^3}{3} - log0 \text{ x } \frac{0^3}{3} \right) - \int_0^2 \frac{x^2}{3}\text{dx}$

$= [\frac{8}{3}\text{log2} - 0 - \frac{1}{3}\frac{x^3}{3}] \begin{matrix} 2 \\ 0 \end{matrix}$

$= \frac{8}{3}\text{log2} - \frac{8}{9}$

(f) Solve $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

Dividing by $(1 + x^2)$ on both sides we get

$\frac{dy}{dx} + \frac{2x}{1+x^2} \times y = \frac{4x^2}{1+x^2}$

$\frac{dy}{dx}$ + py = Q

$P = \frac{2x}{1+x^2}$

$Q = \frac{4x^2}{1+x^2}$

I.F = $e^{\int p\,dx}$ = $e^{\int \frac{2x}{1+x^2}}$ = $e^{\log(1+x^2)}$ = $1 + x^2$

∴Solution is y x IF = $\int Q$ x IF dx

y x $(1 + x^2)$ = $\int \frac{4x^2}{1+x^2}$ x $(1 + x^2)$ dx

y x $(1 + x^2)$ = $\int 4x^2$ dx

y x $(1 + x^2)$ = 4 $\frac{x^3}{3}$+ C

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Evaluate $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$

$\lim_{x\to 2} \frac{x^3-8}{x^2-4} = \frac{3\text{x } 2^{3-1}}{2\text{x } 2^{2-1}}$

$= \frac{3 \text{ x } 4}{2\text{x } 2}$

$= 3$

1

(b) If y = $ae^x + be^{2x}$ prove that $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

y = $ae^x + be^{2x}$

$\frac{dy}{dx}$= $ae^x + 2be^{2x}$

$\frac{d^2y}{dx^2}$= $ae^x + 4be^{2x}$

$\therefore 3\frac{dy}{dx} = 3ae^{x} + 6be^{2x}$

$2y = 2ae^{x} + 2be^{2x}$

$\therefore \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = ae^{x} + 4be^{2x} - 3ae^{x} + 6be^{2x} + 2ae^{x} + 2be^{2x}$

$= 0$

(c) Find

i. $\frac{dy}{dx}\left[\frac{\sin^{-1} x}{x}\right]$

ii. $\frac{dy}{dx}\left[\frac{1-x^2}{1+x^2}\right]$

i. $\frac{dy}{dx}\left[\frac{\sin^{-1} x}{x}\right] = \frac{x \text{ x} \frac{1}{\sqrt{1-x^2}} - \sin^{-1} x}{x^2}$

ii. $\frac{dy}{dx}\left[\frac{1-x^2}{1+x^2}\right] = \frac{(1+x^2)\text{x}-2x-(1-x^2)2x}{(1+x^2)^2}$

$= \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2}$

$= \frac{-4x}{(1+x^2)^2}$

IV.

(a) Find the differential coefficient with respect to x

i. Log (secx + tanx)

ii. $\frac{e^{x} sinx}{1+logx}$

i. Y = log (secx + tanx)

$\frac{dy}{dx} = \frac{1}{secxtanx} \text{x } secxtanx + \sec^2 x$

$= \frac{\sec(secx+tanx)}{secx+tanx}$

$= secx$

ii. $Y = \frac{e^x sinx}{1+logx}$

$$\frac{dy}{dx} = \frac{(1+logx)(e^x cosx+sinx e^x)-e^x sinx \text{ x } \frac{1}{x}}{(1+logx)^2}$$

(b) Find $\frac{dy}{dx}$, if $ax^2 + 2hxy + by^2 = 0$

$$ax^2 + 2hxy + by^2 = 0$$

$$2ax + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} = 0$$

$\frac{dy}{dx}(2hx + 2by) = -2ax - 2hy$

$\frac{dy}{dx} = \frac{-2ax-2hy}{2hx+2by}$

$\frac{dy}{dx} = \frac{-(ax-hy)}{(hx+by)}$

(c) Find $\frac{dy}{dx}$ if $x = asec\theta, y = btan\theta$

$x = asec\theta$

$\frac{dx}{d\theta} = asec\theta tan\theta$

$$y = btan\theta$$

$\frac{dy}{d\theta} = \text{bsec}^2\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\mathrm{bsec}^2\,\theta}{asec\theta tan\theta}$$

$$= \frac{b}{a}\frac{sec\theta}{tan\theta}$$

$$= \frac{b}{a}\frac{cot\,\theta}{cos\theta}$$

$$= \frac{b}{a}cosec\theta$$

V.

(a) Find the maximum and minimum values if $y = 4x^3 + 9x^2 - 12x + 2$

$$y = 4x^3 + 9x^2 - 12x + 2$$

$$\frac{dy}{dx} = 12x^2 + 18x - 12$$

$$\frac{dy}{dx} = 6(2x^2 + 3x - 2)$$

$$\frac{d^2y}{dx^2} = 24x + 18$$

$\frac{dy}{dx}$= 0 ===>$6(2x^2 + 3x - 2) = 0$

===>$(2x^2 + 3x - 2) = 0$

===> x =-2, ½

When x = -2 $\frac{d^2y}{dx^2} = 24\text{ x} - 2 + 18$

= -48 + 18

= -30 < 0

∴Maximum at x = -2

Maximum value, y = $4(-2)^3 + 9(-2)^2 - 12\text{ x}(-2) + 2$

= 32 + 36 + 24 + 2

= 62 – 32

= 30

When x = ½ $\frac{d^2y}{dx^2} = 24 \text{ x} ½ + 18$

$= 12 + 18$

$= 30 > 0$

∴ Minimum at x = ½

Minimum value, $y = 4\left(\frac{1}{2}\right)^3 + 9\left(\frac{1}{2}\right)^2 - 12 \text{ x} \left(\frac{1}{2}\right) + 2$

$= \frac{4}{8} + \frac{9}{4} - \frac{12}{2} + 2$

$= \frac{1}{2} - \frac{12}{2} + \frac{9}{4} + 2$

$= \frac{-11}{2} + \frac{9}{4} + 2$

$= \frac{-22+9+8}{4}$

$= \frac{-5}{4}$

(b) A stone is dropped into still water. The radius of the outermost ripple then found increases at the rate of 6 cm/sec. How that is the surface area increasing when the radius is 16 cm.

Given that $\frac{dr}{dt}$ = 6 cm/sec

Area A = πr^2

$\frac{dA}{dt} = \pi \text{ x } 2r \text{ x } \frac{dr}{dt}$

Given r = 16 cm.

So $\frac{dA}{dt} = \pi \text{ x } 2 \text{ x } 16 \text{ x } 6$

$\frac{dA}{dt} = 192\pi \text{ cm}^2/\text{sec}$

(c) Find the equation to the normal to the curve $y = \sqrt{(25 - x^2)}$ at(4, 3)

$$y = \sqrt{(25 - x^2)}$$

$X_1 = 4$

$Y_1 = 3$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{(25-x^2)}} x \text{ -}2x$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{(25-x^2)}}$$

$$m = \frac{-4}{\sqrt{(25-4^2)}}$$

$$= \frac{-4}{\sqrt{(25-16)}}$$

$$= \frac{-4}{\sqrt{9}}$$

$$= \frac{-4}{3}$$

Equation of normal is $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - 3 = \frac{3}{4}(x - 4)$$

$$4y - 12 = 3x - 12$$

$$3x - 4y = 0$$

VI.

(a) A spherical balloon is inflated with air such that its volume increases at the rate of 5cc per second. Find the rate at which its curved surface is increasing

Volume, $V = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = \frac{4}{3} 3\pi r^2 x \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Given $\frac{dv}{dt} = 5$

$$5 = 4\pi r^2 \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{5}{4\pi r^2}$$

Surface area S = $4\pi r^2$

$$\frac{ds}{dt} = 4\pi \text{ x } 2r \text{ x } \frac{dr}{dt}$$

$$\frac{ds}{dt} = 4\pi \text{ x } 2r \text{ x } \frac{5}{4\pi r^2}$$

$$\frac{ds}{dt} = \frac{10}{r}$$

Given r = 7cm

$$\frac{ds}{dt} = \frac{10}{7} \text{cm/sec}$$

(b) Find the turning points of $2x^3 - 9x^2 + 12x + 2$

Y = $2x^3 - 9x^2 + 12x + 2$

$$\frac{dy}{dx} = 6x^2 - 18x + 12$$

$\therefore \frac{dy}{dx} = 0$

===>$6x^2 - 18x + 12 = 0$

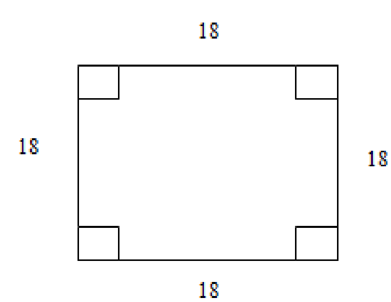
===>$6(x^2 - 3x + 2) = 0$

===>$(x^2 - 3x + 2) = 0$

===> (x – 2) (x –1) = 0

===> x = 1, 2

(c) An open box is to be made out a square sheet of side 18cms. By cutting off equal shares at each and turning up the sides. What size of the squares should be cut in order that the volume of the box may be maximum?



Length of the box = 18 – 2x

Breadth of the box = 18 – 2x

Height = x

∴ Volume v = (18 – 2x)(18 – 2x)x

= $(324 - 36x - 36x + 4x^2)$ x

$= 324x - 72x^2 + 4x^3$

$V = 4x^3 - 72x^2 + 324x$

$\frac{dv}{dx} = 0$

$12x^2 - 144x + 324 = 0$

$12(x^2 - 12x + 27) = 0$

At a maxima or minima

$\frac{dv}{dx} = 0 \Longrightarrow 12(x^2 - 12x + 27) = 0$

$ie\ x^2 - 12x + 27 = 0$

$ie\ (x - 9)(x - 3) = 0$

X = 3, or 9

But x = 9 is not possible

$\therefore x = 3$

VII.

(a) Evaluate $\int \frac{2+3sinx}{\cos^2 x} dx$

$$\int \frac{2 + 3sinx}{\cos^2 x} dx$$

$$= \int \frac{2}{\cos^2 x} dx + 3\int \frac{sinx}{\cos^2 x} dx$$

$$=2 \int \sec^2 x\, dx + 3 \int secxtanx\, dx$$

$$= 2tanx + 3secx + C$$

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$

$$\int_0^{\frac{\pi}{2}} \sin^2 x\ dx = \int_2^{\frac{\pi}{2}} \frac{1-cos2x}{2} dx$$

$$= \int_0^{\frac{\pi}{2}} (1 - cos2x) dx$$

$$= \frac{1}{2}\left[x - \frac{sin2x}{2}\right]_0^{\frac{\pi}{2}}$$

$=\frac{1}{2}\left[\frac{\pi}{2}-\frac{\sin\frac{2\pi}{2}}{2}-0+\frac{\sin 2 \text{ x } 0}{2}\right]$

$=\frac{1}{2}\left[\frac{\pi}{2}-\frac{\sin\pi}{2}-0+0\right]$

$=\frac{1}{2}\left[\frac{\pi}{2}-0\right]$

$=\frac{\pi}{4}$

(c) find $\int \frac{1+cosx}{x+sinx}dx$

U = x + sinx

$\frac{du}{dx}=1+cosx$

du =$1+cosx$ dx

$\int \frac{1+cosx}{x+sinx}dx$

$=\int \frac{du}{u}$

$=\log u$

$=\log(x+sinx)+C$

(d) Evaluate $\int xcos3x\ dx$

$\int xcos3x\ dx = x \text{ x} \frac{sin3x}{3}-\int \frac{sin3x}{3}\text{x } 1dx$

$=\frac{x}{3}sin3x-\frac{1}{3}\frac{-cos3x}{3}$

$=\frac{x}{3}sin3x+\frac{1}{9}cos3x+C$

VIII.

(a) Evaluate $\int logx\ dx$

$\int logx\ dx = logx \text{ x } x-\int x\frac{1}{x}dx$

= x logx - ∫ 1dx

= x logx – x + C

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{1 + sin2x}dx$

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + sin2x}\, dx$$

$= \int_0^{\frac{\pi}{2}} \sqrt{\sin^2 x + \cos^2 x + 2sinxcosx}\ \ dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{(sinx + cosx)^2}\, dx$

$= \int_0^{\frac{\pi}{2}} sinx + cosx\ dx$

$= [sinx + cosx]_0^{\frac{\pi}{2}}$

$= sin\frac{\pi}{2} - cos\frac{\pi}{2} - sin0 + cos0$

$= 1 - 0 - 0 + 1$

$= 2$

(c) Evaluate $\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$

$\int_0^{\frac{\pi}{2}} \cos^3 x\, dx = \int_0^{\frac{\pi}{2}} \frac{cos3x+3cosx}{4} dx$

$= \frac{1}{4}\left[\frac{sin3x}{3} + 3sinx\right]$

$= \frac{1}{4}\left[\frac{-1}{3} + 3\text{x}\,1 - 0 - 3\text{x}\,0\right]$

$= \frac{1}{4}\ \text{x}\ \frac{8}{3}$

$= \frac{2}{3}$

(d) Evaluate $\int_0^{\frac{\pi}{2}} \frac{cosx}{1+sinx} dx$

$$\int_0^{\frac{\pi}{2}} \frac{cosx}{1 + sinx} dx$$

$= \int_0^{\frac{\pi}{2}} \frac{1}{u} dx$

$= logu$

$= \log[u]_1^2$

$= log2 - log1$

$= log2 - 0$

$= \log2$

u = 1 + sinx

$\frac{du}{dx}$ = cosx

du = cosx dx

x = 0, u = 1

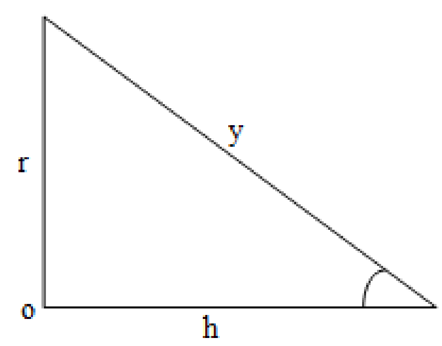
x = $\frac{\pi}{2}$, u = 2

IX.

(a) Show by integration that the volume of aright circular cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$

$m = \frac{r}{h}$

$m =$



Y = mx

Y = $\frac{r}{h}x$

$$V = \pi \int_a^b y^2 dx$$

$= \pi \int_0^h \frac{r^2}{h^2} x^2 dx$

$= \frac{\pi r^2}{h^2} \left[\frac{x^3}{3}\right]_0^h$

$$V = \frac{\pi r^2}{h^2}\left[\frac{h^3}{3}\right]$$

$$V = \frac{1}{3}\pi r^2 h^2$$

(b) Solve $\frac{dy}{dx} + \frac{x\sqrt{1+y^2}}{y\sqrt{1+x^2}}$

$$\frac{dy}{dx} = -\frac{x\sqrt{1+y^2}}{y\sqrt{1+x^2}}$$

Type equation here

$1 + x^2 = u$

$\frac{du}{dx} = 2x$

du = 2x dx

$\frac{du}{2} = x\,dx$

Integrating on both sides we get,

$$\int \frac{y}{\sqrt{1+y^2}}\,dy = -\int \frac{x}{\sqrt{1+x^2}}\,dx$$

$$\int -\frac{x}{\sqrt{1+x^2}}\,dx = \int \frac{\frac{du}{2}}{\sqrt{u}}$$

$$= \frac{1}{2}\int u^{-\frac{1}{2}}du$$

$$= \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= \sqrt{u}$$

$$= \sqrt{1+x^2}$$

$\therefore$Solution is $\sqrt{1+y^2} = -\sqrt{1+x^2} + C$

$$\therefore \sqrt{1+y^2} = -\sqrt{1+x^2} + C$$

(c) Find the area bounded by the curve $y = x^2 + x$ and the x - axis

$y = x^2 + x$

$\therefore x^2 + x = 0$

$x(x + 1) = 0$

$x = 0$ or $x = -1$

$\therefore$ Area A = $\int_a^b y\,dx$

$= \int_{-1}^{0} x^2 + x\,dx$

$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{0}$

$= \frac{0^3}{3} + \frac{0^2}{2} - \frac{(-1)^3}{3} - \frac{(-1)^2}{2}$

$= \frac{-1}{6}$

Area cannot be –ve, so Area = $\frac{1}{6}m^2$

X.

(a) Find the area between the curves $x^2 = 4y$ and $y^2 = 4x$

$Y = 2\sqrt{x}$

$F(x) = 2\sqrt{x}$

$G(x) = \frac{x^2}{4}$

$\therefore f(x) = g(x)$

$2\sqrt{x} = \frac{x^2}{4}$

$8\sqrt{x} = x^2$

$64x = x^4$

$$x^4 - 64x = 0$$

$$x(x^3 - 64) = 0$$

$$x^3 - 64 = 0$$

X = 0

Or

X = 4

$\therefore$ Area A = $\int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)$dx

A = $\left[2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{12}\right]_0^4$

$= \frac{4}{3} 4^{\frac{3}{2}} - \frac{1}{12}$ x 4^3

$= \frac{4}{3}$ x $8 - \frac{64}{12}$

$= \frac{32}{3} - \frac{32}{6}$

$= \frac{32}{3} - \frac{16}{3}$

$= \frac{16}{3}$

V = $\frac{16}{3}$square unit

(b) Solve $\frac{dy}{dx} = e^{x+y} + x^2e^y$

$$\frac{dy}{dx} = e^{x+y} + x^2e^y$$

$$\frac{dy}{dx} = e^y(e^x + x^2)$$

$$e^{-y}dy = (e^x + x^2)dx$$

Integrating on both sides we get

$$\frac{e^{-y}}{-1} = e^x + \frac{x^3}{3} + C$$

(c) Show that volume of the solid generated when the area bounded by the parabola $y = x^2$ then x – axis and the ordinates x = 0 and x = 2 is revolved about the x axis is $\frac{32}{5}\pi$

$y = x^2$

a = 0

b = 2

$$\therefore V = \pi \int_a^b y^2 dx$$

$$V = \pi \int_0^2 x^4 dx$$

$$V = \pi \frac{32}{5}$$

$= \frac{32}{5}$ cubic unit