TED (10)-1015 Reg. No. …………………………

(REVISION-2010) Signature ……..……………………

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2013

TECHNICAL MATHEMATICS- II

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Evaluate $\lim_{x\to 0}\frac{x^2+4}{x+1}$

$\lim_{x\to 0}\frac{x^2+4}{x+1} = \frac{0^2+4}{0+1} = \frac{4}{1} = 4$

(b) Find $\frac{dy}{dx}$ if y = $x^2 sinx$

y = $x^2 sinx$

$\frac{dy}{dx} = x^2 + cosx + sinx$ x 2x

= $x^2cosx + 2xsinx$

(c) Find $\int \tan^2 x\, dx$

$\int \tan^2 x\, dx = \int (\sec^2 x - 1)\, dx$

$= \int \sec^2 x \, dx - \int 1 \, dx$

= tanx – x + C

(d) Solve $\frac{dy}{dx}$= 5y

$\frac{dy}{dx}$= 5y

$\frac{dy}{y} =$ 5dx

Integrating on both sides,

$\int \frac{dy}{y}$ = ∫ 5dx

Logy = 5x + C

===> y = $e^{5x}k$ or y = $k\ e^{5x}$

(e) Find the slope of the tangent to the curve $y = 2x^3 - 39 + 12x - 3$ at the point(1,1)

$$y = 2x^3 - 9x^2 + 12x - 3$$

$\frac{dy}{dx}$= $6x^2 - 18x + 12$

$\left(\frac{dy}{dx}\right)_{x=1}$= 6 x $1^2 - 18$ x $1 + 12$

= 6 – 18 + 12 = 0

Slope = 0, at (1, 1)

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a)

i. Evaluate $\lim_{x\to 0} \frac{1-cosx}{x^2}$

$$\lim_{x\to 0} \frac{1-cosx}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$= \lim_{x\to 0} 2\left[\frac{sinx}{x}\right]^2$

= 2 x 1 = 2

ii. If $y = (2x^2 + 5x + 1)^{10}$ find $\frac{dy}{dx}$

$$y = (2x^2 + 5x + 1)^{10}$$

$\frac{dy}{dx} = 10(2x^2 + 5x + 1)^9(4x + 5)$

(b) If $y = acospx + bsinpx$ where a, p and b are constant. Show that $\frac{d^2y}{dx^2}$ is proportional to y

$$y = acospx + bsinpx$$

$\frac{dy}{dx} = -asinpx$ x p $+ bcospx$ x p

$= -apsinpx + bpcospx$

$\frac{d^2y}{dx^2} = -ap^2cospx + -bp^2sinpx$

$= -p^2[acospx + bsinpx]$

$= -p^2y$

Ie, $\frac{d^2y}{dx^2} = -p^2y$

Hence the result.

(c) Find the equation of the tangent and normal to the curve $y = x^2 + x - 1$ at (2, 7)

$$y = x^2 + x - 1$$

$\left(\frac{dy}{dx}\right)_{x=2}$ = 2 x 2 + 1 = 5

Slope of the tangent at (2, 7) = 5

∴ Slope of normal at (2, 7) = $\frac{-1}{5}$

Equation of the tangent: $y - y_1 = m(x - x_1)$

$y - 7 = 5(x - 2)$

$y - 7 = 5x - 10$

$5x - y = 3$

Equation of the tangent: $y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 7 = -\frac{1}{5}(x - 2)$

$5y - 35 = -x + 2$

$x + 5y = 37$

(d) If $y = 2x^3 - 3x^2 - 36x = 10$. Find its minimum value

$$y = 2x^3 - 3x^2 - 36x = 10$$

$\frac{dy}{dx} = 6x^2 - 6x - 36$

At minimum

$\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

$\frac{dy}{dx} = 0 \Longrightarrow 6x^2 - 6x - 36 = 0$

$$6(x^2 - x - 6) = 0$$

$x = \frac{1 \pm \sqrt{(1+24)}}{2}$

$= \frac{1 \pm 5}{2} = 3, -2$

$\frac{d^2y}{dx^2} = 12x - 6$

At $x = 3$

$\frac{d^2y}{dx^2}$= 12x 3– 6 = 30 > 0

At x = -2

$\frac{d^2y}{dx^2}$= 12x -2– 6 = -24 – 6 = -30 < 0

∴ At x = 3, y is minimum

And the minimum value is

$$y = 2(3)^3 - 3(3)^2 - 36(3) + 10 = -71$$

(e) Obtain

i. $\int \frac{3cosx+4}{\sin^2 x}$dx   ii. $\int \frac{2x^4}{1+x^{10}}\, dx$

i. $\int \frac{3cosx+4}{\sin^2 x}$dx = $\int \frac{3cosx}{\sin^2 x}$dx+ $\int \frac{4}{\sin^2 x}$dx

= $\int 3cotx\ cosecx\ dx$ + $\int 4cosec^2x$dx

= -3cosecx – 4cotx + C

ii. $\int \frac{2x^4}{1+x^{10}}\, dx$

Put u = x^5

$\frac{du}{dt}$ = $5x^4$

du = $5x^4$ dx

$\frac{du}{5} =$ x^4 dx

= $\int \frac{2x^4}{1+\left(x^5\right)^2}\, dx$

= $\frac{1}{5}\int \frac{2du}{1+u^2}$dx

= $\frac{2}{5}\int \frac{du}{1+u^2}$dx

= $\frac{2}{5}$tan^{-1}u + C

= $\frac{2}{5}$tan^{-1}x^5 + C

(f) Evaluate $\int_0^2 x^3 logx\, dx$

$$\int x^3 logx\, dx$$

$= \text{logx}\int x^3 dx - \int \frac{d}{dx} logx. \int x^3 dx\, dx$

$= logx \text{ x } \frac{x^4}{4} - \int \frac{1}{x} . \frac{x^4}{4} \text{dx}$

$= \frac{x^4}{4}\text{logx} - \int \frac{x^3}{4}\text{dx}$

$= \frac{x^4}{4}\text{logx}\frac{x^4}{16} = \text{f(x)}$

$\text{F(2)} = \frac{2^4}{4}\text{log2} - \frac{2^4}{16}$

$= 4\text{log2} – 1$

F(0) = 0

$\int_0^2 x^3 logx\, dx = \text{f(2)} – \text{f(0)}$

$= \text{log}2^4 – 1$

$= \text{log16} – 1$

(g) Solve $(1 + x^2)\frac{dy}{dx} + \text{y} = e^{\tan^{-1} x}$

$(1 + x^2)\frac{dy}{dx} + \text{y} = e^{\tan^{-1} x}$

$$\frac{dy}{dx} + \frac{y}{1 + x^2} = \frac{e^{\tan^{-1} x}}{1 + x^2}$$

$\text{IF} = e^{\int \frac{1}{1+x^2} dx} = e^{\int \tan^{-1} x}$

Solution is y x IF = $\int \frac{e^{\tan^{-1} x}}{1+x^2}$ x IF dx

Ie, y x IF = $\int \frac{e^{\tan^{-1} x}}{1+x^2} e^{\tan^{-1} x}$ dx y x $e^{\tan^{-1}x}$= $\int u\,du$

y x $e^{\tan^{-1}x}$= $\frac{u^2}{2}$+ C

y x $e^{\tan^{-1}x}$= $\frac{\left(e^{\tan^{-1}x}\right)^2}{2}$+ C

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Find $\frac{dy}{dx}$ if y = $\frac{xsecx}{3x+2}$

y = $\frac{xsecx}{3x+2}$ (Use product rule)

$\frac{d}{dx}\left(\frac{u}{v}\right)$= $\frac{v\frac{du}{dx}+u\frac{dv}{dx}}{v^2}$

u = $xsecx$

$\frac{du}{dx} = xsecxtanx + secx$

v = 3x + 2

$\frac{dv}{dx}$= 3

$\frac{dy}{dx}$= $\frac{(3x + 2)(xsecxtanx+secx)-3xsecx}{(3x + 2)^2}$

= $\frac{(3x + 2)(xsecxtanx+secx)-3xsecx}{(3x + 2)^2}$

(b) Evaluate $\lim_{x\to 3}\frac{x^3-27}{x^4-81}$

$$\lim_{x\to a} \frac{x^m - a^m}{x^n - a^n} = \frac{ma^{m-1}}{na^{n-1}}$$

$$\lim_{x\to 3} \frac{x^3-27}{x^4-81} = \frac{3(3)^2}{4(3)^3} = \frac{1}{4}$$

(c) Using 1st principle, find derivatives of cosx

Y = cosx

$$\frac{dy}{dt} = \lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{cos(x+\Delta x)-cos(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{-2sin\left(\frac{2x+\Delta x}{2}\right)sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{-2sin\left(\frac{2x+\Delta x}{2}\right)sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2} \times 2}$$

$$= -sin\left(\frac{2x+0}{2}\right) = -\sin x \qquad [\lim_{x\to 0} \frac{sinx}{x} = 1]$$

IV.

(a) If $x^3 + y^3 = 3axy$ find $\frac{dy}{dx}$

$$x^3 + y^3 = 3axy$$

Differentiating on both sides

$$3x^2 + 3y^2\frac{dy}{dx} = 3a[x\frac{dy}{dx} + y \times 1]$$

ie,

$$x^2 + y^2\frac{dy}{dx} = ax\frac{dy}{dx} + ay$$

$$y^2\frac{dy}{dx} - ax\frac{dy}{dx} = +ay - x^2$$

$$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

(b) If $xy = ax^2 + \frac{b}{x}$ show that $x^2y'' + 2xy' - 2y = 0$

xy = $ax^2 + \frac{b}{x}$

y = $\frac{\left(ax^2+\frac{b}{x}\right)}{x} = ax + \frac{b}{x^2}$

$\frac{dy}{dx} =$ y' = $a - \frac{2b}{x^3}$

y'' = $\frac{6b}{x^4}$

$x^2y'' + 2xy' - 2y$ = $x^2\frac{6b}{x^4} + 2x(a - \frac{2b}{x^3}) - 2(ax + \frac{b}{x^2})$

= $\frac{6b}{x^2} + 2ax - \frac{4b}{x^2} - 2ax - \frac{2b}{x^2}$

= 0

(c) If $y = \frac{sin2x}{1+cos2x}$, find $\frac{dy}{dx}$

U = sin2x

$\frac{du}{dx}$= 2cos2x

V = 1 + cos2x

$\frac{dv}{dx}$= -sin2x x 2

$y = \frac{sin2x}{1+cos2x}$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(1 + \cos 2x)2\cos 2x - \sin 2x - 2\sin 2x}{(1 + \cos 2x)^2}$

$= \frac{2cos2x + 2\,cos^2 x + 2\,sin^2 2x}{(1 + cos2x)^2}$

$= \frac{2cos2x + 2(cos^2 2x + sin^2 2x)}{(1 + cos2x)^2}$

$$= \frac{2cos2x + 2}{(1 + cos2x)^2}$$

$$= \frac{2(1 + cos2x)}{(1 + cos2x)^2}$$

$$= \frac{2}{(1 + cos2x)}$$

V.

(a) Find the velocity and acceleration of a body at $t = 2sec$. If the displacement at time t is given by $s = 2t^3 - 3t^2 + 12t + 6$

$$s = 2t^3 - 3t^2 + 12t + 6$$

Velocity $= \left(\frac{ds}{dt}\right)_{t=2}$

$= 6t^2 - 6t + 12$

$= 6 \times 2^2 - 6 \times 2 + 12$

$= 24 - 12 + 12 = 24$ unit

Acceleration $= \left(\frac{d^2s}{dt^2}\right)_{t=2}$

$= 12t - 6$

$= 12 \times 2 - 6$

$= 24 - 6$

$= 18$

(b) A balloon is spherical in shape. Gas is filling into it at the rate of 10cc/sec. How fast is the surface area increasing when the radius is 15cm?

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 10$cc/sec

$\therefore \frac{dv}{dt} = 10 = 4\pi r^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{10}{4\pi r^2}$

The rate at which the surface area increasing is $\frac{ds}{dt}$

$\frac{ds}{dt} = 8\pi r \times \frac{10}{4\pi r^2}$

$\frac{20}{} \frac{}{15} = \frac{4}{3}$ cm^2/sec

(c) The deflection of a beam is $y = 2(100x - x^2)$. Find the maximum deflection

$$y = 2(100x - x^2)$$

At maximum,

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} < 0$

$\frac{dy}{dx} = 2(100 - 2x) = 0$

===> x = 50

$\frac{d^2y}{dx^2} = -4 < 0$

∴ Maximum deflection is y = 2 (100 x 50 – 50^2)

= 2 (5000 – 2500)

= 5000 units

VI.

(a) Show that the function $x^3 - 3x^2 + 3x + 7$ is increasing for all real values of x

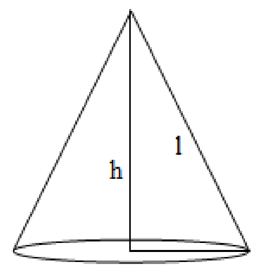
$$\frac{dy}{dx} = 3x^2 - 6x + 3$$

$$= 3(x^2 - 2x + 1)$$

$= 3(x-1)^2 > 0$ for every x

∴y is increasing for every values of x

(b) The sand falls into a conical pile at the rate of 1`0cc/sec and the radius of the pile is always equal to half of its altitude. How fast is the altitude of the pile increasing? When altitude is 150cm?



Volume of the cone = V = $\frac{1}{3}\pi r^2 h$

Given r = $\frac{h}{2}$

$$\therefore V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$= \frac{1}{3}\pi\frac{h^3}{4} = \frac{\pi h^3}{12}$$

$$\frac{dv}{dh} = \frac{\pi}{12} \text{ x } 3h^2 \text{ x } \frac{dh}{dt}$$

$$= \frac{\pi h^2}{4}\frac{dh}{dt}$$

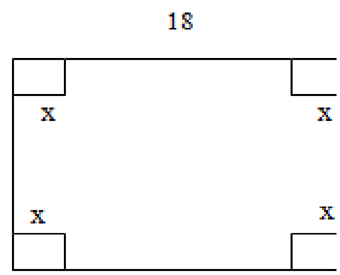
$$\frac{dv}{dt} = 10$$

$$ie, \frac{\pi h^2}{4}\frac{dh}{dt} = 10$$

$$\frac{dh}{dt} = \frac{4 \text{ x } 10}{\pi \text{ x } 150 \text{ x } 150}$$

$$= \frac{2}{1125\pi} \text{cm/sec}$$

(c) An open box is to be made out of a square sheet of side18cm by cutting off equal squares at each corner and turning up the sides. What size of the squares should be cut in order that the volume of the box may be maximum?



Let x be the side of the square sheet,

which is cut from each of the corners

The length of the open box = 18 - 2x

The length of the open box = 18 – 2x

The height of the open box = x

∴ Volume of the open box V = lbh

V = (18 – 2x) (18 – 2x)x

$\frac{dv}{dx} = (18 - 2x)(18 - 2x)x + (18 - 2x)\ \mathrm{x} - 2\ \mathrm{x}\ x + -2\ \mathrm{x}\ (18 - 2\mathrm{x})\mathrm{x}\ x$

$\frac{dv}{dx} = (18 - 2x)(18 - 2x - 4x)$

$= (18 - 2x)(18 - 6x)$

$\frac{d^2v}{dx^2} = (18 - 2x)\ \mathrm{x} - 6 + (18 - 6x)\mathrm{x} - 2$

At maximum

$\frac{dv}{dx}$ =0 and $\frac{d^2v}{dx^2} < 0$

===>$(18 - 2x)(18 - 6x) = 0$

===> x = 9, 3

But x = 9 is not acceptable.

∴ The side of the square sheet cut off = 3cm

VII. Find

(a) $\int(\sec^2 x + e^x - 5)dx$

$$\int (\sec^2 x + e^x - 5)dx$$

$$= \int \sec^2 x\, dx + \int e^x\, dx - \int 5\, dx$$

$$= tanx + e^x - 5x + C$$

(b) $\int \tan^3 x. \sec^2 x\ \ dx$

$\int \tan^3 x. \sec^2 x\ \ dx$

$=\int u^3\ du$ Put u = tanx

$= \frac{u^4}{4} + \text{C}$ du = $\sec^2$x dx

$= \frac{\tan^4 x}{4} + \text{C}$

(c) $\int_1^2 \frac{1}{9x^2-4}\text{dx}$ (Out of syllabus)

$\int_1^2 \frac{1}{9x^2-4}\text{dx} = \int_1^2 \frac{1}{(3x+2)(3x-2)}\text{dx}$

Put $\frac{1}{9x^2-4} = \frac{A}{(3x+2)} + \frac{B}{(3x-2)}$

1 = A(3x – 2) + B(3x + 2)

Solving A = $\frac{-1}{4}$ B = $\frac{1}{4}$

Then $\int \frac{1}{9x^2-4}\text{dx} = \int \frac{\frac{-1}{4}}{3x+2}\text{dx} + \int \frac{\frac{1}{4}}{3x-2}\text{dx} = \frac{-1}{12}\log(3x+2) + \frac{1\log(3x-2)}{12}$ then apply the limits. You will get the answer as $\frac{-1}{12} log \frac{5}{2}$

(d) $\int_0^{\frac{\pi}{2}} xcosx\ dx$

$\int xcosx\,dx = x \text{ x} \int cosx\,dx - \int \frac{dx}{dx} \int cosx\,dx\,dx$

$= x \text{ x } sinx - \int sinx\,dx$

$= xsinx + cosx = f(x)$

$F\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = \frac{\pi}{2}$

F(0) = 0sin0 + cos0 = 1

$\int_0^{\frac{\pi}{2}} xcosx\,dx = F\left(\frac{\pi}{2}\right) - F(0) = \frac{\pi}{2} - 1$

VIII. Find

(a) $\int \left(sinx + \frac{1}{x} + cosec^2x\right) dx$

$\int \left(sinx + \frac{1}{x} + cosec^2x\right) dx = \int sinx\,dx + \int \frac{1}{x}dx + \int cosec^2xdx$

= -cosx + logx – cotx + C

(b) $\int \frac{e^{2x}}{1+e^{2x}}$dx

Put u = $1 + e^{2x}$

du = $2e^{2x}$ dx

$\int \frac{e^{2x}}{1+e^{2x}}$dx = $\frac{1}{2}\int \frac{du}{u}$

= $\frac{1}{2}$logu

= $\frac{1}{2}\log(1 + e^{2x})$ + C

(c) $\int xlogx\ dx$

$\int xlogx\ dx = logx \int x\ dx - \int \frac{d}{dx} logx \int x\ dx\ dx$

= logx$\frac{x^2}{2} - \int \frac{1}{x}\frac{x^2}{2}$dx

=$\frac{x^2}{2}$ logx $- \int \frac{x}{2}$dx

= $\frac{x^2}{2}$ logx $- \frac{x^2}{4}$ + C

(d) $\int_0^1 \sin^{-1} x\ dx$

Put u = $\sqrt{1 - x^2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{1-x^2}}$ x $- 2x$

du = $\frac{-x}{\sqrt{1-x^2}}$dx

$\int \sin^{-1} x\ dx$

= $\int \sin^{-1} x$ x $1\ dx$

= $\sin^{-1} x \int 1\ dx - \int \frac{d}{dx} \sin^{-1} x \int 1\ dx\ \ dx$

= $\sin^{-1} x$ x $x - \int \frac{1}{\sqrt{1-x^2}}$ x $x\ dx$ ____________________ (1)

$\int \frac{x}{\sqrt{1-x^2}}\, dx$

= $-\int du$

= -u

= $-\sqrt{1 - x^2}$

Then (1) becomes

$\int \sin^{-1} x\, dx$ = $x sin^{-1}x + \sqrt{1-x^2}$ = f(x)

F(1) = $\sin^{-1}1$ +$\sqrt{0}$ = $\frac{\pi}{2}$

F(0) = 0 + $\sqrt{1}$ = 1

$\int_0^1 \sin^{-1} x\, dx$ = $\frac{\pi}{2}$– 1

IX.

(a) Find the area enclosed between the curves $x = y^2 - 2y$. Y axis and the ordinate at y = 1 and y = 2

Required area = $\int_a^b x\, dy$

= $\int_1^2 y^2 - 2y\, dy$

$\int y^2 - 2y\, dy$ = $\frac{y^3}{3} - \frac{2y^2}{2}$= $\frac{y^3}{3}$ — y^2 = f(y)

F(2) = $\frac{2^3}{3}$ — 2^2 = = $\frac{8}{3}$ —4 = $\frac{-4}{3}$

F(1) = $\frac{1}{3}$ —1 = $\frac{-2}{3}$

Area = $|f(2) - f(1)|$

= $\left|\frac{-4}{3} - \frac{-2}{3}\right|$

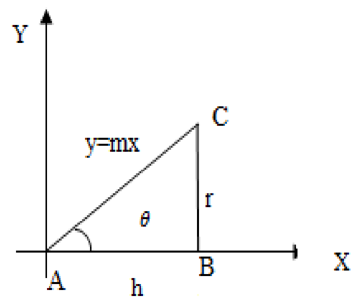
= $\left|\frac{-4}{3} + \frac{2}{3}\right|$

= $\left|\frac{-2}{3}\right|$

= $\frac{2}{3}$

$\therefore$ Required area = $\frac{2}{3}$ $sq\ units$

(b) Show by integration that the volume of a right circular cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$



Slop of AC = $\frac{r}{h} tan\theta$

Equation of AC = y = mx = $\frac{r}{h}x$

$$Y = \frac{r}{h}x$$

Volume of cone = $\pi\int_a^b y^2 dx$

$$= \pi\int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$= \pi\int_0^h \frac{r^2x^2}{h^2}dx$$

$$= \pi\frac{r^2}{h^2}\int_0^h x^2 dx$$

$$= \frac{\pi}{3}\frac{r^2}{h^2} \times h^3$$

$$= \frac{1}{3}\pi r^2 h$$

(c) Solve $\frac{d^2y}{dx^2} = x\,e^x + cosx$

$$\frac{d^2y}{dx^2} = x\,e^x + cosx$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = x\,e^x + cosx$$

Integrating on both sides we get

$\frac{dy}{dx} = \int x\, e^x\, dx + \int cosx\, dx$

$= x \int e^x\, dx + \int 1 \text{ x } e^x dx + sinx$

$= xe^x - e^x + sinx + C_1$

Again integrating on both sides we get

$y = xe^x - e^x - e^x + -cosx + x\, C_1 + C_2$

$y = xe^x - 2e^x + -cosx + C_1x + C_2$

X.

(a) Find the area bounded by the parabola $x^2 = y$ and $y^2 = x$

$x^2 = y$

$y^2 = x$

$y = x^2$ ______________ (1)

$y = \sqrt{x}$ _______________ (2)

$x^2 = \sqrt{x}$ ===> x = 0, x = 1

Area $= \left| \int_a^b f(x) - g(x) dx \right|$

$= \left| \int_0^1 x^2 - \sqrt{x}\, dx \right|$ _____________ (1)

$\int x^2 dx - \int \sqrt{x}\, dx = \frac{x^3}{3} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} =$ f(x)

F(1) $= \frac{1}{3} - \frac{2}{3} = \frac{3-6}{9} = \frac{-3}{9} = \frac{-1}{3}$

F(0) = 0

Required area = $|f(1) - f(0)|$

$= \left|\frac{-1}{3}\right| = \frac{1}{3}$unit^2

(b) Find the volume of solid obtained by rotating the area under the curve$y = x^2 + 1$between x = 0 and x = 1

Volume = $\pi \int_a^b y^2 dx$

= = $\pi \int_0^1 (x^2+1)^2 dx$

$\int (x^2+1)^2 dx$ = $\int x^4 dx$ + 2 $\int x^2 dx + \int 1 dx$

= $\frac{x^5}{5} + \frac{2x^3}{3}$ + x = f(x)

F(1) = $\frac{1}{5} + \frac{2}{3}$ + 1

=$\frac{(3+10)}{15}$+ 1

= $\frac{13+15}{15}$ = $\frac{28}{15}$

F(0) = 0

Volume = = π[F(1) – f(0)]

= π x $\frac{28}{15} = \frac{28}{15}\pi$

(c) Solve$(x^2+1)\frac{dy}{dx} + 2xy$= $4x^2$

$(x^2+1)\frac{dy}{dx} + 2xy$= $4x^2$

$\frac{dy}{dx} + \frac{2x}{(x^2+1)}$y= $\frac{(4x^2)}{(x^2+1)}$

Integrating factor =$e^{\int p\, dx}$

= $e^{\int \frac{2x}{(x^2+1)} dx}$

= $e^{\log(x^2+1)}$

= x^2+1

Solution : y x (x^2+1)

$$= \int (x^2 + 1) \frac{4x^2}{(x^2+1)} \, dx$$

$$= \int 4x^2 \, dx$$

$$= \frac{4x^3}{3} \text{dx}$$

Ie, $y(x^2 + 1) = \frac{4x^3}{3} + C$