

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/  
TECHNOLIGY- MARCH, 2013

TECHNICAL MATHEMATICS- I  
(Common – Except DCP and CABM)

[Time: 3 hours

(Maximum marks: 100)

Marks

PART –A  
(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) If  $\begin{vmatrix} 3x & 7 \\ 2 & 8 \end{vmatrix} = 0$  find the value of x.

$$\begin{vmatrix} 3x & 7 \\ 2 & 8 \end{vmatrix} = 0$$

$$\implies 24x - 14 = 0$$

$$\implies 24x = 14$$

$$\implies x = \frac{14}{24} = \frac{7}{12}$$

(b) If  $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$   $B = \begin{bmatrix} 3 & -3 \\ 0 & 1 \end{bmatrix}$  find  $2A - 3B$

$$2A - 3B = 2 \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} + 3 \begin{bmatrix} 3 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 9 & -9 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -7 & 13 \\ 4 & -5 \end{bmatrix}$$

(c)  $n_{C_{10}} = n_{C_{15}}$  find n

$$n_{C_r} = n_{C_s} \implies r = s \quad \text{or} \quad r + s = n$$

Then  $n = r + s$

$$n = 10 + 15 = 25$$

(d) Evaluate  $\cos\theta$  and  $\tan\theta$  if  $\sin\theta = \frac{1}{2}$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - (1/2)^2$$

$$\cos^2\theta = 1 - (1/4)$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

(e) Find the slope of the line whose inclination to the x axis is  $45^\circ$

$$\text{Slope} = \tan\theta = \tan 45^\circ = 1$$

#### PART -B

Answer any five questions. Each question carries 6 marks

II.

(a) If  $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$  show that  $AA^T$  is symmetric.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix} \quad A^T = \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}$$

$$\text{Clearly } (AA^T)^T = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}^T = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}$$

$\therefore AA^T$  is symmetric.

(b) Solve using determinants:

$$3x + y - z = 3$$

$$-x + y + z = 1$$

$$x + y + z = 3$$

$$AX = B$$

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{3(0) - 1(1-3) - (1-3)}{3(0) - (-1-1) - (-1-1)}$$

$$= \frac{2+2}{2+2} = \frac{4}{4} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 3 & -1 \\ -1 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix}}{4}$$

$$= \frac{3(1-3) - 3(-1-1) - (-3-1)}{4}$$

$$= \frac{-6+6+4}{4} = \frac{4}{4} = 1$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & 3 \\ -1 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix}}{4}$$

$$= \frac{3(3-1) - (-3-1) + 3(-1-1)}{4}$$

$$= \frac{6+4-6}{4} = \frac{4}{4} = 1$$

(c) Find the term independent of  $x$  in the expression of  $(3x - y^2)^5$

$$T_{r+1} = n C_r a^{n-r} b^r$$

$$= 5 C_r (3x)^{5-r} (-y^2)^r$$

We have to find

$$T_4 = 5 C_3 (3x)^2 (-y^2)^3$$

$$= 5 C_3 \cdot 3^2 x^2 (-1)^3 y^6$$

$$= -90x^2y^6$$

- (d) Prove that  $\sin\theta + \sin3\theta + \sin5\theta + \sin7\theta = 4 \cos\theta \cdot \cos2\theta \cdot \sin4\theta$

$$\begin{aligned} \sin\theta + \sin3\theta + \sin5\theta + \sin7\theta & \quad \left[ \sin C + \sin D = 2 \cdot \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right] \\ &= (\sin\theta + \sin7\theta) + (\sin3\theta + \sin5\theta) \\ &= 2\sin4\theta \cdot \cos3\theta + 2\sin4\theta \cdot \cos\theta \\ &= 2\sin4\theta(\cos3\theta + \cos\theta) \quad \left[ \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2} \right] \\ &= 2\sin4\theta \cdot 2(\cos2\theta \cdot \cos\theta) \\ &= 2\sin4\theta \cdot \cos2\theta \cdot \cos\theta, \text{ hence the result.} \end{aligned}$$

- (e) Prove that  $\frac{\sin3\theta}{\sin\theta} + \frac{\cos3\theta}{\cos\theta} = 4\cos2\theta$

$$\begin{aligned} \frac{\sin3\theta}{\sin\theta} + \frac{\cos3\theta}{\cos\theta} &= \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta} + \frac{4\cos^3\theta - 3\cos\theta}{\cos\theta} \\ &= 3 - 4\sin^2\theta + 4\cos^2\theta - 3 = 4(\cos^2\theta - \sin^2\theta) \\ &= 4\cos2\theta \end{aligned}$$

- (f) Find the equation of the line passing through the point (2, -1) and (-6, 3). Also find the slope of the line.

Two points of a line is given by,

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$(x_1, y_1) = (2, -1)$$

$$(x_2, y_2) = (-6, 3)$$

$$\frac{y-(-1)}{3-(-1)} = \frac{x-2}{-6-2}$$

$$\frac{y+1}{4} = \frac{x-2}{-8}$$

$$-2(y+1) = x - 2$$

$$-2y - 2 = x - 2$$

$$x + 2y = 0$$

$$\text{Slope of the line } x + 2y = 0 \text{ is } m = -\frac{a}{b} = -\frac{1}{2}$$

(g) Find the value of  $k$  so that the following lines are concurrent.

$$5x + 2y - 4 = 0$$

$$2x + ky + 11 = 0$$

$$3x - 4y - 18 = 0$$

$$5x + 2y - 4 = 0$$

$$2x + ky + 11 = 0$$

$$3x - 4y - 18 = 0$$

Since the lines are concurrent

$$\therefore \begin{vmatrix} 5 & 2 & -4 \\ 2 & k & 11 \\ 3 & -4 & -18 \end{vmatrix} = 0$$

$$5 \begin{vmatrix} k & 11 \\ -4 & -18 \end{vmatrix} - 2 \begin{vmatrix} 2 & 11 \\ 3 & -18 \end{vmatrix} - 4 \begin{vmatrix} 2 & k \\ 3 & -4 \end{vmatrix} = 0$$

$$5(-18k + 44) - 2(-36 - 33) - 4(-8 - 3k) = 0$$

$$-90k + 220 - 2 \times -69 + 32 + 12k = 0$$

$$-78k + 220 + 138 + 32 = 0$$

$$390 = 78k$$

$$K = \frac{390}{78} = 5$$

PART –C  
(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If  $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$  and I unit matrix of same order, then find  $A^3 - 3A^2 + 2A + I$

$$A^3 = A^2 \cdot A$$

$$\begin{aligned} A^3 &= A^2 \cdot A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} A \\ &= \begin{bmatrix} 4 & 9 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 21 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A^3 - 3A^2 + 2A + I &= \begin{bmatrix} 8 & 21 \\ 0 & 1 \end{bmatrix} - 3 \begin{bmatrix} 4 & 9 \\ 0 & 1 \end{bmatrix} + 2 \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

(b) If  $A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$  show that  $(AB)^{-1} = B^{-1}A^{-1}$

$$AB = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 47 & 34 \\ 22 & 16 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 47 & 34 \\ 22 & 16 \end{vmatrix} = 4$$

$$\text{Cofactor matrix} = \begin{bmatrix} 16 & -22 \\ -34 & 47 \end{bmatrix}$$

$$\text{Adj. (AB)} = \begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}$$

$$\text{Inverse of AB} = \frac{\text{Adj. AB}}{|AB|} = \frac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4} = \begin{bmatrix} 4 & -\frac{17}{2} \\ -\frac{11}{2} & \frac{47}{4} \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \quad |A| = 4$$

$$\text{Cofactor matrix of A} = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\text{Adj. (A)} = \begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj.}A}{|A|} = \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$$

$$B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \quad |B| = 1$$

$$\text{Cofactor matrix of } B = \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$$

$$\text{Adj. } (B) = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$$

$$B^{-1} = \frac{\text{Adj.}B}{|B|} = \frac{\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}}{1}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4} = \begin{bmatrix} 4 & -\frac{17}{2} \\ -\frac{11}{2} & \frac{47}{4} \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

(c) Find x if  $\begin{vmatrix} 2 & x & 3 \\ 4 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & 2 \\ 2 & 3 \end{vmatrix}$

$$\begin{vmatrix} 2 & x & 3 \\ 4 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} - x \begin{vmatrix} 4 & 2 \\ 0 & 1 \end{vmatrix} + 3 \begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 2(1-2) - x(4) + 3(4)$$

$$= -2 - 4x + 12 = 10 - 4x$$

$$\begin{vmatrix} x & 2 \\ 2 & 3 \end{vmatrix} = 3x - 4$$

$$\therefore 3x - 4 = 10 - 4x$$

$$\implies 7x = 14$$

$$\implies x = 2$$

IV.

(a) If  $A = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 1 \\ 6 & -5 \end{bmatrix}$ , show that  $(A + B)^T = A^T + B^T$

$$A + B = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 6 & -5 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 10 & 4 \end{bmatrix}$$

$$(A + B)^T = \begin{bmatrix} 5 & 10 \\ 3 & 4 \end{bmatrix} \text{ ————— } \textcircled{1}$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 4 & 1 \\ 6 & -5 \end{bmatrix}^T = \begin{bmatrix} 4 & 6 \\ 1 & -5 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 5 & 10 \\ 3 & 4 \end{bmatrix} \quad \text{--- (2)}$$

From (1) & (2) it is clear that  $(A + B)^T = A^T + B^T$

- (b) Find  $k$ , if the following system of equation are consistent  
 $x + y + 1 = 0$ ,  $x + 2y + 1 = 0$ ,  $2x + 3y + k = 0$

If the system is consistent then,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & k \end{vmatrix} = 0$$

$$1 \begin{vmatrix} 2 & 1 \\ 3 & k \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 2 & k \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 0$$

$$1(2k - 3) - (k - 2) + (3 - 4) = 0$$

$$2k - 3 - k + 2 - 1 = 0$$

$$k - 2 = 0$$

$$k = 2$$

- (c) Solve using inverse of the coefficient matrix

$$x + y + z = 1,$$

$$2x + 2y + 3z = 6,$$

$$x + 4y + 9z = 3$$

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

Calculation for  $A^{-1}$

$$m_{11} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6$$

$$m_{12} = \begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix} = 15$$

$$m_{13} = \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 6$$



$$m_{21} = \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = 5$$

$$m_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8$$

$$m_{23} = \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = 3$$

$$m_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$$

$$m_{32} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$$

$$m_{33} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$$

$$|A| = -3$$

$$\text{Minor matrix} = \begin{bmatrix} 6 & 15 & 6 \\ 5 & 8 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\text{Cofactor matrix} = \begin{bmatrix} 6 & -15 & 6 \\ -5 & 8 & -3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\text{Adjoint matrix} = \begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$$

$$\text{So inverse matrix, } A^{-1} = \frac{\text{Adj.}A}{|A|} = \frac{\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}}{-3}$$

$$X = A^{-1}B = \frac{\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}}{-3} = \frac{\begin{bmatrix} -21 \\ 30 \\ -12 \end{bmatrix}}{-3}$$

$$x = -\frac{21}{-3} = 7$$

$$y = \frac{30}{-3} = -10$$

$$z = -\frac{12}{-3} = 4$$

V.

(a) If  ${}^{20}C_r = {}^{20}C_{r+2}$  find r

$${}^nC_r = {}^nC_s \quad \implies r = s \quad \text{or } r + s = n$$

$$\text{ie, } r + r + 2 = 20$$

$$2r + 2 = 20$$

$$2r = 18, \quad r = 9$$

(b) Find middle term in the expansion of  $(x^2 + 3/x)^{20}$

$$T_{r+1} = n c_r a^{n-r} b^r, \quad n = 20$$

$$n + 1 = 21, \text{ odd.}$$

$$\therefore \left(\frac{n+1}{2}\right)^{\text{th}} = 22/2 = 11^{\text{th}} \text{ term is the middle term}$$

$$\begin{aligned} \text{ie, } T_{11} &= 20 c_{10} (x^2)^{10} (3/x)^{10} \\ &= 20 c_{10} x^{20} 3^{10} x^{-10} \\ &= 20 c_{10} x^{10} 3^{10} \\ &= 20 c_{10} 3^{10} x^{10} \end{aligned}$$

(c) Prove that  $\frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = 2 - \sqrt{3}$

$$\frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\frac{(\sqrt{3} - 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{(\sqrt{3} - 1)^2}{3 - 1} = \frac{3 + 1 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

VI.

(a) Expand  $\left(3x - \frac{y}{2}\right)^4$  binomially

$$(a + b)^n = a^n + n c_1 a^{n-1} b + n c_2 a^{n-2} b^2 + \dots + n c_n b^n$$

$$\begin{aligned} \left(3x - \frac{y}{2}\right)^4 &= (3x)^4 - 4 c_1 (3x)^3 \left(\frac{y}{2}\right) + 4 c_2 (3x)^2 \left(\frac{y}{2}\right)^2 - 4 c_3 (3x) \left(\frac{y}{2}\right)^3 + 4 c_4 \left(\frac{y}{2}\right)^4 \\ &= 81x^4 - 4 \times 27x^3 \left(\frac{y}{2}\right) + 6 \times 9x^2 \times \left(\frac{y^2}{4}\right) - 4 \times 3 \times x \times \left(\frac{y^3}{8}\right) + \left(\frac{y^4}{16}\right) \\ &= 81x^4 - 54x^3y + \left(\frac{27}{2}\right)x^2y^2 - \left(\frac{3xy^3}{2}\right) + \frac{y^4}{16} \end{aligned}$$

(b) Find the constant term in the expansion of  $(\sqrt{x} + \frac{2}{x^2})^{10}$

$$T_{r+1} = n c_r a^{n-r} b^r$$

$$T_{r+1} = 10 c_r (\sqrt{x})^{10-r} \left(\frac{2}{x^2}\right)^r$$

$$= 10 c_r x^{\frac{10-r}{2}} 2^r x^{-2r}$$

$$= 10 c_r x^{\frac{10-r}{2}} x^{-2r} 2^r$$

$$= 10 c_r x^{\frac{10-r}{2}-2r} 2^r$$

$$= 10 c_r x^{\frac{10-5r}{2}} 2^r$$

$$\text{Then } \frac{10-5r}{2} = 0 \quad \implies 10 - 5r = 0$$

$$\implies r = 2$$

$$\text{Therefore } T_3 = 10 c_2 2^2$$

$$= 45 \times 4 = 180 \text{ is the constant term.}$$

(c) Prove that  $\sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \cdot \operatorname{cosec}^2 \theta$

$$\sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

$$\sec^2 \theta + \operatorname{cosec}^2 \theta = \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$= \frac{1}{\sin^2 \theta \cos^2 \theta}$$

$$= \sec^2 \theta \cdot \operatorname{cosec}^2 \theta$$

VII. (a) Prove that in  $\Delta ABC$ ,  $\sum a(\sin B - \sin C) = 0$

in  $\Delta ABC$ , we know that  $\sin B = \frac{b}{2R}$  (by sine rule)

$$\text{and } \sin C = \frac{c}{2R}$$

$$\begin{aligned}\sum a(\sin B - \sin C) &= \sum a \left( \frac{b}{2R} - \frac{c}{2R} \right) \\ &= \frac{1}{2R} (\sum a(b - c)) \\ &= \frac{1}{2R} [a(b - c) + b(c - a) + c(a - b)] \\ &= \frac{1}{2R} [ab - ac + bc - ba + ca - cb] \\ &= \frac{1}{2R} \times 0 = 0\end{aligned}$$

(b) Prove that  $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$

$$\text{We have } \cos 60^\circ = \frac{1}{2}$$

$$\text{ie, } \frac{1}{2} \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ =$$

$$= \frac{1}{2} \cos 20^\circ \cdot \frac{1}{2} [\cos 120^\circ - \cos(-40^\circ)]$$

$$= \frac{1}{4} \cdot \cos 20^\circ \left[ -\frac{1}{2} + \cos 40^\circ \right]$$

$$= -\frac{1}{8} \cdot \cos 20^\circ + \frac{1}{4} \cos 20^\circ \cdot \cos 40^\circ$$

$$= -\frac{1}{8} \cdot \cos 20^\circ + \frac{1}{4} \times (\cos 60^\circ + \cos 20^\circ)$$

$$= -\frac{1}{8} \cdot \cos 20^\circ + \frac{1}{8} \cos 60^\circ + \frac{1}{8} \cos 20^\circ$$

$$= \frac{1}{8} \cdot \cos 60^\circ = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$$

(c) Show that  $\sin 120^\circ \cdot \cos 330^\circ + \cos 240^\circ \cdot \sin 330^\circ = 1$

$$\sin 120^\circ = \sin(1 \times 90^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 330 = \cos(360 - 30) = \cos(4 \times 90 - 30) = \cos 30 = \frac{\sqrt{3}}{2}$$

$$\cos 240 = \cos(270 - 30) = \cos(3 \times 90 - 30) = -\sin 30 = \frac{-1}{2}$$

$$\sin 330 = \sin(360 - 30) = \sin(4 \times 90 - 30) = -\sin 30 = \frac{-1}{2}$$

$$\sin 120 \cdot \cos 330 + \cos 240 \cdot \sin 330$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{-1}{2} \times \frac{-1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

VIII.

- (a) Express  $3\cos\theta + 4\sin\theta$  in the form of  $R\sin(\theta + \alpha)$  where  $\alpha$  is acute.

$$\sqrt{3}\cos x + \sin x = R \cdot \sin(x + \alpha)$$

$$= R \cdot \sin x \cdot \cos \alpha + R \cos x \cdot \sin \alpha$$

Equating the similar terms on both sides,

$$\sqrt{3}\cos x = R \sin \alpha \cdot \cos \alpha$$

$$\sin x = R \sin x \cdot \cos \alpha$$

$$\implies \sqrt{3} = R \sin \alpha \quad \text{--- (1)}$$

$$\implies 1 = R \cos \alpha \quad \text{--- (2)}$$

Squaring and adding (1) & (2)

$$3 + 1 = R^2 \sin^2 \alpha + \cos^2 \alpha$$

$$4 = R^2 \implies R = \pm 2$$

$$\frac{(1)}{(2)} \implies \sqrt{3} = \frac{\sin \alpha}{\cos \alpha} \implies \tan \alpha = \sqrt{3}$$

$$\implies \alpha = \tan^{-1}(\sqrt{3})$$

$$\implies \alpha = 60^\circ$$

- (b) Prove that  $\sin(A + B) \cdot \sin(A - B) = \sin^2 A - \sin^2 B$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$\begin{aligned}
\sin(A+B).\sin(A-B) &= (\sin A.\cos B + \cos A.\sin B) (\sin A.\cos B - \cos A.\sin B) \\
&= \sin^2 A.\cos^2 B - \sin A.\cos A.\sin B.\cos B \\
&\quad + \sin A.\cos A.\sin B.\cos B - \cos^2 A.\sin^2 B \\
&= \sin^2 A.\cos^2 B - \cos^2 A.\sin^2 B \\
&= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B \\
&= \sin^2 A - \sin^2 A.\sin^2 B - \sin^2 B + \sin^2 A.\sin^2 B \\
&= \sin^2 A - \sin^2 B
\end{aligned}$$

(c) In any  $\Delta ABC$ , show that  $(b + c)\sin A/2 = a.\cos(\frac{B-C}{2})$

$$\text{LHS} = (a + b).\sin\frac{C}{2}$$

$$= (2R\sin A + 2R\sin B)\sin\frac{C}{2} \left[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \right]$$

$$= 2R(\sin A + \sin B)\sin\frac{C}{2}$$

$$= 2R.2.\sin\frac{A+B}{2}.\cos\frac{A-B}{2}.\sin\frac{C}{2}$$

$$= \cos\frac{A-B}{2}.4R.\sin\frac{A+B}{2}.\sin\frac{C}{2}$$

$$= \cos\frac{A-B}{2}.4R.\sin(90 - \frac{C}{2}).\sin\frac{C}{2}$$

$$= \cos\frac{A-B}{2}.2R.(2\cos\frac{C}{2}.\sin\frac{C}{2})$$

$$= \cos\frac{A-B}{2}.2R.\sin C$$

$$= \cos\frac{A-B}{2}.c = \text{RHS}$$

IX.

(a) Solve  $\Delta ABC$  if  $a = 5\text{cm}$ ,  $B = 30^\circ$  &  $c = 8\text{cm}$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$\frac{A-B}{2} = \tan^{-1} \left[ \frac{a-b}{a+b} \cot \frac{C}{2} \right]$$

$$\frac{A-B}{2} = \tan^{-1} \left[ \frac{5-8}{13} \cot \frac{30}{2} \right]$$

$$\frac{A-B}{2} = \tan^{-1} \left[ \frac{-3}{13} \cot 15^\circ \right]$$

$$\frac{A-B}{2} = \tan^{-1} [-0.8612] = -40.736$$

$$A - B = -81.473$$

$$A + B = 180 - 30 = 150$$

$$\text{Solving } \textcircled{1} + \textcircled{2}$$

$$A = 34.2635 = 34^\circ 16'$$

$$B = 150 - 34.2635 = 115^\circ 44'$$

Now we have to find 'C'

We have 
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{5}{\sin 34^\circ 16'} = \frac{c}{\sin 30^\circ}$$

$$c = \frac{5}{0.5629996} \times \sin 30^\circ = 4.44 \text{ cm}$$

(b) Find the slope and intercept of the line  $3x + 4y = 12$

Slope of  $3x + 4y = 12$  is  $-\frac{a}{b} = -\frac{3}{4}$

Intercept form of a line is  $\frac{x}{a} + \frac{y}{b} = 1$

$$3x + 4y = 12$$

$$\frac{3x}{12} + \frac{4y}{12} = 1$$

$$\implies \frac{x}{4} + \frac{y}{3} = 1$$

X intercept = 4

Y intercept = 3

(c) Find k so that the lines  $kx + 2y - 10 = 0$ ,  $2x - 4y + 15 = 0$  are

(i). Perpendicular to each other.

(ii). Parallel to each other.

(i).  $m_1 \times m_2 = -1$

$$-\frac{k}{2} \times -\frac{2}{-4} = -1-1$$

$$\frac{2k}{-8} = \implies 2k = 8 \implies k = 4$$

(ii).  $m_1 = m_2$

$$-\frac{k}{2} = -\frac{2}{-4}$$

$$-\frac{k}{2} = -\frac{1}{-2}$$

$$k = -1$$

X.

(a) Solve  $\Delta ABC$  using Napier's formula, given  $a = 87\text{cm}$ ,  $b = 53\text{cm}$  and  $C = 70^\circ$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$A - B = 2 \tan^{-1} \left[ \frac{a-b}{a+b} \cot \frac{C}{2} \right]$$

$$A - B = 2 \tan^{-1} \left[ \frac{87-53}{87+53} \cot 35^\circ \right]$$

$$= 2 \tan^{-1} \left[ \frac{34}{140} \cot 15^\circ \right]$$

$$= 2 \tan^{-1} [0.3469] = 2 \times 19^\circ 08' = 38^\circ 16'$$

$$A + B = 180 - 70 = 110^\circ$$

$$2A = 148^\circ 16' / 2 = 74^\circ 08'$$

$$A = B = 110$$

$$B = 110 - 74^\circ 08' = 35^\circ 52'$$

Now we have to find 'c', we have

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{87}{\sin 74^\circ 08'} = \frac{c}{\sin 70^\circ}$$



$$\implies c = \frac{\sin 70^\circ \times 87}{\sin 74^\circ 08'} = \frac{0.9397 \times 87}{0.9619} = 84.99 \text{ cm}$$

- (b) Find the equation to the line passing through the point of intersection of  $x - y + 1 = 0$  and  $2x - 3y + 2 = 0$  and perpendicular to the line  $x + y - 6 = 0$

Given

$$x - y + 1 = 0$$

$$2x - 3y + 2 = 0$$

$$\therefore x - y = -1$$

$$2x - 3y = -2$$

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\begin{aligned} \therefore x &= \frac{\begin{vmatrix} -1 & -1 \\ -2 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}} \\ &= \frac{3-2}{-3+2} = \frac{1}{-1} = -1 \end{aligned}$$

$$\begin{aligned} Y &= \frac{\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}} \\ &= \frac{-2+2}{-3+2} = \frac{0}{1} = 0 \end{aligned}$$

$\therefore$  Point of intersection =  $(-1, 0)$

$\therefore$  Given the line is,

$$x + y - 6 = 0$$

$$a = 1, b = 1, c = -6$$

$\therefore$  Perpendicular line is,

$$bx - ay + k = 0$$

$$x - y + k = 0 \quad \text{--- (1)}$$

$\therefore$  (1) Passes through  $(-1, 0)$

$$\begin{aligned} \text{(1)} \implies -1 - 0 + k &= 0 \\ K &= 1 \end{aligned}$$

$\therefore$  (1)  $\implies x - y + 1 = 0$

(c) A line passes through  $(-6, 3)$ . The X-intercept of the line is 3 times its Y-intercept. Find the equation of the line.

$$a = 3b \text{ (given)}$$

The equation of a line is  $\frac{x}{a} + \frac{y}{b} = 1$

Ie,  $\frac{x}{3b} + \frac{y}{b} = 1 \dots\dots\dots(1)$

(1) Pass through  $(-6, 3)$

(1) Implies,  $\frac{-6}{3b} + \frac{3}{b} = 1$

$$\frac{-2}{b} + \frac{3}{b} = 1$$

$$\frac{1}{b} = 1, \quad b = 1$$

(1) Implies  $\frac{x}{3} + \frac{y}{1} = 1$

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