TED (10)-1002 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- OCTOBER, 2011

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time*: 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) If $\begin{vmatrix} 3x & 7 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix}$ find x.

$\begin{vmatrix} 3x & 7 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix}$

===>9x – 14 = 8 – 4

===>9x – 14 = 4

===>9x = 18

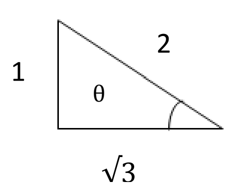
===> x = 18/9 = 2

(b) If $n_{c_{20}} = n_{c_{23}}$ find n

$n_{c_r} = n_{c_s}$ ===> r = s or r + s = n

Then n = r + s = 20 + 23 = 43

(c) If cosθ = $\sqrt{3}/2$ find sinθ&tanθ



sinθ = 1/2 tanθ = $1/\sqrt{3}$

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(d) If sinθ =a find sin3θ

$\text{Sin}3\theta = 3\sin\theta - 4\sin^3\theta$

$\text{Sin}3\theta = 3a - 4a^3$

(e) Find the slope of the linejoining the vertices (2, 6), (4, 0)

$$\text{Slope} = \frac{y2-y1}{x2-x1} = \frac{0-6}{4-2} = \frac{-6}{2} = -3$$

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Solve using determinants:

x + y + z = 3
2x + 3y + z = –6
x + y - z = –3

AX = B

$$\begin{bmatrix}1 & 1 & 1\\2 & 3 & 1\\1 & 1 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\6\\-3\end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix}3 & 1 & 1\\-6 & 3 & 1\\-3 & 1 & -1\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\2 & 3 & 1\\1 & 1 & -1\end{vmatrix}}$$

$$= \frac{(-3-1)-(-2-1)+2-3}{-2} \qquad = \frac{-18}{-2} = 9$$

$$y = \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix}1 & 3 & 1\\2 & -6 & 1\\1 & -3 & -1\end{vmatrix}}{-2}$$

$$= \frac{18}{-2} = -9$$

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$$z = \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & -6 \\ 1 & 1 & -3 \end{vmatrix}}{-2}$$

$$= \frac{-6}{-2} = 3$$

(b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ evaluate AB & BA

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1+2+6 & 3+4+9 \\ 8 & -4+5+12 & 12+10+18 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 16 \\ 8 & 13 & 40 \\ 2 & -1 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4+3 & 4-5 & 6-6 \\ 4+2 & 5 & 6 \\ 8+3 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 6 & 5 & 6 \\ 11 & 10 & 12 \end{bmatrix}$$

(c) Find the middle term of $(x^2 + 1/x)^{12}$

$T_{r+1} = nc_r\, a^{n-r} b^r$, n= 12

$T_{r+1} = 12c_r (x^2)^{12-r} (1/x)^r$

n + 1 = 13, odd.

$\therefore \left(\frac{n+1}{2}\right)^{th} = \frac{12}{2} + 1 = 6 + 1 = 7^{th}$ term is the middle term

ie, $T_7 = 12c_6\ (x^2)^6 (1/x)^6$

$= 12c_{10} x^{12} x^{-6}$

$= 12c_6 x^6$

$= 924x^6$

(d) Prove that $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 4 \cos\theta . \cos 2\theta . \sin 4\theta$

$\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta$ $\left[\sin C + \sin D = 2 \sin{(C+D)}/_2). \cos{(C-D)}/_2)\right]$

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$= (\sin\theta + \sin 7\theta) + (\sin 3\theta + \sin 5\theta)$

$= 2\sin 4\theta.\cos 3\theta + 2\sin 4\theta.\cos\theta$

$= 2\sin 4\theta(\cos 3\theta + \cos\theta)$ $\left[\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)\right]$

$= 2\sin 4\theta \,.\, 2(\cos 2\theta \,.\, \cos\theta)$

$= 2\sin 4\theta \,.\, \cos 2\theta \,.\, \cos\theta$, hence the result.

(e) Find the equation of the line parallel and perpendicular to 2x – 3y + 10 = 0 and passing through (1, 1).

2x – 3y + 10 = 0

a = 2, b = -3, c = 10

<u>Case I</u>

Equation of a line parallel is ax + by + k = 0

2x – 3y +k = 0 —— (1)

(1) Passes through (1, 1)

(1) ===> 2 x 1 – 3 x 1 + k = 0

2 – 3 + k = 0

-1 + k = 0

K = 1

So, (1) ===>2*x*– 3y + 1 = 0

<u>Case II</u>

Equation of a line perpendicular is, bx- ay + k = 0

-3x – 2y +k = 0 —— (2)

(2) Passing through (1, 1)

(2) ===> -3 x 1 – 2 x 1 + k = 0

-3 – 2 + k = 0

-5 + k = 0

K = 5

So, (1) ===>-3*x*– 2y + 5 = 0

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(f) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$

$\text{Cos}3A = \cos(2A + A)$

$= \cos 2A.\cos A - \sin 2A.\sin A$

$= (2\cos^2 A - 1)\cos A - 2\sin A.\cos A.\sin A$

$= 2\cos^3 A - \cos A - 2\sin^2 A.\cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$

(g) The straight line through (4, 3) makes intercepts of 4a and 3a on the X axis and Y axis respectively. Find a

Equation of a line in intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{4a} + \frac{y}{3a} = 1$ —— (1)

(1) Passing through (4, 3) then $\frac{x}{4a} + \frac{3}{3a} = 1$

$\Longrightarrow \frac{1}{a} + \frac{1}{a} = 1$

$\Longrightarrow a = 2$

Then (1) becomes,

$\frac{x}{8} + \frac{y}{6} = 1$

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PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Find x if $\begin{vmatrix} 2 & 3 & 5 \\ 2 & x & 5 \\ 3 & -1 & 2 \end{vmatrix} = 0$

$$\begin{vmatrix} 2 & 3 & 5 \\ 2 & x & 5 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

===> 2(2x + 5) – 3(4 - 15) + 5(-2 - 3x) = 0

===> 4x + 10 – 12 + 45 + - 10 – 15x = 0

===> -11x = -33

x = -33/-11 = 3

(b) Express the matrix A = $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

$$\frac{A+A^T}{2} = \frac{\begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}}{2}$$

$$= \frac{\begin{bmatrix} 2 & 6 & 8 \\ 6 & 4 & 4 \\ 8 & 4 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$ —— (1) a symmetric matrix

$$\frac{A-A^T}{2} = \frac{\begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}}{2}$$

$$= \frac{\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 2 \\ -2 & -2 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$ —— (2) a skew symmetric matrix

Adding (1) & (2)

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$$\begin{bmatrix}1 & 3 & 4\\3 & 2 & 2\\4 & 2 & 0\end{bmatrix}+\begin{bmatrix}0 & 1 & 1\\-1 & 0 & 1\\-1 & -1 & 0\end{bmatrix}=\begin{bmatrix}1 & 4 & 5\\2 & 2 & 3\\3 & 1 & 0\end{bmatrix}=A$$

Hence the result.

(c) Find the inverse of $\begin{bmatrix}1 & 2 & 3\\0 & 5 & 0\\2 & 4 & 3\end{bmatrix}$

$A=\begin{bmatrix}1 & 2 & 3\\0 & 5 & 0\\2 & 4 & 3\end{bmatrix}$

<u>Minors</u>

$m_{11}=\begin{vmatrix}5 & 0\\4 & 3\end{vmatrix}=15$

$m_{12}=\begin{vmatrix}0 & 0\\2 & 3\end{vmatrix}=0$

$m_{13}=\begin{vmatrix}0 & 5\\2 & 4\end{vmatrix}=-10$

$m_{21}=\begin{vmatrix}2 & 3\\4 & 3\end{vmatrix}=6$

$m_{22}=\begin{vmatrix}1 & 3\\2 & 3\end{vmatrix}=3-6=-3$

$m_{23}=\begin{vmatrix}1 & 2\\2 & 4\end{vmatrix}=0$

$m_{31}=\begin{vmatrix}2 & 3\\5 & 0\end{vmatrix}=-15$

$m_{32}=\begin{vmatrix}1 & 3\\0 & 0\end{vmatrix}=0$

$m_{33}=\begin{vmatrix}1 & 2\\0 & 5\end{vmatrix}=5$

Minor matrix of A= $\begin{bmatrix}15 & \mathbf{0} & -10\\ \mathbf{-6} & -3 & \mathbf{0}\\-15 & \mathbf{0} & 5\end{bmatrix}$

Cofactor matrix of A= $\begin{bmatrix}15 & 0 & -10\\6 & -3 & 0\\-15 & 0 & 5\end{bmatrix}$

Adj A = $\begin{bmatrix}15 & 6 & -15\\0 & -3 & 0\\-10 & 0 & 5\end{bmatrix}$

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$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{vmatrix} = 1\begin{vmatrix} 5 & 0 \\ 4 & 3 \end{vmatrix} - 2\begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} + 3\begin{vmatrix} 0 & 5 \\ 2 & 4 \end{vmatrix}$$

$$= 15 \text{ x } 2 \text{ x } 0 + 3 \text{ x } -10 = 15 - 30 = -15$$

$$\text{So inverse matrix} = \frac{\text{Adj.A}}{|A|} = \frac{\begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}}{-15}$$

IV.

(a) Find the value of 'p'

$2x + 3y + 9 = 0$

$4x + py + 13 = 0$

$px - 2y - 25 = 0$

Since the system is consistent,

$$\text{The eliminant, } \begin{vmatrix} 2 & 3 & 9 \\ 4 & p & 13 \\ p & -2 & -25 \end{vmatrix} = 0$$

$2(-25p + 26) - 3(-100 - 13p) + 9(-8 - p^2) = 0$

$-50p + 52 + 300 + 39p - 72 - 9p^2 = 0$

$-9p^2 - 11p + 280 = 0$

$$P = \frac{-11 \pm \sqrt{11^2 - 4 \text{ x } 9 \text{ x} -280}}{2 \text{ x } 9} = \frac{-11 \pm \sqrt{10201}}{18} = \frac{-11 \pm 101}{18}$$

$$P = \frac{-112}{18} = \text{ or } p = \frac{90}{18} = 5$$

(b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ show that $A^2 - 4A - 5I = 0$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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(c) If A = $\begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$ show that $(AB)^{-1} = B^{-1}A^{-1}$

AB = $\begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$

AB = $\begin{bmatrix} 47 & 34 \\ 22 & 16 \end{bmatrix}$

<u>To find $(AB)^{-1}$</u>

<u>Minors</u>

m_{11}=16

m_{12} = 22

m_{21} = 34

m_{22} = 47

∴ Minor matrix = $\begin{bmatrix} 16 & \mathbf{22} \\ \mathbf{34} & 47 \end{bmatrix}$

Cofactor of AB = $\begin{bmatrix} 16 & -22 \\ -34 & 47 \end{bmatrix}$

Adj AB = $\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}$

∴ |AB|= $\begin{vmatrix} 47 & 34 \\ 22 & 16 \end{vmatrix}$

= 752 – 748

= 4

$(AB)^{-1} = \frac{\text{adj AB}}{|AB|}$

$= \frac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4}$

<u>To find B^{-1}</u>

B = $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$

<u>Minors</u>

m_{11}=3

m_{12} = 4

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$m_{21} = 5$

$m_{22} = 7$

$\therefore$ Minor matrix = $\begin{bmatrix} 3 & \mathbf{4} \\ \mathbf{5} & 7 \end{bmatrix}$

Cofactor of B = $\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

AdjB = $\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$

$\therefore$ $|B| = \begin{vmatrix} 7 & 5 \\ 4 & 3 \end{vmatrix}$

$= 1$

$B^{-1} = \frac{\text{adj } B}{|B|}$

$= \frac{\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}}{1}$

<u>To find A^1</u>

$A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$

<u>Minors</u>

$m_{11}=2$

$m_{12} = 2$

$m_{21} = 3$

$m_{22} = 5$

$\therefore$ Minor matrix = $\begin{bmatrix} 2 & \mathbf{2} \\ \mathbf{3} & 5 \end{bmatrix}$

Cofactor of A = $\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

AdjA = $\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$

$\therefore$ $|A| = \begin{vmatrix} 5 & 3 \\ 2 & 2 \end{vmatrix}$

$= 10 – 6$

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$= 4$

$A^{-1} = \frac{\text{adj } A}{|A|}$

$= \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$

$B^{-1} \times A^{-1} = \frac{\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}}{1} \times \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$

$B^{-1} \times A^{-1} = \frac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4}$

$\therefore\ (AB)^{-1} = B^{-1}A^{-1}$

V.

(a) Expand $(x + 1/\sqrt{x})^5$ binomially

$(a + b)^n = a^n + nc_1\, a^{n-1}\, b + nc_2\, a^{n-2}\, b + \ldots\ldots\ldots + nc_n b^n$

$(x + 1/\sqrt{x})^5 = x^5 + 5c_1\, x^4(1/\sqrt{x}) + 5c_2\, x^3(1/\sqrt{x})^2 + 5c_3\, x^2(1/\sqrt{x})^3 + 5c_4 x\, (1/\sqrt{x})^4 + (1/\sqrt{x})^5$

$= x^5 + 5x^4\, (1/\sqrt{x}) + 10x^3\, (1/x) + 10x^2\, (1/x\sqrt{x}) + 5x\, (1/x^2) + (1/x^2\sqrt{x})$

$= x^5 + 5x^4\, x^{-1/2} + 10x^2 + 10x.x^{-1/2} + 5x^{-1} + x^{-5/2}$

$= x^5 + 5x^{7/2} + 10x^2 + 10x^{1/2} + 5x^{-1} + x^{-5/2}$

(b) Find the 10^{th} term in the expansion of $(x^2 + 1/x)^{20}$

$T_{r+1} = nc_r\, a^{n-r} b^r \quad , n = 20$

$T_{r+1} = 20c_r\, (x^2)^{20-r}\, (1/x^2)^r$

$= 20c_r\, x^{40-2r}\, x^{-2r}$

$= 20c_r\, x^{40-4r}$

ie, $T_{10} = 20c_9\, x^{40-36} = 20c_9\, x^4$

(c) Prove that $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$

$\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)\sin A}$

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$$= \frac{\sin^2 A+1+\cos^2 A+2\cos A}{(1+\cos A)\sin A}$$

$$= \frac{1+1+2\cos A}{(1+\cos A)\sin A} = \frac{2+2\cos A}{(1+\cos A)\sin A}$$

$$= \frac{2(1+\cos A)}{(1+\cos A)\sin A} = 2\text{cosec}A$$

VI.

(a) Find the middle term of $(2x + 3/x)^9$

$T_{r+1} = nc_r\, a^{n-r}b^r$, n= 9

$T_{r+1} = 9c_r(2x)^{9-r}(3/x)^r$

n + 1 = 10, even.

∴5^{th} term and 6^{th} term are the middle term.

ie,$T_5 = 9c_4\,(2x)^5(3/x)^4$

$= 9c_4 2^5 x^5 3^4 x^{-4}$

$=9c_4 2^5 x 3^4$

$= 9c_4 2^5\, 3^4\, x$

ie,$T_6 = 9c_5\,(2x)^4(3/x)^5$

$= 9c_5 2^4 x^4 3^5 x^{-5}$

$=9c_5 2^4 x^{-1} 3^5$

$= 9c_5 2^4\, 3^5\, x^{-1}$

(b) Find the coefficient of x^4 in the expansion of $(x^4 - 1/x^3)^{15}$

$T_{r+1} = nc_r\, a^{n-r}b^r$,   n = 11

$T_{r+1} = 15c_r\,(x^4)^{15-r}\,(-1/x^3)^r$

$= 15c_r x^{60-4r}\,(-1)^r x^{-3r}$

$= 15c_r x^{60-7r}\;(-1)^r$

Now 60 – 7r = 4

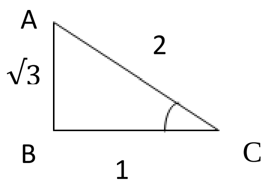
-7r =-56

r = 56/7 = 8

$T_9 = 15c_8 x^4\,(-1)^8$

∴Coefficient of x^4 is $15c_8$

(c) If sinA = $1/2$, A lies in first quadrant. Find all t- functions.



$AB = \sqrt{2^2 - 1^2} = \sqrt{3}$

sinA = ½

cosA = $\sqrt{3}/2$

secA = $2/\sqrt{3}$

tanA = $1/\sqrt{3}$

cotA = $\sqrt{3}$

cosecA = 2

VII.

(a) If tanA = 18/17, tanB 1/35, prove that A – B = 45^o

$$\tan(A + B) = \frac{\tan A + \tan B}{(1 - \tan A . \tan B)}$$

$$= \frac{\frac{18}{17} - \frac{1}{35}}{1 - \frac{18}{17} \text{ x } \frac{1}{35}}$$

$$= \frac{\frac{630 - 17}{595}}{\frac{595 + 18}{595}}$$

$$= \frac{\frac{613}{595}}{\frac{613}{595}} = 1$$

(b) Prove that $\frac{\cos(90+A).\sec(360+A).\tan(10-A)}{\sec(A-720).\sin(540+A).\cot(A-90)} = 1$

Cos(90 + A) = -sinA

Sec(360 + A) = 1/cos(360 + A) = 1/cosA = secA

Tan(180 – A) = -tanA

Sec(A – 720) = sec –(720 – A) = secA

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Sin(540 + A) = sin(360 + 180 + A) = sin(180 + A) = -sinA

Cot(A – 90) = cot(90 – A) = -tanA

Substituting the above values

$$\frac{\cos(90+A).\sec(360+A).\tan(10-A)}{\sec(A-720).\sin(540+A).\cot(A-90)} = \frac{-\sin A.\sec A.-\tan A}{\sec A.-\sec A.-\tan A} = 1$$

(c) Prove that in ΔABC, (a + b)sinC/2 = c.cos$\left(\frac{A-B}{2}\right)$.

LHS = (a + b).sinC/2

= (2RsinA + 2RsinB)sinC/2 $\left[\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

=2R(sinA + sinB)sinC/2

= 2R.2.sin$\left(\frac{A+B}{2}\right)$.cos$\left(\frac{A-B}{2}\right)$.sinC/2

= cos$\left(\frac{A-B}{2}\right)$.4R.sin$\left(\frac{A+B}{2}\right)$.sinC/2

= cos$\left(\frac{A-B}{2}\right)$.4R.sin (90 - $\frac{C}{2}$).sinC/2

= cos$\left(\frac{A-B}{2}\right)$.2R.(2cosC/2.sinC/2) [sinA = 2 sinA/2.cosA/2]

= cos$\left(\frac{A-B}{2}\right)$.2R.sinC

= cos$\left(\frac{A-B}{2}\right)$.c = RHS

VIII.

(a) Express $\sqrt{3}$cosx + sinx in the form of $R\sin(x + \alpha)$ where α is acute.

$\sqrt{3}$cosx + sin$x$ = R.sin$(x + \alpha)$

= R.sin$x$.cosα + Rcos$x$.sinα

Equating the similar terms on both sides,

$\sqrt{3}$cosx = Rsinα.cosα

Sin$x$ = Rsin$x$.cosα

===>$\sqrt{3}$ = Rsinα —— (1)

===> 1 = Rcosα —— (2)

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Squaring and adding (1) & (2)

$3 + 1 = R^2 \sin^2\alpha + \cos^2\alpha$

$4 = R^2$ ===> $R = \pm 2$

(1) ÷ (2) ===> $\sqrt{3} = \frac{\sin\alpha}{\cos\alpha}$ ===> $\tan\alpha = \sqrt{3}$

===> $\alpha = \tan^{-1}(\sqrt{3})$

===> $\alpha = 60^o$

(b) Show that cos20.cos40.cos60.cos80 = 1/16

We have cos60 = ½

ie, ½cos20.cos40.cos80 =

$= {}^1/_2.\cos20.\ {}^1/_2\ [\cos120 - \cos(-40)]$ [cosA.cosB = 1/2[cos(A+B)-cos(A-B)]

$= {}^1/_4.\cos20[{}^{-1}/_2 + \cos40]$

$= {}^{-1}/_8\ .\ \cos20 + {}^1/_4\ \cos20.\cos40$

$= {}^{-1}/_8\ .\ \cos20 + {}^1/_4\ x\ (\cos60 + \cos20)$

$= {}^{-1}/_8\ .\ \cos20 + {}^1/_8\ \cos60 + {}^1/_8\cos20$

$= {}^1/_8\ .\ \cos60 = {}^1/_8\ x\ {}^1/_2 = {}^1/_{16}$

(c) In a ΔABC, $A = 30^o, C = 45^o$, a = 2cm find c

$A = 30^o, C = 45^o$ $B = 180 - (A + C)$
$= 180 - 75$
$= 105^o$

Using sine rule we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

===> $\frac{a}{\sin A} = \frac{c}{\sin C}$

===> $\frac{2}{\sin 30} = \frac{c}{\sin 45}$

===> $c = \frac{2}{1/2} x ½$

$c = 4\ x\ \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828$

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IX.

(a) Solve ΔABC if a = 24.5cm, b = 18.6cm & c = 26.4cm

We have $\tan\frac{A}{2} = \frac{\sqrt{((s-b)(s-c))}}{s(s-a)}$ where $s = \frac{a+b+c}{2} = \frac{24.5+18.6+26.4}{2} = 34.75$

$= \frac{\sqrt{((34.75-18.6)(34.75-26.4))}}{34.75(34.75-24.5)}$

$= 0.6153$

$\tan\frac{A}{2} = 0.6153$

===> $A/2 = \tan^{-1}(0.6153) = 31^{o}36^{'}$

$A = 2 \times 31^{o}36^{'} = 63^{o}12^{'}$

$\tan B/2 = \frac{\sqrt{((s-a)(s-c))}}{s(s-c)} = \frac{\sqrt{((34.75-24.5)(34.75-26.4))}}{34.75(34.75-18.6)}$

$= 0.3905$

(b) The x – intercept of a line is 3times its y – intercept. The line passes through (-2, 3). Find its equation.

Intercept form of a line is

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{3b} + \frac{y}{b} = 1$

The equation passes through (-2, 3)

$\frac{-2}{3b} + \frac{3}{b} = 1$

$-2b + 9b = 3b^2$

$7b = 3b^2$

===> $b = 7/3$

∴ Equation of the line is

$\frac{x}{7} + \frac{y}{7/3} = 1$

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(c) Find the value of k so that the following lines are concurrent.

5x + 2y – 4 = 0
2x + ky + 11 = 0
3x – 4y – 18 = 0

Eliminant = 0

$$\begin{bmatrix} 5 & 2 & -4 \\ 2 & k & 11 \\ 3 & -4 & 18 \end{bmatrix} = 0$$

$$5\begin{vmatrix} k & 11 \\ -4 & -18 \end{vmatrix} - 2\begin{vmatrix} 2 & 11 \\ 3 & -18 \end{vmatrix} - 4\begin{vmatrix} 2 & k \\ 3 & -4 \end{vmatrix} = 0$$

5(-18k + 44) – 2(-36 – 33) – 4(-8 – 3k) = 0

-90k + 220 + 72 + 66 + 32 +12k = 0

-78k + 390 = 0

-78k = -390

$K = \frac{-390}{-78} = 5$

X.

(a) Solve ΔABC a = 4, b = 5 and C = 50^o

$\tan(\frac{A-B}{2}) = \frac{a-b}{a+b} \cot \frac{c}{2}$

$A - B = 2\tan^{-1}[\frac{a-b}{a+b}\cot\frac{c}{2}]$

$A - B = 2\tan^{-1}[\frac{4-5}{4+5} \cot 50/2]$

$= 2\tan^{-1}[\frac{-1}{9} \cot 25^o]$

$= 2\tan^{-1}[-0.111 \text{ x} 2.145]$

A – B = -26.7849 ——— (1)

A + B = 180 – 50 = 130 ——— (2)

Solving & (1) (2)

A – B = -26.7849

A + B = 130

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2A = 103.215

A = 51.6075 = $51^o 36^{'}$

B = 130 – A=130 - $51^o 36^{'}$= $78^o 24^{'}$

Now we have to find ‘C’, we have

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\frac{4}{\sin 51^o 36'} = \frac{c}{\sin 50^o}$$

$$===>c = \frac{\sin 50^o \text{ x } 4}{\sin 51^o 36'} = 3.909$$

(b) Find the equation to the line passing through the point of intersection of x – y + 1 = 0 and 2x +3y +2 = 0 and perpendicular to the line x + y – 6 = 0

Given x – y + 1 = 0

2x – 3y + 2 = 0

∴x – y = -1

2x – 3y = -2

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\therefore x = \frac{\begin{vmatrix} -1 & -1 \\ -2 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}}$$

$$= \frac{3-2}{-3+2} = \frac{1}{-1} = -1$$

$$Y = \frac{\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}}$$

$$= \frac{-2+2}{-3+2} = \frac{0}{-1} = 0$$

∴ Point of intersection = (-1, 0)

∴ Given line is

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x + y – 6 = 0

a = 1, b = 1, c = -6

∴ Perpendicular line is

bx – ay + k = 0

x – y + k = 0 ——— (1)

∴ (1) Pass through (-1, 0)

(1) ===> -1 – 0 + k = 0

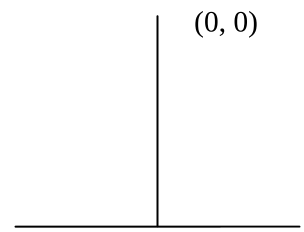
K = 1

∴ (1) ===> x – y + 1 = 0

(c) Find the foot of the perpendicular from the origin to the line 3x – 2y – 13 = 0

3x – 2y – 13 = 0

Slope of the line perpendicular to the line 3x – 2y = 13 is -2/3



$m = \frac{-3}{2} = \frac{3}{2}$

Equation of the line perpendicular to 3x – 2y = 13 is

y – 0 =-2/3 (x – 0)

y = -2/3 x

3y = -2x

2x + 3y = 0

Foot of the perpendicular is obtained by solutions

3x – 2y = 13 ——— (1)

2x + 3y = 0 ——— (2)

6x – 4y = 26

6x + 9y = 0

-13y = 26

$y = \frac{26}{-13} = -2$

3 ×*x* – 2 ×-2 = 13

3x = 13 – 4

x = 9/3 = 3

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∴ Foot perpendicularis (3, -2)

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