TED (10)-1002 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- OCTOBER, 2012

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time*: 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Which of the following matrices is singular

$\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}$, $\begin{bmatrix} 5 & -1 \\ 0 & 5 \end{bmatrix}$, $\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

$\begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix}$ = 6 + 6 = 12 $\begin{vmatrix} 5 & -1 \\ 0 & 5 \end{vmatrix}$ = 25 $\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix}$ = 6 – 6 = 0

$\therefore \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ is singular.

(b) Fins the value of 20 c_{18}

20 c_{18} = 20 $c_2 = \frac{20 \text{ x } 19}{2 \text{ x } 1}$ = 190

(c) State the identities for sin(A – B)

Sin(A – B) = sinAcosB–cosASinB

(d) State Napier’s formula.

In anyΔABC,

$$\text{Tan}\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right) \cot\frac{A}{2}$$

(e) Find the third side of a triangle given b = 2cm, c = 3 cm and A = 60^o

Using cosine rule $a^2 = b^2 + c^2 – 2bc.\cos A$

$a^2 = 2^2 + 3^2 – 2 \times 2 \times 3\cos 60$

$= 4 + 9 – 12 \times ½$

$= 13 – 6 = 17$

$a = \sqrt{7}$cm

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Solve the equations: 2x + y + z = 1, x -2y - z = 3/2&3 y -5z = 9 by finding the inverse of the coefficient matrix.

$2x + y + z = 1$

$x - 2y – z= \frac{3}{2}$

$3y – 5z = 9$

$\therefore A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

AX = B

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

<u>To find A^{-1}</u>

$m_{11} = \begin{vmatrix} -2 & -1 \\ 3 & -5 \end{vmatrix} = 13$

$m_{12} = \begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix}$ = -5

$m_{13} = \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix}$ = 3

$m_{21} = \begin{vmatrix} 1 & -1 \\ 3 & -5 \end{vmatrix}$= -8

$m_{22} = \begin{vmatrix} 2 & 1 \\ 0 & -5 \end{vmatrix}$ = -10

$m_{23} = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}$ = 6

$m_{31} = \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix}$ = 1

$m_{32} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$ = -3

$m_{33} = \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix}$ = -5

Minor matrix = $\begin{bmatrix} 13 & \mathbf{-5} & 3 \\ \mathbf{-8} & -10 & \mathbf{6} \\ 1 & \mathbf{-3} & 5 \end{bmatrix}$

Cofactor matrix = $\begin{bmatrix} 13 & 5 & 3 \\ 8 & -10 & -6 \\ 1 & 3 & -5 \end{bmatrix}$

Adjoint matrix = $\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$

∴ $|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$

$= 2\begin{vmatrix} -2 & -1 \\ 3 & -5 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix}$

= 2(10 + 3) – 1(-5 – 0) + 1(3 – 0)

= 2 x 13 – 1 x -5 + 3

= 26 + 5 + 3

= 34

∴ $A^{-1} = \dfrac{\text{Adj.A}}{|A|} = \dfrac{\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}}{34}$

$$\therefore X = A^{-1} \times B = \frac{\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}}{34}$$

$$= \frac{\begin{bmatrix} 34 \\ -17 \\ -51 \end{bmatrix}}{34} = \begin{bmatrix} 1 \\ -\frac{17}{34} \\ -\frac{3}{2} \end{bmatrix}$$

$x = 1 \qquad y = -\frac{17}{34} \qquad z = -\frac{3}{2}$

$x = 1 \qquad y = -\frac{1}{2} \qquad z = -\frac{3}{2}$

(b) If $A = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$ find A^{-1} and show that $A.A^{-1} = A^{-1}A = I$

$A = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$

<u>Cofactors</u>

$m_{11} = 5$

$m_{21} = -1$

$m_{12} = -6$

$m_{22} = 4$

Cofactor matrix $= \begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj.A}}{|A|} = \frac{\begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}}{14}$$

$$A.A^{-1} = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix} \frac{\begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}}{14}$$

$$= \frac{\begin{bmatrix} 20-6 & 0 \\ 30-30 & -6+20 \end{bmatrix}}{14}$$

$$= \frac{\begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}}{14}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{-1}A = \frac{\begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}}{14} \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 20-6 & 0 \\ -24+24 & -6+20 \end{bmatrix}}{14}$$

$$= \frac{\begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}}{14}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore A.A^{-1} = A^{-1} A = I$

(c) Find the middle term of $(2x + \frac{3}{x})^9$

$T_{r+1} = nc_r\ a^{n-r}b^r$, n= 9

$T_{r+1} = 9c_r(2x)^{9-r}(\frac{3}{x})^r$

n + 1 = 10, even.

$\therefore$ 5^{th} term and 6^{th} term are the middle term.

$ie, T_5 = 9c_4\ (2x)^5(\frac{3}{x})^4$

$= 9c_4 2^5 x^5 3^4 x^{-4}$

$=9c_4 2^5 x 3^4$

$= 9c_4 2^5\ 3^4\ x$

$ie, T_6 = 9c_5\ (2x)^4(\frac{3}{x})^5$

$= 9c_5 2^4 x^4 3^5 x^{-5}$

$=9c_5 2^4 x^{-1} 3^5$

$= 9c_5 2^4\ 3^5\ x^{-1}$

(d) Prove that $\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A.\sin A + \cos 7A.\sin 3A} = \tan 8A$

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A.\sin A + \cos 7A.\sin 3A}$$

$$= \frac{-\frac{1}{2}[\cos 12A - \cos 10A] - \frac{1}{2}[\cos 10A - \cos 4A]}{\frac{1}{2}[\sin 2A - \sin 10A] + \frac{1}{2}[\sin 10A - \sin 4A]}$$

$$= \frac{\frac{1}{2}[\cos 4A - \cos 12A]}{\frac{1}{2}[\sin 12A - \sin 4A]}$$

$$= \frac{-2\sin 8A.\sin(-4A)}{2\cos 8A.\sin(4A)} \qquad [\sin(-\theta) = -\sin\theta]$$

$= \tan 8A$

(e) State and prove Projection formula.

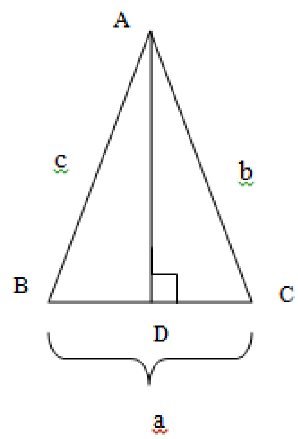
Statement

In any ΔABC, $a = b.\cos C + c.\cos B$

$b = a.\cos C + c.\cos A$

$c = a.\cos B + b.\cos A$

Proof



Consider a ΔABC . AD is the perpendicular to BC. We can see two right angled triangles in the figure. From ΔABC,

$\cos B = \frac{BD}{AB} = \frac{BD}{C}$

$\therefore BD = c.\cos B$ —— 1

From the ΔABC,

$\cos C = \frac{CD}{AC} = \frac{CD}{b}$

$\therefore CD = b.\cos C$ —— 2

Now consider the side BC.

BC = BD + DC

a = BD + DC

$a = c.\cos B + b.\cos C$ [from (1) & (2)]

Similarly we can prove the other two results.

(f) Solve ΔABC given b = 50cm, c = 80cm and A = 132°

We know that, $\tan(\frac{A-B}{2}) = \frac{a-b}{a+b}\cot\frac{C}{2}$

$\tan(\frac{B-C}{2}) = \frac{b-c}{b+c}\cot\frac{A}{2}$

$\tan(\frac{B-C}{2}) = \frac{b-c}{b+c}\cot\frac{A}{2}$

$\tan(\frac{B-C}{2}) = \frac{50-80}{50+80}\cot\frac{132}{2}$

$\tan(\frac{B-C}{2}) = \frac{-30}{130}\cot 66^o$

$B - C = 2\tan^{-1}[\frac{-30}{130}\cot\ 66^O\ ]$

$B - C = 2\tan^{-1}[\frac{-30}{130} x\ 0.44522]$

$= -11.732322$ ——— 1

$B + C = 180 - 132 = 48$ ——— 2

Solving 1 & 2

$2B = 36.267678$

$B = 18.1338 = 18^o8^{'}$

$C = 29.8662 = 29^o52^{'}$

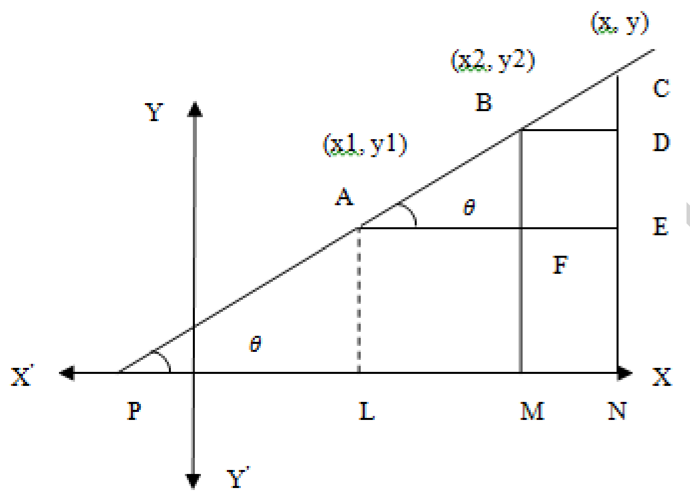
Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 132} = \frac{50}{\sin 18^o8'}$$

$===> a = {}^{50}/_{\sin 18^o8'}\ x\ \sin 132 = 119.385cm$

(g) Derive equation of a straight line of the form $\frac{y-y1}{y1-y2} = \frac{x-x1}{x1-x2}$



Let A be the point (x1, y1) and B be the point (x2, y2).

Then clearly ΔAFB&ΔAEC are similar.

$\frac{AF}{AE} = \frac{BF}{CE}$ ——— 1 [since the corresponding sides are proportional]

AF = x2 – x1 BF = y2 – y1

AE = x – x1 CE = y – y1

Then 1 becomes

$\frac{x2-x1}{x-x1} = \frac{y2-y1}{y-y1}$ ——— 2

===> $\frac{y-y1}{y1-y2} = \frac{x-x1}{x1-x2}$

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \end{bmatrix}$ C = $\begin{bmatrix} 2 & -1 & -1 \\ -2 & 2 & 3 \end{bmatrix}$ verify that A (B + C) = AB + AC

$$A(B+C) = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\left(\begin{bmatrix}1 & 2 & 3\\3 & 0 & 1\end{bmatrix} + \begin{bmatrix}2 & -1 & -1\\-2 & 2 & 3\end{bmatrix}\right)$$

$$= \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}_{2\times 2}\left(\begin{bmatrix}3 & 1 & 2\\1 & 2 & 4\end{bmatrix}\right)_{2\times 3}$$

$$= \begin{bmatrix}5 & 5 & 10\\7 & 4 & 8\end{bmatrix} \quad \text{—— } 1$$

$$AB + AC = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\3 & 0 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}2 & -1 & -1\\-2 & 2 & 3\end{bmatrix}$$

$$= \begin{bmatrix}7 & 2 & 5\\5 & 4 & 7\end{bmatrix} + \begin{bmatrix}-2 & 3 & 5\\2 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}5 & 5 & 10\\7 & 4 & 8\end{bmatrix} \quad \text{—— } 2$$

$\therefore$ A(B + C) = AB + AC

(b) If $A = \begin{bmatrix}1 & 0 & 5\\-2 & 1 & 6\\3 & 2 & 7\end{bmatrix}$ show that $A.A^T$ and $A^T.A$ are symmetric.

$$A.A^T = \begin{bmatrix}1 & 0 & 5\\-2 & 1 & 6\\3 & 2 & 7\end{bmatrix}\begin{bmatrix}1 & -2 & 3\\0 & 1 & 2\\5 & 6 & 7\end{bmatrix}$$

$$= \begin{bmatrix}26 & 28 & 28\\28 & 41 & 38\\38 & 38 & 62\end{bmatrix}$$

$$A^T.A = \begin{bmatrix}1 & -2 & 3\\0 & 1 & 2\\5 & 6 & 7\end{bmatrix}\begin{bmatrix}1 & -2 & 3\\0 & 1 & 2\\5 & 6 & 7\end{bmatrix}$$

$$= \begin{bmatrix}14 & 4 & 14\\4 & 5 & 20\\14 & 20 & 110\end{bmatrix}$$

Clearly $(A\,A^T)^T = \begin{bmatrix}26 & 28 & 28\\28 & 41 & 38\\38 & 38 & 62\end{bmatrix}$

$$(A^TA) = \begin{bmatrix}14 & 4 & 14\\4 & 5 & 20\\14 & 20 & 110\end{bmatrix}$$

$\therefore$ $A.A^T$ and $A^T.A$ are symmetric.

(c) Solve using determinants:

x + y – z = 3

2x +3y + z = 6

x – y – z = -3

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 6 & 3 & 1 \\ -3 & -1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix}}$$

$$= \frac{3(-3+1)-1(-6+3)+(-6+9)}{1(-3+1)-(-2-1)+(-2-3)}$$

$$= \frac{-6+3+3}{-2+3-5} \quad = \frac{0}{-4} = 0$$

$$y = \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 2 & 6 & 1 \\ 1 & -3 & 1 \end{vmatrix}}{-4}$$

$$= \frac{1(6+3)-3(2-1)+(-6-6)}{-4}$$

$$= \frac{9-3-12}{-4} = \frac{3}{2} = 1$$

$$z = \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & 6 \\ 1 & -1 & -3 \end{vmatrix}}{-4}$$

$$= \frac{(-9+6)-(-6-6)+3(-2-3)}{-4}$$

$$= -\frac{6}{4} = \frac{3}{2}$$

IV.

(a) If I is unit matrix of order 3 and A = $\begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$ find $A^2 - 3A + I$

$$A^2 = A.A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 18 \\ 21 & 12 & 30 \\ 3 & 9 & 15 \end{bmatrix}$$

$$A^2 - 3A + I = \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 18 \\ 21 & 12 & 30 \\ 3 & 9 & 15 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 & 38 \\ 24 & 49 & 102 \\ 24 & 20 & 47 \end{bmatrix}$$

(b) Shows that every square matrix can be expressed as the sum of two matrices of which one is symmetric and the other is skew symmetric.

Suppose A is a square matrix.

Then consider $X = \frac{A+A^T}{2}$

Then $X^T = \left(\frac{A+A^T}{2}\right)^T$

$$= \frac{A^T+(A^T)^T}{2} = \frac{A^T+A}{2} = X$$

$\therefore \frac{A+A^T}{2}$ is symmetric.

Consider $Y = \frac{A-A^T}{2} = \frac{A^T-(A^T)^T}{2}$

$$= \frac{A^T-A}{2} = -Y$$

$\therefore$ Y is skew symmetric.

Also $\frac{A+A^T}{2} + \frac{A-A^T}{2} = \frac{A+A^T+A-A^T}{2} = A$

Hence the result.

(c) Find k, if the following system of equation are consistent
$x + y + 1 = 0$, $x + 2y + 1 = 0$, $2x+3y + k = 0$

If the system is consistent then,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & k \end{vmatrix} = 0$$

$$1\begin{vmatrix} 2 & 1 \\ 3 & k \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & k \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 0$$

$1(2k - 3) - (k - 2) + (3 - 4) = 0$

$2k - 3 - k + 2 - 1 = 0$

$k - 2 = 0$

$k = 2$

V.

(a) Expand $(x^2 - \frac{3}{x})^5$ using binomial theorem.

$(a + b)^n = a^n + nc_1\, a^{n-1}\, b + nc_2\, a^{n-2}\, b + \ldots\ldots\ldots + nc_n b^n$

$(a - b)^n = a^n - nc_1\, a^{n-1}\, b + nc_2\, a^{n-2}\, b - \ldots\ldots\ldots + (-1)^n b^n$

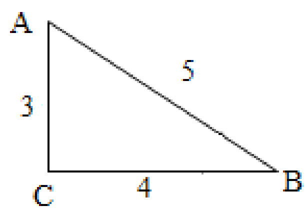
Now

$$\left(x^2 - \frac{3}{x}\right)^5 = (x^2)^5 - 5c_1 (x^2)^4 \left(\frac{3}{x}\right) + 5c_2 (x^2)^3 \left(\frac{3}{x}\right)^2$$
$$- 5c_3 (x^2)^2 \left(\frac{3}{x}\right)^3 + 5c_4 (x^2)^1 \left(\frac{3}{x}\right)^4 - \left(\frac{3}{x}\right)^5$$

$$= x^{10} - 5x^8\left(\frac{3}{x}\right) + 10x^3\left(\frac{9}{x^2}\right) - 10x^4\left(\frac{27}{x^3}\right) + 5x^2\left(\frac{81}{x^4}\right) - \left(\frac{243}{x^5}\right)$$

$$= x^{10} - 15x^7 + 90x^4 - 270x + 405x^{-2} 243x^{-5}$$

(b) If $\cos x = -4/5$ & x is in the second quadrant. Find the remaining trigonometric functions of x.



AB = $\sqrt{AC^2 - BC^2} = \sqrt{25 - 16} = 3$

$\sin x = \dfrac{3}{5}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{3}{-4} = -\dfrac{3}{4}$

$\operatorname{cosec} x = \dfrac{5}{3}$

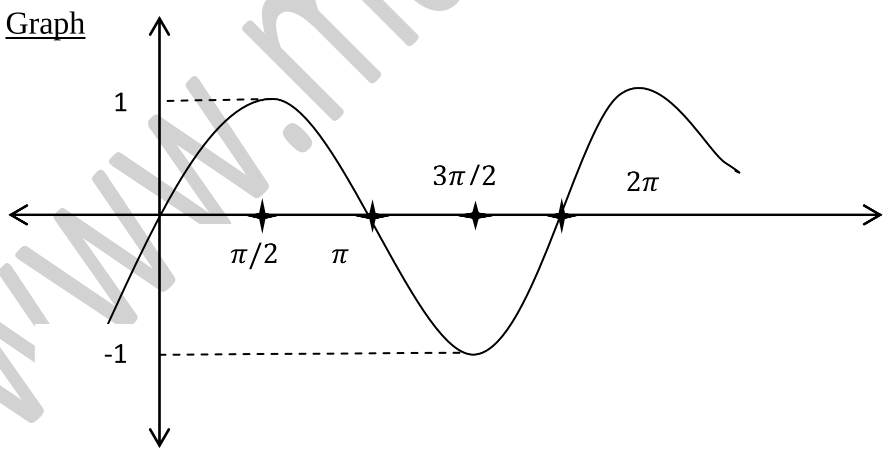
$\cot x = -\dfrac{4}{3}$

$\sec x = -\dfrac{5}{4}$

(c) Draw the graph of y = $\sin 3x$

X : 0	90^o $\frac{\pi}{2}$	180^o π	270^o $3\frac{\pi}{2}$	360^o 2π
Y : 0	1	0	-1	0

<u>Graph</u>



VI.

(a) Find the coefficient of x^{11} in the expansion of $(x^4 - \frac{1}{x^3})^{15}$

$T_{r+1} = (-1)^r nc_r\, a^{n-r} b^r$,

$T_{r+1} = 15c_r\, (x^4)^{15-r} (\frac{1}{x^3})^r$

$=(-1)^r 15c_r x^{60-4r}\ x^{-3r}$

$= (-1)^r 15c_r x^{60-7r}$

Now $60 - 7r = 11$

$-7r = -60 + 11$

$-7\ r = -49$

$r = 7$

$T_8=(-1)^7 15c_7 x^{11} \quad = -\ 15c_7$

$\therefore$ Coefficient of x^{11} is $-15c_7$

(b) Write the sign of following

(i) Cot(1080 + x) $\quad 0 < x < 90$
(ii) Cot(– 97)
(iii) Sec(360 – x) $\quad 0 < x < 90$

(i) Cot(1080 + x) = cot(12 x 90 + x) $\quad$ n = 12 even
= cot*x* $\quad$ (1st quadrant)

$\therefore$ Sign is positive.

(ii) Cot (-97) = -cot (1 x 90 + 7) $\quad$ n = 1 odd.

= -tan7 $\quad$ but $97^o \in 2^{nd}$ quadrant, $\therefore$ sign is –ve
Sign is positive.

(iii) Sec(360 – x) $\quad 0 < x < 90$

$\text{Sec}(360 - x) = \frac{1}{\cos(360-x)} = \frac{1}{\cos(4 \times 90-x)}$

$= \frac{1}{\cos x}$

Sign is positive.

(c) Prove that $\tan^2 30 + \tan^2 45 + \tan^2 60 = 13/3$

Tan30 = $1/\sqrt{3}$ $\quad$ tan45 = 1 $\quad$ tan60 = $\sqrt{3}$

$\tan^2 30 + \tan^2 45 + \tan^2 60 = (1/\sqrt{3})^2 + (1)^2 + (\sqrt{3})^2$

$= 1/3 + 1 + 3 = 13/3$

VII.

(a) Express $\sqrt{3}\cos x + \sin x$ in the form of $R\sin(x + \alpha)$

$\sqrt{3}\cos x + \sin x = R.\sin(x + \alpha)$

$= R.\sin x.\cos\alpha + R\cos x.\sin\alpha$

Equating the similar terms on both sides,

$\sqrt{3}\cos x = R\sin\alpha.\cos\alpha$

$\text{Sin}x = R\sin x.\cos\alpha$

$\Longrightarrow \sqrt{3} = R\sin\alpha$ 1

$\Longrightarrow 1 = R\cos\alpha$ 2

Squaring and adding 1 & 2

$3 + 1 = R^2 \sin^2\alpha + \cos^2\alpha$

$4 = R^2$ $\Longrightarrow R = \pm 2$

1 ÷ 2 $\Longrightarrow \sqrt{3} = \frac{\sin\alpha}{\cos\alpha} \Longrightarrow \tan\alpha = \sqrt{3}$

$\Longrightarrow \alpha = \tan^{-1}(\sqrt{3})$

$\Longrightarrow \alpha = 60^o$

(b) Prove identity for sin3A

$\text{Sin}3A = \sin(2A + A)$

$= \sin 2A\cos A + \cos 2A\sin A$

$= 2\sin A\cos A.\cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A\cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

(c) Show that $2(bc.\cos A + ca.\cos B + ab.\cos C) = a^2 + b^2 + c^2$

Using cosine rule,

$a^2 = b^2 + c^2 - 2bc.\cos A$ 1

$b^2 = a^2 + c^2 - 2ac.\cos B$ 2

$c^2 = a^2 + b^2 - 2ab.\cos C$ 3

Adding 1 , 2 and 3 we get

$a^2 + b^2 + c^2 = 2(b^2 + c^2 + a^2) - 2(bc.\cos A + ac.\cos B + ab.\cos C)$

$\Longrightarrow 2(b^2 + c^2 + a^2) - (a^2 + b^2 + c^2) = 2(bc.\cos A + ac.\cos b + ab.\cos C)$

$\Longrightarrow a^2 + b^2 + c^2 \quad = \quad 2(bc.\cos A + ca.\cos B + ab.\cos C)$

VIII.

(a) Show that tan15 + cot15 = 4

Tan15 = tan (45 - 30)

$$= \frac{\tan 45 - \tan 30}{1 + \tan 45.\tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\text{Cot15} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\tan 15 + \cot 15 = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2 + \left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2 - 1^2}$$

$$= \frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{2} \quad = 8/2 = 4$$

(b) If tanA = 0.38 find tan2A

$$\text{Tan2A} = \frac{2\tan A}{1 - \tan^2 A} = \frac{2 \text{ x } (0.38)}{1-(0.38)^2}$$

$$= 0.8882$$

(c) Prove that cos55+.cos65+.cos175 = 0

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

$\cos 55 + \cos 65 = 2\cos\frac{120}{2}.\cos\frac{-10}{2}$

$= 2 \times \cos 60.\cos 5$ [cos(-A) = cosA]

$= 2 \times ½ \times \cos 5 = \cos 5$

Then cos55 + cos65 + cos175

$= \cos 5 + \cos 175 = 2\cos 90.\cos 85 = 0$

IX.

(a) Find the equation of the line passes through the point (-4, 5) and whose intercepts are equal in magnitude but opposite in sign.

Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

Let $\frac{x}{a} + \frac{y}{-a} = 1$ ——— 1

Since 1 passes through (-4, 5)

$\frac{-4}{a} + \frac{5}{-a} = 1$

$\frac{-4}{a} - \frac{5}{a} = 1$ ===> $\frac{-9}{a} = 1$

===> a = -9

The equation of the line is

$\frac{x}{-9} + \frac{y}{9} = 1$

===> $\frac{-x}{9} + \frac{y}{9} = 1$

===> -x + y = 9

===> y – x = 9

(b) Find the slop and intercept of the line 3x +y –15 = 0

3x + 4y = 15 ——— 1

Slop = $\frac{-a}{b} = \frac{-3}{4}$

From 1 we have,

$\frac{3x}{15}+\frac{4y}{15}= 1$

$\frac{x}{5}+\frac{y}{\frac{15}{4}}= 1$

$\therefore$x intercept = 5

y intercept = $\frac{15}{4}$

(a) Find the angle between the lines ax + by + c = 0 and dx + ey + f = 0

ax + by + c = 0 ——— 1

dx + ey + f = 0 ——— 2

Slop of ax + by + c = 0 is $m_1 = \frac{-a}{b}$

Slop of dx + ey + f = 0 is $m_2 = \frac{-d}{e}$

Angle between 1 and 2 is

$$\text{Tan}\theta = \left|\frac{m1-m2}{1+m1m2}\right|$$

$$= \left|\frac{-\frac{a}{b}+\frac{d}{e}}{1+-\frac{a}{b}x\frac{d}{e}}\right| = \left|\frac{-ae+bd}{be+ad}\right|$$

$$= \left|\frac{bd-ae}{be+ad}\right|$$

$$\theta = \tan^{-1}\left(\left|\frac{bd-ae}{be+ad}\right|\right)$$

X.

(a) Find the condition for two lines (i) Parallel (ii) Perpendicular.

Consider two lines ax + by + c = 0 and dx + ey + f = 0

Case I(Parallel)

Slope of ax + by + c = 0 is $m_1 = \frac{-a}{b}$

Slop of $dx + ey + f = 0$ is $m_2 = \frac{-d}{e}$

<u>Condition for parallism.</u>

$m_1 = m_2$

$\frac{-a}{b} = \frac{-d}{e}$ ===>$ae = bd$ or $ae - bd = 0$

<u>Case II (perpendicular)</u>

$m_1 = \frac{-a}{b}$ $m_2 = \frac{-d}{e}$ $m_1.m_2 = -1$

$m_1 \text{ x } m_2 = \frac{-a}{b} \text{x} \frac{-d}{e} = -1$

$\frac{ad}{be} = -1$

$ad = -be$

===>$ad + be = 0$

(b) The straight line through (4, 3) makes intercepts 4a and 3a on the X and Y axis respectively find a.

Intercept form $= \frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{4a} + \frac{y}{3a} = 1$ ——— 1

1 Passes through (4, 3)

Then $\frac{4}{4a} + \frac{3}{3a} = 1$

===>$\frac{1}{a} + \frac{1}{a} = 1$

===>$\frac{2}{a} = 1$ ===>$a = 2$

(c) Show that the point of intersection of the lines $5x - 12y = 2$ and $3x - 8y + 2 = 0$ lies on $2x - 3y = 8$.

$5x - 12y = 2$ ——— 1

$3x - 8y = -2$ ——— 2

Solving& 1 2

$$x = \frac{\begin{vmatrix} 2 & -12 \\ -2 & -8 \end{vmatrix}}{\begin{vmatrix} 5 & -12 \\ 3 & -8 \end{vmatrix}} = \frac{-16-24}{-40+36} = \frac{-40}{-4} = 10$$

Then 1 becomes

$$5 \times 10 - 12y = 2$$

$$50 - 12y = 2$$

$$-12y = -48$$

$$y = 4$$

Put $x = 10$ & $y = 4$ in $2x - 3y = 8$

$2 \times 10 - 3 \times 4 = 20 - 12 = 8$. Satisfied.

Hence the result.