

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/  
TECHNOLIGY- OCTOBER, 2012

TECHNICAL MATHEMATICS- I  
(Common – Except DCP and CABM)

[Time: 3 hours

(Maximum marks: 100)

Marks

PART –A  
(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Which of the following matrices is singular

$$\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}, \begin{bmatrix} 5 & -1 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix} = 6 + 6 = 12$$

$$\begin{vmatrix} 5 & -1 \\ 0 & 5 \end{vmatrix} = 25$$

$$\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = 6 - 6 = 0$$

$\therefore \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$  is singular.

(b) Find the value of  ${}^{20}C_{18}$

$${}^{20}C_{18} = 20 \quad {}^{20}C_2 = \frac{20 \times 19}{2 \times 1} = 190$$

(c) State the identities for  $\sin(A - B)$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

(d) State Napier's formula.

In any  $\Delta ABC$ ,

$$\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right) \cot\frac{A}{2}$$

(e) Find the third side of a triangle given  $b = 2\text{cm}$ ,  $c = 3\text{ cm}$  and  $A = 60^\circ$

Using cosine rule

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$a^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \cos 60$$

$$= 4 + 9 - 12 \times \frac{1}{2}$$

$$= 13 - 6 = 7$$

$$a = \sqrt{7}\text{cm}$$

### PART -B

Answer any five questions. Each question carries 6 marks

II.

(a) Solve the equations:  $2x + y + z = 1$ ,  $x - 2y - z = \frac{3}{2}$  &  $3y - 5z = 9$  by finding the inverse of the coefficient matrix.

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

$$\therefore A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$$AX = B$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

To find  $A^{-1}$

$$m_{11} = \begin{vmatrix} -2 & -1 \\ 3 & -5 \end{vmatrix} = 13$$

$$m_{12} = \begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix} = -5$$

$$m_{13} = \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} = 3$$

$$m_{21} = \begin{vmatrix} 1 & -1 \\ 3 & -5 \end{vmatrix} = -8$$

$$m_{22} = \begin{vmatrix} 2 & 1 \\ 0 & -5 \end{vmatrix} = -10$$

$$m_{23} = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6$$

$$m_{31} = \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} = 1$$

$$m_{32} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$$

$$m_{33} = \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -5$$

$$\text{Minor matrix} = \begin{bmatrix} 13 & -5 & 3 \\ -8 & -10 & 6 \\ 1 & -3 & 5 \end{bmatrix}$$

$$\text{Cofactor matrix} = \begin{bmatrix} 13 & 5 & 3 \\ 8 & -10 & -6 \\ 1 & 3 & -5 \end{bmatrix}$$

$$\text{Adjoint matrix} = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$\begin{aligned} \therefore |A| &= \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix} \\ &= 2 \begin{vmatrix} -2 & -1 \\ 3 & -5 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix} + 1 \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} \\ &= 2(10 + 3) - 1(-5 - 0) + 1(3 - 0) \\ &= 2 \times 13 - 1 \times -5 + 3 \\ &= 26 + 5 + 3 \\ &= 34 \end{aligned}$$

$$\therefore A^{-1} = \frac{\text{Adj.}A}{|A|} = \frac{\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}}{34}$$

$$\therefore X = A^{-1} \times B = \frac{\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 9 \end{bmatrix}}{34}$$

$$= \frac{\begin{bmatrix} 34 \\ -17 \\ -51 \end{bmatrix}}{34} = \begin{bmatrix} 1 \\ -\frac{17}{34} \\ -\frac{3}{2} \end{bmatrix}$$

$$x = 1 \quad y = -\frac{17}{34} \quad z = -\frac{3}{2}$$

$$x = 1 \quad y = -\frac{1}{2} \quad z = -\frac{3}{2}$$

(b) If  $A = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$  find  $A^{-1}$  and show that  $A \cdot A^{-1} = A^{-1} \cdot A = I$

$$A = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$$

Cofactors

$$m_{11} = 5$$

$$m_{21} = -1$$

$$m_{12} = -6$$

$$m_{22} = 4$$

$$\text{Cofactor matrix} = \begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj.}A}{|A|} = \frac{\begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}}{14}$$

$$A \cdot A^{-1} = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix} \frac{\begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}}{14}$$

$$= \frac{\begin{bmatrix} 20-6 & 0 \\ 30-30 & -6+20 \end{bmatrix}}{14}$$

$$= \frac{\begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}}{14}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\begin{aligned}
 A^{-1}A &= \frac{\begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}}{14} \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix} \\
 &= \frac{\begin{bmatrix} 20-6 & 0 \\ -24+24 & -6+20 \end{bmatrix}}{14} \\
 &= \frac{\begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}}{14} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

$$\therefore A \cdot A^{-1} = A^{-1} A = I$$

(c) Find the middle term of  $(2x + \frac{3}{x})^9$

$$T_{r+1} = n c_r a^{n-r} b^r, n = 9$$

$$T_{r+1} = 9c_r (2x)^{9-r} \left(\frac{3}{x}\right)^r$$

$$n + 1 = 10, \text{ even.}$$

$\therefore$  5<sup>th</sup> term and 6<sup>th</sup> term are the middle term.

$$\text{ie, } T_5 = 9c_4 (2x)^5 \left(\frac{3}{x}\right)^4$$

$$= 9c_4 2^5 x^5 3^4 x^{-4}$$

$$= 9c_4 2^5 x 3^4$$

$$= 9c_4 2^5 3^4 x$$

$$\text{ie, } T_6 = 9c_5 (2x)^4 \left(\frac{3}{x}\right)^5$$

$$= 9c_5 2^4 x^4 3^5 x^{-5}$$

$$= 9c_5 2^4 x^{-1} 3^5$$

$$= 9c_5 2^4 3^5 x^{-1}$$

(d) Prove that  $\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \cdot \sin A + \cos 7A \cdot \sin 3A} = \tan 8A$

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \cdot \sin A + \cos 7A \cdot \sin 3A}$$

$$= \frac{-\frac{1}{2}[\cos 12A - \cos 10A] - \frac{1}{2}[\cos 10A - \cos 4A]}{\frac{1}{2}[\sin 2A - \sin 10A] + \frac{1}{2}[\sin 10A - \sin 4A]}$$

$$= \frac{\frac{1}{2}[\cos 4A - \cos 12A]}{\frac{1}{2}[\sin 12A - \sin 4A]}$$

$$= \frac{-2\sin 8A \cdot \sin(-4A)}{2\cos 8A \cdot \sin(4A)}$$

$$[\sin(-\theta) = -\sin \theta]$$

$$= \tan 8A$$

(e) State and prove Projection formula.

Statement

In any  $\Delta ABC$ ,  
 $a = b \cdot \cos C + c \cdot \cos B$   
 $b = a \cdot \cos C + c \cdot \cos A$   
 $c = a \cdot \cos B + b \cdot \cos A$

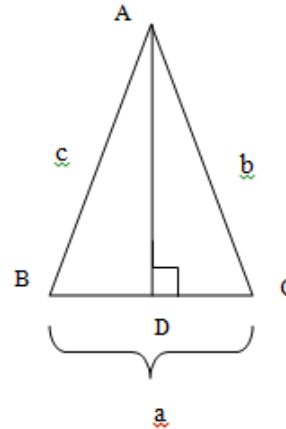
Proof

Consider a  $\Delta ABC$ .  $AD$  is the perpendicular to  $BC$ . We can see two right angled triangles in the figure. From  $\Delta ABD$ ,

$$\cos B = \frac{BD}{AB} = \frac{BD}{c}$$

$$\therefore BD = c \cdot \cos B$$

①



From the  $\Delta ADC$ ,

$$\cos C = \frac{CD}{AC} = \frac{CD}{b}$$

$$\therefore CD = b \cdot \cos C$$

②

Now consider the side  $BC$ .

$$BC = BD + DC$$

$$a = BD + DC$$

$$a = c \cdot \cos B + b \cdot \cos C \quad [\text{from (1) \& (2) }]$$

Similarly we can prove the other two results.

(f) Solve  $\Delta ABC$  given  $b = 50\text{cm}$ ,  $c = 80\text{cm}$  and  $A = 132^\circ$

$$\text{We know that, } \tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{50-80}{50+80} \cot \frac{132}{2}$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{-30}{130} \cot 66^\circ$$

$$B - C = 2 \tan^{-1} \left[ \frac{-30}{130} \cot 66^\circ \right]$$

$$B - C = 2 \tan^{-1} \left[ \frac{-30}{130} \times 0.44522 \right]$$

$$= -11.732322$$

$$B + C = 180 - 132 = 48$$

Solving (1) & (2)

$$2B = 36.267678$$

$$B = 18.1338 = 18^\circ 8'$$

$$C = 29.8662 = 29^\circ 52'$$

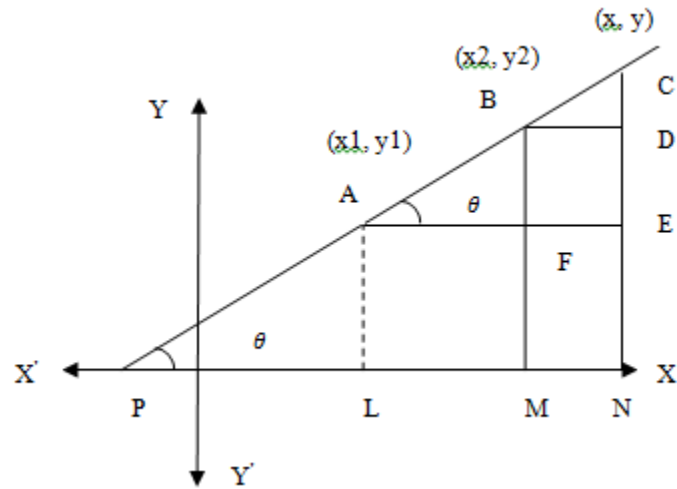
Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 132} = \frac{50}{\sin 18^\circ 8'}$$

$$\implies a = \frac{50}{\sin 18^\circ 8'} \times \sin 132 = 119.385\text{cm}$$

(g) Derive equation of a straight line of the form  $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$



Let A be the point  $(x_1, y_1)$  and B be the point  $(x_2, y_2)$ .

Then clearly  $\Delta AFB$  &  $\Delta AEC$  are similar.

$$\frac{AF}{AE} = \frac{BF}{CE} \quad \text{--- (1) [since the corresponding sides are proportional]}$$

$$AF = x_2 - x_1 \quad BF = y_2 - y_1$$

$$AE = x - x_1 \quad CE = y - y_1$$

Then (1) becomes

$$\frac{x_2 - x_1}{x - x_1} = \frac{y_2 - y_1}{y - y_1} \quad \text{--- (2)}$$

$$\implies \frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

### PART - C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$   $B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \end{bmatrix}$   $C = \begin{bmatrix} 2 & -1 & -1 \\ -2 & 2 & 3 \end{bmatrix}$  verify that  $A(B + C) = AB + AC$



$$\begin{aligned}
 A(B+C) &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left( \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -1 \\ -2 & 2 & 3 \end{bmatrix} \right) \\
 &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2 \times 2} \left( \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \right)_{2 \times 3} \\
 &= \begin{bmatrix} 5 & 5 & 10 \\ 7 & 4 & 8 \end{bmatrix} \quad \text{--- (1)}
 \end{aligned}$$

$$\begin{aligned}
 AB+AC &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -2 & 2 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 7 & 2 & 5 \\ 5 & 4 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 3 & 5 \\ 2 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 5 & 5 & 10 \\ 7 & 4 & 8 \end{bmatrix} \quad \text{--- (2)}
 \end{aligned}$$

$$\therefore A(B+C) = AB+AC$$

(b) If  $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix}$  show that  $A \cdot A^T$  and  $A^T \cdot A$  are symmetric.

$$\begin{aligned}
 A \cdot A^T &= \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} 26 & 28 & 28 \\ 28 & 41 & 38 \\ 38 & 38 & 62 \end{bmatrix}
 \end{aligned}$$

$$A^T \cdot A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 4 & 14 \\ 4 & 5 & 20 \\ 14 & 20 & 110 \end{bmatrix}$$

$$\text{Clearly } (A \cdot A^T)^T = \begin{bmatrix} 26 & 28 & 28 \\ 28 & 41 & 38 \\ 38 & 38 & 62 \end{bmatrix}$$

$$(A^T \cdot A) = \begin{bmatrix} 14 & 4 & 14 \\ 4 & 5 & 20 \\ 14 & 20 & 110 \end{bmatrix}$$

$\therefore A \cdot A^T$  and  $A^T \cdot A$  are symmetric.

(c) Solve using determinants:

$$x + y - z = 3$$

$$2x + 3y + z = 6$$

$$x - y - z = -3$$

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$$

$$\begin{aligned} x &= \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 6 & 3 & 1 \\ -3 & -1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix}} \\ &= \frac{3(-3+1) - 1(-6+3) + (-6+9)}{1(-3+1) - (-2-1) + (-2-3)} \\ &= \frac{-6+3+3}{-2+3-5} = \frac{0}{-4} = 0 \end{aligned}$$

$$\begin{aligned} y &= \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 2 & 6 & 1 \\ 1 & -3 & 1 \end{vmatrix}}{-4} \\ &= \frac{1(6+3) - 3(2-1) + (-6-6)}{-4} \\ &= \frac{9-3-12}{-4} = \frac{3}{2} = 1 \end{aligned}$$

$$\begin{aligned} z &= \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & 6 \\ 1 & -1 & -3 \end{vmatrix}}{-4} \\ &= \frac{(-9+6) - (-6-6) + 3(-2-3)}{-4} \\ &= -\frac{6}{4} = \frac{3}{2} \end{aligned}$$

IV.

(a) If I is unit matrix of order 3 and  $A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$  find  $A^2 - 3A + I$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix}$$

$$3A = 3 \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 18 \\ 21 & 12 & 30 \\ 3 & 9 & 15 \end{bmatrix}$$

$$A^2 - 3A + I = \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 18 \\ 21 & 12 & 30 \\ 3 & 9 & 15 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 & 38 \\ 24 & 49 & 102 \\ 24 & 20 & 47 \end{bmatrix}$$

(b) Shows that every square matrix can be expressed as the sum of two matrices of which one is symmetric and the other is skew symmetric.

Suppose A is a square matrix.

$$\text{Then consider } X = \frac{A + A^T}{2}$$

$$\text{Then } X^T = \left( \frac{A + A^T}{2} \right)^T$$

$$= \frac{A^T + (A^T)^T}{2} = \frac{A^T + A}{2} = X$$

$\therefore \frac{A + A^T}{2}$  is symmetric.

$$\text{Consider } Y = \frac{A - A^T}{2} = \frac{A^T - (A^T)^T}{2}$$

$$= \frac{A^T - A}{2} = -Y$$

$\therefore Y$  is skew symmetric.

$$\text{Also } \frac{A + A^T}{2} + \frac{A - A^T}{2} = \frac{A + A^T + A - A^T}{2} = A$$

Hence the result.

(c) Find k, if the following system of equation are consistent

$$x + y + 1 = 0, x + 2y + 1 = 0, 2x + 3y + k = 0$$

If the system is consistent then,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & k \end{vmatrix} = 0$$

$$1 \begin{vmatrix} 2 & 1 \\ 3 & k \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 2 & k \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 0$$

$$1(2k - 3) - (k - 2) + (3 - 4) = 0$$

$$2k - 3 - k + 2 - 1 = 0$$

$$k - 2 = 0$$

$$k = 2$$

V.

(a) Expand  $(x^2 - \frac{3}{x})^5$  using binomial theorem.

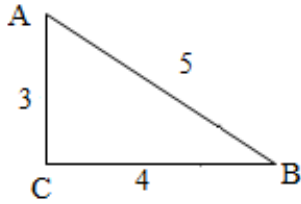
$$(a + b)^n = a^n + n c_1 a^{n-1} b + n c_2 a^{n-2} b^2 + \dots + n c_n b^n$$

$$(a - b)^n = a^n - n c_1 a^{n-1} b + n c_2 a^{n-2} b^2 - \dots + (-1)^n b^n$$

Now

$$\begin{aligned} (x^2 - \frac{3}{x})^5 &= (x^2)^5 - 5c_1 (x^2)^4 (\frac{3}{x}) + 5c_2 (x^2)^3 (\frac{3}{x})^2 \\ &\quad - 5c_3 (x^2)^2 (\frac{3}{x})^3 + 5c_4 (x^2)^1 (\frac{3}{x})^4 - (\frac{3}{x})^5 \\ &= x^{10} - 5x^8 (\frac{3}{x}) + 10x^3 (\frac{9}{x^2}) - 10x^4 (\frac{27}{x^3}) + 5x^2 (\frac{81}{x^4}) - (\frac{243}{x^5}) \\ &= x^{10} - 15x^7 + 90x^4 - 270x + 405x^{-2} - 243x^{-5} \end{aligned}$$

(b) If  $\cos x = -4/5$  & x is in the second quadrant. Find the remaining trigonometric functions of x.



$$AB = \sqrt{AC^2 - BC^2} = \sqrt{25 - 16} = 3$$

$$\sin x = \frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{3}{-4} = -\frac{3}{4}$$

$$\operatorname{cosec} x = \frac{5}{3}$$

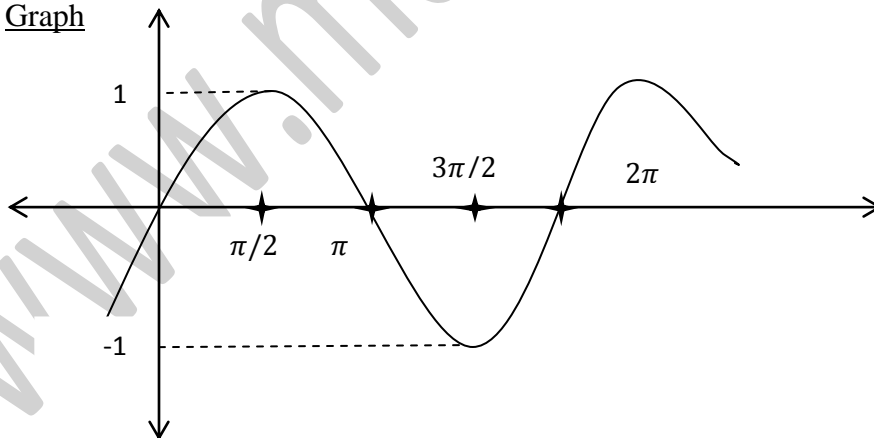
$$\cot x = -\frac{4}{3}$$

$$\sec x = -\frac{5}{4}$$

(c) Draw the graph of  $y = \sin 3x$

|       |                 |       |                  |        |
|-------|-----------------|-------|------------------|--------|
| X : 0 | 90°             | 180°  | 270°             | 360°   |
|       | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| Y : 0 | 1               | 0     | -1               | 0      |

Graph



VI.

(a) Find the coefficient of  $x^{11}$  in the expansion of  $(x^4 - \frac{1}{x^3})^{15}$

$$T_{r+1} = (-1)^r n C_r a^{n-r} b^r,$$

$$\begin{aligned} T_{r+1} &= 15C_r (x^4)^{15-r} \left(\frac{1}{x^3}\right)^r \\ &= (-1)^r 15C_r x^{60-4r} x^{-3r} \\ &= (-1)^r 15C_r x^{60-7r} \end{aligned}$$

$$\text{Now } 60 - 7r = 11$$

$$-7r = -60 + 11$$

$$-7r = -49$$

$$r = 7$$

$$T_8 = (-1)^7 15C_7 x^{11} = -15C_7$$

$\therefore$  Coefficient of  $x^{11}$  is  $-15C_7$

(b) Write the sign of following

(i)  $\text{Cot}(1080 + x)$        $0 < x < 90$

(ii)  $\text{Cot}(-97)$

(iii)  $\text{Sec}(360 - x)$        $0 < x < 90$

(i)  $\text{Cot}(1080 + x) = \text{cot}(12 \times 90 + x)$      $n = 12$  even  
 $= \text{cot} x$     (1<sup>st</sup> quadrant)

$\therefore$  Sign is positive.

(ii)  $\text{Cot}(-97) = -\text{cot}(1 \times 90 + 7)$        $n = 1$  odd.

$= -\tan 7$       but  $97^\circ \in 2^{\text{nd}}$  quadrant,  $\therefore$  sign is -ve

Sign is positive.

(iii)  $\text{Sec}(360 - x)$        $0 < x < 90$

$$\begin{aligned} \text{Sec}(360 - x) &= \frac{1}{\cos(360 - x)} = \frac{1}{\cos(4 \times 90 - x)} \\ &= \frac{1}{\cos x} \end{aligned}$$

Sign is positive.

(c) Prove that  $\tan^2 30 + \tan^2 45 + \tan^2 60 = 13/3$

$$\tan 30 = 1/\sqrt{3} \quad \tan 45 = 1 \quad \tan 60 = \sqrt{3}$$

$$\tan^2 30 + \tan^2 45 + \tan^2 60 = (1/\sqrt{3})^2 + (1)^2 + (\sqrt{3})^2$$

$$= 1/3 + 1 + 3 = 13/3$$

VII.

(a) Express  $\sqrt{3}\cos x + \sin x$  in the form of  $R\sin(x + \alpha)$

$$\begin{aligned}\sqrt{3}\cos x + \sin x &= R.\sin(x + \alpha) \\ &= R.\sin x.\cos\alpha + R\cos x.\sin\alpha\end{aligned}$$

Equating the similar terms on both sides,

$$\sqrt{3}\cos x = R\sin\alpha.\cos\alpha$$

$$\sin x = R\sin x.\cos\alpha$$

$$\implies \sqrt{3} = R\sin\alpha \quad \text{--- (1)}$$

$$\implies 1 = R\cos\alpha \quad \text{--- (2)}$$

Squaring and adding (1) & (2)

$$3 + 1 = R^2 \sin^2\alpha + \cos^2\alpha$$

$$4 = R^2 \implies R = \pm 2$$

$$(1) \div (2) \implies \sqrt{3} = \frac{\sin\alpha}{\cos\alpha} \implies \tan\alpha = \sqrt{3}$$

$$\implies \alpha = \tan^{-1}(\sqrt{3})$$

$$\implies \alpha = 60^\circ$$

(b) Prove identity for  $\sin 3A$

$$\begin{aligned}\sin 3A &= \sin(2A + A) \\ &= \sin 2A \cos A + \cos 2A \sin A \\ &= 2\sin A \cos A \cos A + (1 - 2\sin^2 A) \sin A \\ &= 2\sin A \cos^2 A + \sin A - 2\sin^3 A \\ &= 2\sin A (1 - \sin^2 A) + \sin A - 2\sin^3 A \\ &= 3\sin A - 4\sin^3 A\end{aligned}$$

(c) Show that  $2(bc.\cos A + ca.\cos B + ab.\cos C) = a^2 + b^2 + c^2$

Using cosine rule,

$$a^2 = b^2 + c^2 - 2bc.\cos A \quad \text{--- (1)}$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B \quad \text{---} \quad \textcircled{2}$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C \quad \text{---} \quad \textcircled{3}$$

Adding  $\textcircled{1}$ ,  $\textcircled{2}$  and  $\textcircled{3}$  we get

$$a^2 + b^2 + c^2 = 2(b^2 + c^2 + a^2) - 2(bc \cdot \cos A + ac \cdot \cos B + ab \cdot \cos C)$$

$$\implies 2(b^2 + c^2 + a^2) - (a^2 + b^2 + c^2) = 2(bc \cdot \cos A + ac \cdot \cos B + ab \cdot \cos C)$$

$$\implies a^2 + b^2 + c^2 = 2(bc \cdot \cos A + ca \cdot \cos B + ab \cdot \cos C)$$

VIII.

(a) Show that  $\tan 15 + \cot 15 = 4$

$$\tan 15 = \tan (45 - 30)$$

$$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \cdot \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\cot 15 = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\tan 15 + \cot 15 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} + \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}}{2} = \frac{8}{2} = 4$$

(b) If  $\tan A = 0.38$  find  $\tan 2A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \times (0.38)}{1 - (0.38)^2}$$

$$= 0.8882$$

(c) Prove that  $\cos 55 + \cos 65 + \cos 175 = 0$

$$\cos C + \cos D = 2 \cos \left( \frac{C+D}{2} \right) \cdot \cos \left( \frac{C-D}{2} \right)$$



$$\begin{aligned} \cos 55 + \cos 65 &= 2 \cos \frac{120}{2} \cdot \cos \frac{-10}{2} \\ &= 2 \times \cos 60 \cdot \cos 5 && [\cos(-A) = \cos A] \\ &= 2 \times \frac{1}{2} \times \cos 5 = \cos 5 \end{aligned}$$

$$\begin{aligned} \text{Then } \cos 55 + \cos 65 + \cos 175 \\ = \cos 5 + \cos 175 = 2 \cos 90 \cdot \cos 85 = 0 \end{aligned}$$

IX.

- (a) Find the equation of the line passes through the point  $(-4, 5)$  and whose intercepts are equal in magnitude but opposite in sign.

Intercept form:  $\frac{x}{a} + \frac{y}{b} = 1$

Let  $\frac{x}{a} + \frac{y}{-a} = 1$  ————— (1)

Since (1) passes through  $(-4, 5)$

$$\frac{-4}{a} + \frac{5}{-a} = 1$$

$$\frac{-4}{a} - \frac{5}{a} = 1 \implies \frac{-9}{a} = 1$$

$$\implies a = -9$$

The equation of the line is

$$\frac{x}{-9} + \frac{y}{9} = 1$$

$$\implies \frac{-x}{9} + \frac{y}{9} = 1$$

$$\implies -x + y = 9$$

$$\implies y - x = 9$$

- (b) Find the slope and intercept of the line  $3x + y - 15 = 0$

$$3x + 4y = 15 \text{ ————— (1)}$$

$$\text{Slope} = \frac{-a}{b} = \frac{-3}{4}$$

From (1) we have,

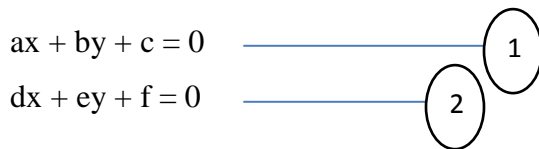
$$\frac{3x}{15} + \frac{4y}{15} = 1$$

$$\frac{x}{5} + \frac{y}{\frac{15}{4}} = 1$$

$$\therefore x \text{ intercept} = 5$$

$$y \text{ intercept} = \frac{15}{4}$$

(a) Find the angle between the lines  $ax + by + c = 0$  and  $dx + ey + f = 0$



$$\text{Slope of } ax + by + c = 0 \text{ is } m_1 = \frac{-a}{b}$$

$$\text{Slope of } dx + ey + f = 0 \text{ is } m_2 = \frac{-d}{e}$$

Angle between (1) and (2) is

$$\begin{aligned} \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{\frac{-a}{b} + \frac{d}{e}}{1 + \frac{-a}{b} \times \frac{d}{e}} \right| = \left| \frac{-ae + bd}{be + ad} \right| \\ &= \left| \frac{bd - ae}{be + ad} \right| \end{aligned}$$

$$\theta = \tan^{-1} \left( \left| \frac{bd - ae}{be + ad} \right| \right)$$

X.

(a) Find the condition for two lines (i) Parallel (ii) Perpendicular.

Consider two lines  $ax + by + c = 0$  and  $dx + ey + f = 0$

Case I(Parallel)

$$\text{Slope of } ax + by + c = 0 \text{ is } m_1 = \frac{-a}{b}$$

Slop of  $dx + ey + f = 0$  is  $m_2 = \frac{-d}{e}$

Condition for parallism.

$$m_1 = m_2$$

$$\frac{-a}{b} = \frac{-d}{e} \implies ae = bd \quad \text{or } ae - bd = 0$$

Case II (perpendicular)

$$m_1 = \frac{-a}{b} \qquad m_2 = \frac{-d}{e} \qquad m_1 \cdot m_2 = -1$$

$$m_1 \times m_2 = \frac{-a}{b} \times \frac{-d}{e} = -1$$

$$\frac{ad}{be} = -1$$

$$ad = -be$$

$$\implies ad + be = 0$$

- (b) The straight line through (4, 3) makes intercepts 4a and 3a on the X and Y axis respectively find a.

Intercept form =  $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{4a} + \frac{y}{3a} = 1 \quad \text{--- (1)}$$

(1) Passes through (4, 3)

$$\text{Then } \frac{4}{4a} + \frac{3}{3a} = 1$$

$$\implies \frac{1}{a} + \frac{1}{a} = 1$$

$$\implies \frac{2}{a} = 1 \qquad \implies a = 2$$

- (c) Show that the point of intersection of the lines  $5x - 12y = 2$  and  $3x - 8y + 2 = 0$  lies on  $2x - 3y = 8$ .

$$5x - 12y = 2 \quad \text{--- (1)}$$

$$3x - 8y = -2 \quad \text{--- (2)}$$

Solving (1) (2)

$$x = \frac{\begin{vmatrix} 2 & -12 \\ -2 & -8 \end{vmatrix}}{\begin{vmatrix} 5 & -12 \\ 3 & -8 \end{vmatrix}} = \frac{-16 - 24}{-40 + 36} = \frac{-40}{-4} = 10$$

Then (1) comes

$$5x - 12y = 2$$

$$50 - 12y = 2$$

$$-12y = -48$$

$$y = 4$$

Put  $x = 10$  &  $y = 4$  in  $2x - 3y = 8$

$$2 \times 10 - 3 \times 4 = 20 - 12 = 8. \text{ Satisfied.}$$

Hence the result.

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