TED (10)-1002 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- OCTOBER, 2013

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time*: 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Which of the following matrices is symmetric

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 3 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is symmetric.

(b) Find the value of r, if $^{20}c_r = {}^{20}c_{r+2}$

$^{20}c_r = {}^{20}c_{r+2}$

$^{n}c_r = {}^{n}c_s = r + s = n$ or $r = s$.

$\therefore r + r + 2 = 20$

$2r + 2 = 20$

$2r = 18$

$r = 9$

(c) State the identities for tan(A – B)

$$\tan(A - B) = \frac{\tan A - \tan B}{(1 + \tan A.\tan B)}$$

(d) State projection formula.

In any ΔABC, $a = b.\cos C + c.\cos B$ or
$b = a.\cos C + c.\cos A$ or
$c = a.\cos B + b.\cos A$

(e) Define slope of a straight line.

If a straight line is inclined at an angle 'θ' with the x – axis, the slope of that straight line is given by $\tan\theta$. It is represented by $m = \tan\theta$.

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Solve the equations: $3x + y - z = 3$
$-x + y + z = 1$
$x + y + z = 3$ by find the inverse of the coefficient matrix.

$3x + y - z = 3$
$-x + y + z = 1$
$x + y + z = 3$

AX = B

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

<u>Calculations for A^{-1}</u>

$$m_{11} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$m_{12} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$$

$$m_{13} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$$

$m_{21} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$

$m_{22} = \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} = 4$

$m_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$

$m_{31} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$

$m_{32} = \begin{vmatrix} -3 & -1 \\ -1 & 1 \end{vmatrix} = 2$

$m_{33} = \begin{vmatrix} 3 & 1 \\ 1 - & 1 \end{vmatrix} = 4$

Minor matrix = $\begin{bmatrix} 0 & -\mathbf{2} & -2 \\ \mathbf{2} & 4 & \mathbf{2} \\ 2 & \mathbf{2} & 4 \end{bmatrix}$

$\therefore$ Cofactor of A = $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$

$\therefore$ Adj A = $\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} -1\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$

$= 3\,(1 - 1) - 1(-1 - 1) - 1(-1 - 1)$

$\therefore |A| = 4$

$\therefore A^{-1} = \dfrac{\text{Adj A}}{|A|}$

$= \dfrac{\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}}{4}$

$\therefore X = A^{-1}B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}}{4}$

$= \dfrac{\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}}{4} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\therefore x = 1 \qquad y = 1 \qquad z = 1$

(b) If $A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$ show that $(AB)^{-1} = B^{-1}A^{-1}$

$AB = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 47 & 34 \\ 22 & 16 \end{bmatrix}$

$|AB| = \begin{vmatrix} 47 & 34 \\ 22 & 16 \end{vmatrix} = 4$

Cofactor matrix $= \begin{bmatrix} 16 & -22 \\ -34 & 47 \end{bmatrix}$

Adj. (AB) $= \begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}$

Inverse of AB $= \dfrac{\text{Adj.AB}}{|AB|} = \dfrac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4} = \begin{bmatrix} 4 & -\frac{17}{2} \\ -\frac{11}{2} & \frac{47}{4} \end{bmatrix}$

$A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \qquad |A| = 4$

Cofactor matrix of A $= \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

Adj. (A) $= \begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj.A}}{|A|} = \dfrac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$

$B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \qquad |B| = 1$

Cofactor matrix of B $= \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

Adj. (B) $= \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$

$B^{-1} = \dfrac{\text{Adj.B}}{|B|} = \dfrac{\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}}{1}$

$B^{-1}A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \dfrac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4} = \begin{bmatrix} 4 & -\frac{17}{2} \\ -\frac{11}{2} & \frac{47}{4} \end{bmatrix}$

$\therefore \ (AB)^{-1} = B^{-1}A^{-1}$

(c) Prove that $nc_r + nc_{r-1} = (n+1)c_r$

$$nc_r = \frac{n!}{(n-r)!r!} \qquad nc_{r-1} = \frac{n!}{(n-r+1)!(r-1)!}$$

$$nc_r + nc_{r-1} = \frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!}$$

$$= n!\left[\frac{1}{(n-r)!r!} + \frac{1}{(n-r+1)(n-r)!(r-1)!}\right]$$

$$= n!\left[\frac{1}{(n-r)!r(r-1)!} + \frac{1}{(n-r+1)(n-r)!(r-1)!}\right]$$

$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{1}{r} + \frac{1}{(n-r+1)}\right]$$

$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{n!(n+1)}{(n-r)!(n-r+1)r(r-1)!}$$

$$= \frac{(n+1)!}{(n+1-r)r!} = (n+1)c_r$$

(d) Prove that $\frac{\cos4x+\cos3x+\cos2x}{\sin4x+\sin3x+\sin2x} = \cot3x$

$$\text{L.H.S} = \frac{\cos4x+\cos3x+\cos2x}{\sin4x+\sin3x+\sin2x} = \frac{\cos3x+\cos4x+\cos2x}{\sin3x+\sin4x+\sin2x}$$

$$= \frac{\cos3x + 2\cos3x.\cos x}{\sin3x+2\sin3x.\cos x}$$

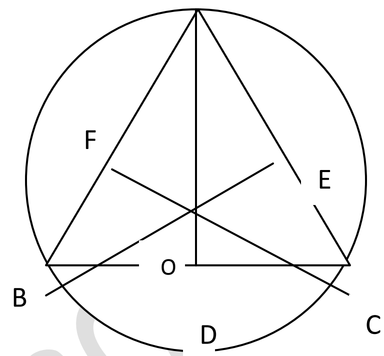
$$= \frac{\cos3x(1+2\cos x)}{\sin3x(1+2\cos x)}$$

$$= \cot3x$$

(e) State and prove sine rule.

<u>Statement</u>

In any ΔABC $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$



<u>Proof</u>

Consider the circumcircle of ΔABC.

The perpendicular bisectors of the sides $\overline{BC}$, $\underline{CA}$, and $\overline{AB}$ intersect at 'O'. Therefore 'O' is the circumcentre such that OA = OB = OC = R

we have $< BOC = 2 < BAC = 2A$

So $< BOD = < COD = A$

In ΔODB , sin<BOD = sinA = BD/OB = $\frac{\frac{a}{2}}{R}$

$\sin A = \frac{a}{2R} \Longrightarrow a = 2R\sin A$

similarly,

$\sin B = \frac{b}{2R} \Longrightarrow b = 2R\sin B$

$\text{siCB} = \frac{c}{2R} \Longrightarrow c = 2R\sin C$

it is clear that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

(f) Using Napier's formula, find the values of the angles A , B in ΔABC , if a = 5cm, b = 8cm and $C = 30^o$

$\tan(\frac{A-B}{2}) = \frac{a-b}{a+b} \cot \frac{c}{2}$

$\frac{A-B}{2} = \tan^{-1}[\frac{a-b}{a+b} \cot \frac{c}{2}]$

$\frac{A-B}{2} = \tan^{-1}[\frac{5-8}{13} \cot \frac{30}{2}]$

$\frac{A-B}{2} = \tan^{-1}[\frac{-3}{13} \cot 15^o]$

$\frac{A-B}{2} = \tan^{-1}[-0.8612] = -40.736$

A – B = -81.473 ——— 1

A + B = 180 – 30 =150 ——— 2

Solving 1 + 2

A = 34.2635 = $34^{O}16'$ B = 150 - 34.2635 = $115^{O}44'$

Now we have to find 'c'

We have $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{5}{\sin 34^{o}16'} = \frac{c}{\sin 30^{o}}$$

$$c = \frac{5}{0.5629996} \times \sin 30^{o} = 4.44\text{cm}$$

(g) Find the equation to the line passing through (4, 5) which is (i) parallel (ii) Perpendicular to the line 2x + 3y = 4

Case I

Equation of a parallel line is

ax + by + k = 0

a = 2

b = 3

2x + 3y + k = 0 ——— 1

1 sses through (4, 5)

∴ 1 ===>2 x 4 + 3 x 5 + k = 0

8 + 15 + k = 0

23 + k = 0

K = -23

∴ 1 ===> 2x + 3y – 23 = 0

Case II

Equation of perpendicular line is

bx – ay + k = 0

a = 2, b = 3

3x – 2y + k = 0 ——— 2

∴ 2 Passes through (4, 5)

∴ 2 ===> 3 x 4 – 2 x 5 + k = 0

12 – 10 + k = 0

2 + k = 0

K = -2

∴ 2 ===> 3x – 2y – 2 = 0

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If A = $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ B = $\begin{bmatrix} -1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix}$ verify that A (B – C) = AB – AC

$$A(B-C) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -3 & 1 & 2 \\ -5 & 2 & -2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 7 & -3 & 6 \\ -5 & 2 & -2 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 & 1 \\ -3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 5 & -5 \\ 2 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -3 & 6 \\ -5 & 2 & -2 \end{bmatrix}$$

Clearly A(B – C) = AB – AC

(b) If A = $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$ show that $(AB)^T = B^T A^T$

$$AB = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 24 \\ 32 & 54 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 14 & 32 \\ 24 & 54 \end{bmatrix} \tag{1}$$

$$B^T A^T = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 14 & 32 \\ 24 & 54 \end{bmatrix} \tag{2}$$

Clearly $(AB)^T = B^T A^T$

(c) Show that the eliminant of $lx + my + n = 0$, $mx + ny + 1 = 0$ and $nx + ly + m = 0$ is $l^3 + m^3 + n^3 = 3mn$

$$\text{Eliminant} = \begin{vmatrix} l & m & n \\ m & n & l \\ n & l & m \end{vmatrix} = 0$$

$\Longrightarrow 1(mn - 1) - m(m^2 - n) + n(m - n^2) = 0$

$\Longrightarrow mn - 1 - m^3 + mn + nm - n^3 = 0$

$\Longrightarrow m^3 + n^3 + 1 = 3mn$

$\Longrightarrow m^3 + n^3 + 1^3 = 3mn$

IV.

(a) If $A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$ find $A^2 - 8A - 20I$

$$A^2 = A.A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix}$$

$$8A = 8\begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 16 & 48 \\ 56 & 32 & 80 \\ 8 & 24 & 40 \end{bmatrix}$$

$$20I = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$A^2 - 8A - 20I = \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix} - \begin{bmatrix} 8 & 16 & 48 \\ 56 & 32 & 80 \\ 8 & 24 & 40 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 12 & 8 \\ -11 & 8 & 52 \\ 19 & 5 & 1 \end{bmatrix}$$

(b) Express the matrix A = $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$ as the sum of two matrices of which one issymmetric and the other is skew symmetric.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}$$

$\frac{A+A^T}{2} = \frac{\begin{bmatrix} 2 & 6 & 8 \\ 6 & 4 & 4 \\ 8 & 4 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix}$, this is a symmetric matrix.

$\frac{A-A^T}{2} = \frac{\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 2 \\ -2 & -2 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$, this is a skew symmetric matrix.

Clearly $\begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = A$

(c) Solve using determinants: x + 2y – z = -1
3x – y – 2z = 5
x – y – 3z = 0

x + 2y – z = -1

3x – y – 2z = 5

x – y – 3z = 0

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} -1 & 2 & -1 \\ 5 & -1 & -2 \\ 0 & -1 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & -2 \\ 1 & -1 & -3 \end{vmatrix}}$$

$$= \frac{-(3-2)-2(-15)-(-5)}{(3-2)-2(-9+2)-(-3+1)}$$

$$= \frac{-1+30+5}{1+14+2} = \frac{34}{17} = 2$$

$$Y = \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix} 1 & -1 & -1 \\ 3 & 5 & -2 \\ 1 & 0 & -3 \end{vmatrix}}{17}$$

$$= \frac{(-15)+(-9+2)-(-5)}{17}$$

$$= \frac{-15-7+5}{17} = \frac{-17}{17} = -1$$

$$z = \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 5 \\ 1 & -1 & 0 \end{vmatrix}}{17}$$

$$= \frac{(5)-2(-5)-(-3+1)}{17}$$

$$= \frac{5+10+2}{17} = \frac{17}{17} = 1$$

V.

(a) Expand $(x+\frac{1}{x})^7$ using binomial theorem.

$(a + b)^n = a^n + nc_1 a^{n-1} b + nc_2 a^{n-2} b + \ldots\ldots\ldots + nc_n b^n$

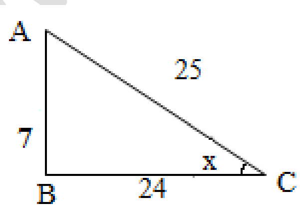
Now

$$(x - \frac{1}{x})^7 = x^7 + 7c_1 x^6 (\frac{1}{x}) + 7c_2 x^5 (\frac{1}{x})^2 + 7c_3 x^4 (\frac{1}{x})^3 + 7c_4 x^3 (\frac{1}{x})^4$$

$$+ 7c_5 x^2 (\frac{1}{x})^5 + 7c_6 x (\frac{1}{x})^6 + 7c_7 (\frac{1}{x})^7$$

$$= x^7 + 7x^5 21x^3 + 35x + 35x^{-1} + 21x^{-3} + 7x^5 + x^{-7}$$

(b) If $\tan x = 7/24$ & x is in the 3rd quadrant. Find the value of $3\sin x - 4\cos x$.



$\sin x = -7/25$ [$x \in 3^{rd}$ quadrant.]

cosx = -24/25

3sinx – 4cosx = 3 x (-7/25) – 4 x (-24/25)

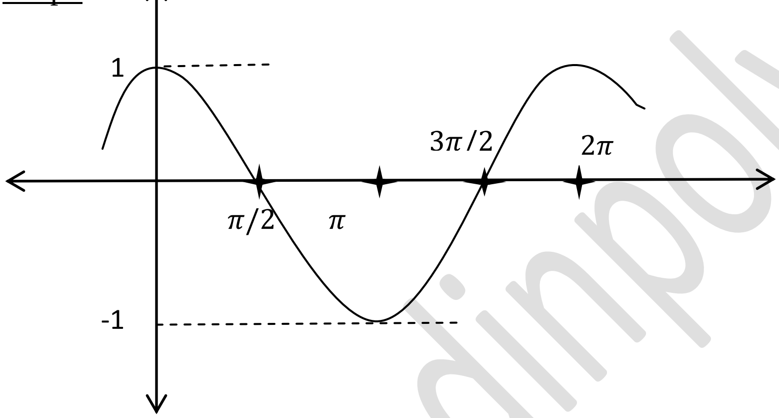
= (-21/25) + (96/25) = 75/25 = 3

(c) Draw the graph of y = cos*3x*

The graph of y = cosx

X :	0	90^o	180^o	270^o	360^o
Y :	1	0	-1	0	1

<u>Graph</u>



VI.

(a) Find the term independent of x in the expansion of $(x + 3/x^3)^{10}$

$T_{r+1} = nc_r\, a^{n-r} b^r$,

$T_{r+1} = 10c_r\, (x)^{10-r}\, (3/x)^r$

$= 10c_r 3^r x^{10-r}\ -x^{-r}$

$= 10c_r 3^r\ x^{10-2r}$

10 – 2r = 0

===> r = 5

∴The term independent of x

$= 10c_5 3^5\ x^0$

= 61236

(b) Write the sign of

(i) Cot($7\pi/4$)

(ii) tan(500)

(iii)cosec(280).

(i) Cot($7\pi/4$) = cot(7 x 45^o) = cot(315)

= cot(3 x 90 + 45)

= -tan45 = -1

∴ Sign is negative.

(ii) tan(500) = tan(5 x 90 + 50)

= -cot50

∴ Sign is negative. .

(iii)Cosec(280)

= cosec(3 x 90 + 10) = $\frac{1}{\sin(3 \text{ x } 90+10)} = \frac{1}{-\cos 10}$

∴ Sign is negative.

(c) Prove that $\frac{\tan 45+\tan 30}{(1+\tan 45.\tan 30)}=2-\sqrt{3}$

$$\frac{\tan 45+\tan 30}{(1+\tan 45.\tan 30)} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\frac{(\sqrt{3}-1)\ (\sqrt{3}-1)}{(\sqrt{3}+1)\ (\sqrt{3}-1)} = \frac{\left(\sqrt{3}-1\right)^2}{\sqrt{3}^2+1^2}$$

$$= \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

VII.

(a) Prove the formula for cos3A

Sin3A = cos (2A + A)

= cos2AcosA –sin2AcosA

= ($2\cos^2$A – 1)cosA – 2sinA.cosA.sinA

$= 2\cos^3A – \cos A – 2\sin^2A.\cos A$

$= 2\cos^3A – \cos A – 2\sin^2A.\cos A$

$= 2\cos^3A – \cos A – 2(1 – \cos^2A).\cos A$

$= 2\cos^3A – \cos A – 2\cos A + 2\cos^3A$

$= 4\cos^3A – 3\cos A$

(b) If sin18 = (√5 -1)/4, find cos36 and sin54

Put $\theta = 18^o$

$Cos36^o = \cos 2 \times (18) = \cos 2\theta = 1 – 2\sin^2\theta$

$1 – 2\sin^2 18 = 1 - 2\left[\frac{\sqrt{5}-1}{4}\right]^2$

$= 1 – 2 \times 0.9549$

$= 0.8090$

$Sin54^o = \sin 3 \times 18 = \sin 3\theta$

$= 3\sin\theta – 4\sin^3\theta$

$3\sin 18 – 4\sin^3 18 = 3\left[\frac{\sqrt{5}-1}{4}\right] – 4\left[\frac{\sqrt{5}-1}{4}\right]^3$

$= 0.8090$

OR $\sin 54^o = \sin(90 – 36) = \cos 36 = 0.8090$

(c) Prove that $\cos\frac{\pi}{8} + \cos3\frac{\pi}{8} + \cos5\frac{\pi}{8} + \cos7\frac{\pi}{8} = 0$

$\cos\frac{\pi}{8} + \cos3\frac{\pi}{8} + \cos5\frac{\pi}{8} + \cos7\frac{\pi}{8}$

$= \cos\frac{\pi}{8} + \cos7\frac{\pi}{8} + \cos3\frac{\pi}{8} + \cos5\frac{\pi}{8}$

$= 2\cos\left(\frac{\frac{\pi}{8}+\frac{7\pi}{8}}{2}\right).\ \cos\left(\frac{\frac{\pi}{8}-\frac{7\pi}{8}}{2}\right) + 2\cos\left(\frac{\frac{3\pi}{8}+\frac{5\pi}{8}}{2}\right).2\cos\left(\frac{\frac{3\pi}{8}-\frac{5\pi}{8}}{2}\right)$

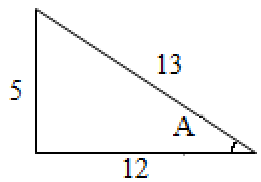
$= 2\cos\frac{\pi}{2}\ .\ \cos(-\ \frac{3\pi}{8}) + 2\cos\frac{\pi}{2}\ .\ \cos(\frac{-\frac{\pi}{4}}{2})$

$= 0$ $\qquad\qquad\qquad\qquad$ $[\cos\frac{\pi}{2} = 0]$

VIII.

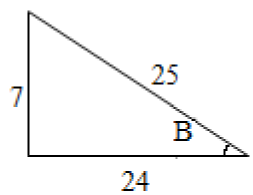
(a) If cosA = -12/13, cotB = 24/7 and A is in the quadrant II and B is in quadrant I. find cos(A –B)

$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$



Given $\cos A = -\frac{12}{13}$

$\therefore \sin A = \frac{5}{13}$ $\qquad$ [$A \in 2^{nd}$ quadrant]



$\cot B = \frac{24}{7}$

$\therefore \sin B = \frac{7}{25}$ $\qquad$ [$B \in 1^{st}$ quadrant]

$\cos B = \frac{24}{25}$

$\therefore \cos(A - B) = \cos A.\cos B + \sin A.\sin B$

$$= -\frac{12}{13} \times \frac{24}{25} + \frac{5}{13} \times \frac{7}{25}$$

$$= \frac{-288+35}{325} = \frac{-253}{325}$$

(b) Prove that cotA – cot2A = cosec2A

$\cot A = \frac{\cos A}{\sin A}$ $\qquad\qquad$ $\cot 2A = \frac{\cos 2A}{\sin 2A}$

$$\cot A - \cot 2A = \frac{\cos A}{\sin A} - \frac{\cos 2A}{\sin 2A}$$

$$= \frac{\cos A.\sin 2A - \cos 2A.\sin A}{\sin A.\sin 2A}$$

$$= \frac{\sin 2A.\cos A - \cos 2A.\sin A}{\sin A.\sin 2A}$$

$$= \frac{\sin(2A - A)}{\sin A.\sin 2A} = \frac{\sin A}{\sin A.\sin 2A} = \text{cosec}2A$$

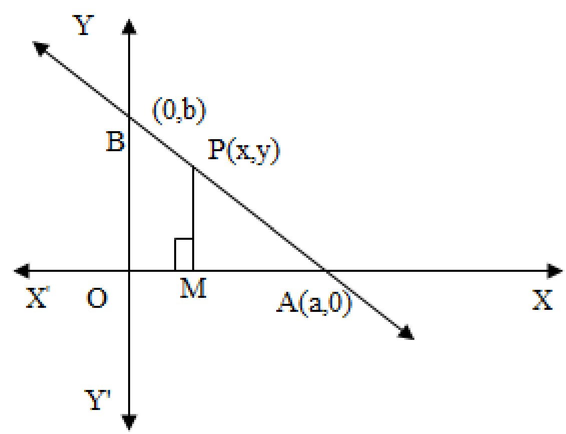
(c) Show that $\left(\frac{a+b}{c}\right)\sin^2 c/2 = \cos\left(\frac{A+B}{C}\right)$

Question is wrong.

IX.

(a) Derive the equation of a straight line of the form $\frac{x}{a} + \frac{y}{b} = 1$

The x intercept 'a' and y intercept 'b' are given, consider a straight line AB having x–intercept 'a' and y-intercept 'b'



Hence OA = a and OB= b

Let (x, y) be any point on the line.

From the figure it is clear that ΔAMP and ΔAOBare similar.

∴Their corresponding sides are proportional.

$$\frac{OA}{MA} = \frac{OB}{MP} \qquad (1)$$

MA = OA – OM = a – x

MP = y

Substituting these in (1) we have

$$\frac{a}{a-x} = \frac{b}{y}$$

===>ay = ab – bx

===>bx + ay = ab

$$\frac{ay}{ba} + \frac{bx}{ab} = \frac{ba}{ab}$$

$===> \frac{x}{a} + \frac{y}{b} = 1$

(b) Find the slope and intercept of the line 5x – 3y +15 = 0

5x–3y = -15

Slope = $\frac{-a}{b} = \frac{-5}{-3} = \frac{5}{3}$

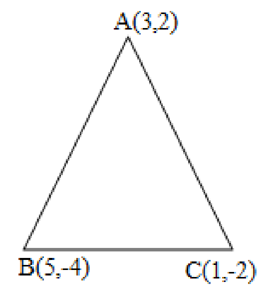
5x – 3y = -15

$\frac{5x}{-15} + \frac{-3y}{-15} = 1$

$===> \frac{x}{-3} + \frac{y}{5} = 1$

$\therefore$Intercept form = $\frac{x}{-3} + \frac{y}{5} = 1$

(c) Find the angle of triangle having vertices (3, 2) , (5, -4) and (1, -2)



Slope of AB = $m_1 = \frac{6}{-2} = -3$

Slope of AC = $m_2 = \frac{4}{2} = 2$

Slope of BC = $m_3 = \frac{2}{-4} = -\frac{1}{2}$

Angle between AB & AC = $\theta = \tan^{-1}\left|\frac{m1-m2}{1+m1m2}\right|$

$= \tan^{-1}\left|\frac{-3-2}{1+-6}\right| = \tan^{-1}\left|\frac{-5}{-5}\right|$

$= \tan^{-1}(1) = \frac{\pi}{4} = 45^o$

Angle between AC & BC = $\theta = \tan^{-1}\left|\frac{m2-m3}{1+m2m3}\right|$

$= \tan^{-1}\left|\frac{2+\frac{1}{2}}{1+2\text{ x}\frac{1}{2}}\right| = \tan^{-1}\left|\frac{\frac{5}{2}}{0}\right|$

$= \tan^{-1}(\infty) = \frac{\pi}{2} = 90^o$

Angle between AB & AC = 180 – (90 + 45) = 45^o

X.

(a) Find the values of P if the lines (2P + 1)x – (5 – P)y = 8 and (5P – 1)x – (P + 1)y = 3 are parallel.

(2P + 1)x – (5 – P)y = 8 ———— 1

(5P – 1)x – (P + 1)y = 3 ———— 2

Slope of 1 is $\frac{-(2P+1)}{-(5-P)}$

Slope of 2 is $\frac{-(5P-1)}{-(P+1)}$

Since the lines are parallel, we have

$$\frac{-(2P+1)}{-(5-P)} = \frac{-(5P-1)}{-(P+1)}$$

(2P + 1) (P + 1) = (5P – 1) (5 – P)

$2P^2 + 2P + P + 1 = 25P - 5P^2 - 5 + P$

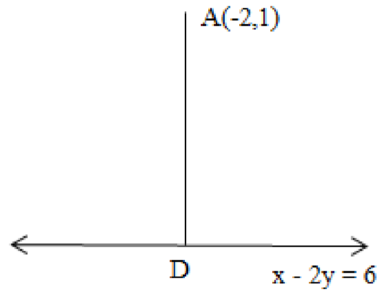
$7P^2 - 23P + 6 = 0$

$P = \frac{23\pm\sqrt{361}}{14} = \frac{23\pm19}{14}$

= 3, 4/14

3 or 2/7

(b) Find the foot of the perpendicular from(-2, 1) on the line x -2y = 6



Slope of $x - 2y = 6$ is

$$m_1 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$$

Slope of AD = -2

Equation of AD: $y - y1 = m(x - x1)$

$$y - 1 = -2(x + 2)$$
$$y - 1 = -2x - 4$$
$$2x + y = -3$$

Solving $x - 2y = 6$ & $2x + y = -3$

$$x = \frac{\begin{vmatrix} 6 & -2 \\ -3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}} = \frac{6-6}{1+4} = 0$$

Now $2 \times 0 + y = -3$ ===> $y = -3$

∴ Foot of perpendicular is(0, -3)

(c) Straight line cuts off on the axes of coordinates positive intercept whose sum is 5. Given that the line passes through (-4, 9), find its equation.

Intercept form of a line is given by

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad 1$$

Given a + b = 5

===> b = 5 – a

Equation 1 passes through (-4, 9)

$$\therefore \frac{-4}{a} + \frac{9}{5-a} = 1$$

===> $-4(5 - a) + 9a = a(5 - a)$

===> $-20 + 4a + 9a = 5a - a^2$

===> $a^2 + 8a - 20 = 0$

$$a = \frac{-8 \pm \sqrt{64+80}}{2} = \frac{-8 \pm \sqrt{144}}{2} = \frac{-8 \pm 12}{2} = -10, 2$$

$\therefore$ a = 2 b = 3