

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/  
TECHNOLIGY- OCTOBER, 2013

TECHNICAL MATHEMATICS- I  
(Common – Except DCP and CABM)

[Time: 3 hours

(Maximum marks: 100)

Marks

PART –A  
(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Which of the following matrices is symmetric

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  is symmetric.

(b) Find the value of r, if  ${}^{20}C_r = 20$   ${}^{20}C_{r+2}$

$${}^{20}C_r = 20 \quad {}^{20}C_{r+2}$$

$${}^nC_r = n \quad {}^nC_s = r + s = n \quad \text{or } r = s.$$

$$\therefore r + r + 2 = 20$$

$$2r + 2 = 20$$

$$2r = 18$$

$$r = 9$$

(c) State the identities for  $\tan(A - B)$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(d) State projection formula.

$$\begin{aligned} \text{In any } \triangle ABC, \quad a &= b \cos C + c \cos B \quad \text{or} \\ b &= a \cos C + c \cos A \quad \text{or} \\ c &= a \cos B + b \cos A \end{aligned}$$

(e) Define slope of a straight line.

If a straight line is inclined at an angle ' $\theta$ ' with the x - axis, the slope of that straight line is given by  $\tan\theta$ . It is represented by  $m = \tan\theta$ .

### PART -B

Answer any five questions. Each question carries 6 marks

II.

(a) Solve the equations:  $3x + y - z = 3$   
 $-x + y + z = 1$   
 $x + y + z = 3$  by find the inverse of the coefficient matrix.

$$3x + y - z = 3$$

$$-x + y + z = 1$$

$$x + y + z = 3$$

$$AX = B$$

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

Calculations for  $A^{-1}$

$$m_{11} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$m_{12} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$$

$$m_{13} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$$

$$m_{21} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$m_{22} = \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} = 4$$

$$m_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$$

$$m_{31} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$m_{32} = \begin{vmatrix} -3 & -1 \\ -1 & 1 \end{vmatrix} = 2$$

$$m_{33} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 4$$

$$\text{Minor matrix} = \begin{bmatrix} 0 & -2 & -2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

$$\therefore \text{Cofactor of A} = \begin{bmatrix} 0 & 2 & -2 \\ -2 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

$$\therefore \text{Adj A} = \begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 3 \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 3 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$
$$= 3(1-1) - 1(-1-1) - 1(-1-1)$$

$$\therefore |A| = 4$$

$$\therefore A^{-1} = \frac{\text{Adj A}}{|A|}$$

$$= \frac{\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}}{4}$$

$$\therefore X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}}{4}$$

$$= \frac{\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}}{4} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore x = 1 \quad y = 1 \quad z = 1$$

(b) If  $A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$  show that  $(AB)^{-1} = B^{-1}A^{-1}$

$$AB = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 47 & 34 \\ 22 & 16 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 47 & 34 \\ 22 & 16 \end{vmatrix} = 4$$

$$\text{Cofactor matrix} = \begin{bmatrix} 16 & -22 \\ -34 & 47 \end{bmatrix}$$

$$\text{Adj. (AB)} = \begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}$$

$$\text{Inverse of AB} = \frac{\text{Adj. AB}}{|AB|} = \frac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4} = \begin{bmatrix} 4 & -\frac{17}{2} \\ -\frac{11}{2} & \frac{47}{4} \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \quad |A| = 4$$

$$\text{Cofactor matrix of A} = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\text{Adj. (A)} = \begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj. A}}{|A|} = \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$$

$$B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \quad |B| = 1$$

$$\text{Cofactor matrix of B} = \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$$

$$\text{Adj. (B)} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$$

$$B^{-1} = \frac{\text{Adj. B}}{|B|} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4} = \begin{bmatrix} 4 & -\frac{17}{2} \\ -\frac{11}{2} & \frac{47}{4} \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

(c) Prove that  $nc_r + nc_{r-1} = (n+1)c_r$

$$\begin{aligned}
 nc_r &= \frac{n!}{(n-r)!r!} & nc_{r-1} &= \frac{n!}{(n-r+1)!(r-1)!} \\
 nc_r + nc_{r-1} &= \frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!} \\
 &= n! \left[ \frac{1}{(n-r)!r!} + \frac{1}{(n-r+1)(n-r)!(r-1)!} \right] \\
 &= n! \left[ \frac{1}{(n-r)!r(r-1)!} + \frac{1}{(n-r+1)(n-r)!(r-1)!} \right] \\
 &= \frac{n!}{(n-r)!(r-1)!} \left[ \frac{1}{r} + \frac{1}{(n-r+1)} \right] \\
 &= \frac{n!}{(n-r)!(r-1)!} \left[ \frac{n-r+1+r}{r(n-r+1)} \right] \\
 &= \frac{n!(n+1)}{(n-r)!(n-r+1)r(r-1)!} \\
 &= \frac{(n+1)!}{(n+1-r)r!} = (n+1)c_r
 \end{aligned}$$

(d) Prove that  $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \frac{\cos 3x + \cos 4x + \cos 2x}{\sin 3x + \sin 4x + \sin 2x} \\
 &= \frac{\cos 3x + 2\cos 3x \cdot \cos x}{\sin 3x + 2\sin 3x \cdot \cos x} \\
 &= \frac{\cos 3x(1+2\cos x)}{\sin 3x(1+2\cos x)} \\
 &= \cot 3x
 \end{aligned}$$

(e) State and prove sine rule.

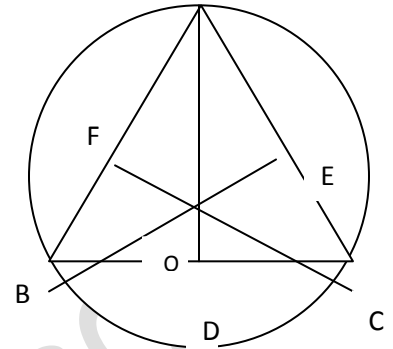
**Statement**

$$\text{In any } \Delta ABC \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

**Proof**

Consider the circumcircle of  $\Delta ABC$ .

The perpendicular bisectors of the sides  $BC$ ,  $CA$ , and  $AB$  intersect at 'O'. Therefore 'O' is the circumcentre such that  $OA = OB = OC = R$



we have  $\angle BOC = 2\angle BAC = 2A$

So  $\angle BOD = \angle COD = A$

$$\text{In } \Delta ODB, \sin \angle BOD = \sin A = \frac{BD}{OB} = \frac{\frac{a}{2}}{R}$$

$$\sin A = \frac{a}{2R} \implies a = 2R \sin A$$

similarly,

$$\sin B = \frac{b}{2R} \implies b = 2R \sin B$$

$$\sin C = \frac{c}{2R} \implies c = 2R \sin C$$

$$\text{it is clear that } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

(f) Using Napier's formula, find the values of the angles A, B in  $\Delta ABC$ , if  $a = 5\text{cm}$ ,  $b = 8\text{cm}$  and  $C = 30^\circ$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$\frac{A-B}{2} = \tan^{-1}\left[\frac{a-b}{a+b} \cot \frac{C}{2}\right]$$

$$\frac{A-B}{2} = \tan^{-1}\left[\frac{5-8}{13} \cot \frac{30}{2}\right]$$

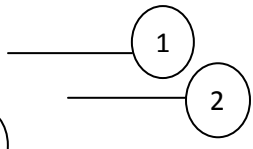
$$\frac{A-B}{2} = \tan^{-1}\left[\frac{-3}{13} \cot 15^\circ\right]$$

$$\frac{A-B}{2} = \tan^{-1}[-0.8612] = -40.736$$

$$A - B = -81.473$$

$$A + B = 180 - 30 = 150$$

Solving (1) + (2)



$$A = 34.2635 = 34^{\circ}16'$$

$$B = 150 - 34.2635 = 115^{\circ}44'$$

Now we have to find 'c'

We have

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{5}{\sin 34^{\circ}16'} = \frac{c}{\sin 30^{\circ}}$$

$$c = \frac{5}{0.5629996} \times \sin 30^{\circ} = 4.44 \text{ cm}$$

(g) Find the equation to the line passing through (4, 5) which is (i) parallel (ii) Perpendicular to the line  $2x + 3y = 4$

Case I

Equation of a parallel line is

$$ax + by + k = 0$$

$$a = 2$$

$$b = 3$$

$$2x + 3y + k = 0 \quad \text{--- (1)}$$

(1) passes through (4, 5)

$$\therefore (1) \implies 2 \times 4 + 3 \times 5 + k = 0$$

$$8 + 15 + k = 0$$

$$23 + k = 0$$

$$K = -23$$

$$\therefore (1) \implies 2x + 3y - 23 = 0$$

Case II

Equation of perpendicular line is

$$bx - ay + k = 0$$

$$a = 2, b = 3$$

$$3x - 2y + k = 0 \quad \text{--- (2)}$$

$\therefore$  (2) Passes through (4, 5)

$\therefore$  (2)  $\implies 3 \times 4 - 2 \times 5 + k = 0$

$$12 - 10 + k = 0$$

$$2 + k = 0$$

$$K = -2$$

$\therefore$  (2)  $\implies 3x - 2y - 2 = 0$

### PART - C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If  $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix}$  verify that  $A(B - C) = AB - AC$

$$A(B - C) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \left( \begin{bmatrix} -1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 & 2 \\ -5 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -3 & 6 \\ -5 & 2 & -2 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 & 1 \\ -3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 5 & -5 \\ 2 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -3 & 6 \\ -5 & 2 & -2 \end{bmatrix}$$

Clearly  $A(B - C) = AB - AC$

(b) If  $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$  show that  $(AB)^T = B^T A^T$

$$AB = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 24 \\ 32 & 54 \end{bmatrix}$$



$$(AB)^T = \begin{bmatrix} 14 & 32 \\ 24 & 54 \end{bmatrix} \text{ ————— } \textcircled{1}$$

$$B^T A^T = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 14 & 32 \\ 24 & 54 \end{bmatrix} \text{ ————— } \textcircled{2}$$

Clearly  $(AB)^T = B^T A^T$

- (c) Show that the eliminant of  $lx + my + n = 0$ ,  $mx + ny + 1 = 0$  and  $nx + ly + m = 0$  is  $l^3 + m^3 + n^3 = 3mn$

$$\text{Eliminant} = \begin{vmatrix} l & m & n \\ m & n & 1 \\ n & 1 & m \end{vmatrix} = 0$$

$$\implies 1(mn - 1) - m(m^2 - n) + n(m - n^2) = 0$$

$$\implies mn - 1 - m^3 + mn + nm - n^3 = 0$$

$$\implies m^3 + n^3 + 1 = 3mn$$

$$\implies m^3 + n^3 + 1^3 = 3mn$$

IV.

(a) If  $A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$  find  $A^2 - 8A - 20I$

$$A^2 = A.A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix}$$

$$8A = 8 \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 16 & 48 \\ 56 & 32 & 80 \\ 8 & 24 & 40 \end{bmatrix}$$

$$20I = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$A^2 - 8A - 20I = \begin{bmatrix} 21 & 28 & 56 \\ 45 & 60 & 132 \\ 27 & 29 & 61 \end{bmatrix} - \begin{bmatrix} 8 & 16 & 48 \\ 56 & 32 & 80 \\ 8 & 24 & 40 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 12 & 8 \\ -11 & 8 & 52 \\ 19 & 5 & 1 \end{bmatrix}$$

(b) Express the matrix  $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$  as the sum of two matrices of which one is symmetric and the other is skew symmetric.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}$$

$$\frac{A+A^T}{2} = \frac{\begin{bmatrix} 2 & 6 & 8 \\ 6 & 4 & 4 \\ 8 & 4 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix}, \text{ this is a symmetric matrix.}$$

$$\frac{A-A^T}{2} = \frac{\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 2 \\ -2 & -2 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}, \text{ this is a skew symmetric matrix.}$$

$$\text{Clearly } \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = A$$

(c) Solve using determinants:  $x + 2y - z = -1$   
 $3x - y - 2z = 5$   
 $x - y - 3z = 0$

$$x + 2y - z = -1$$

$$3x - y - 2z = 5$$

$$x - y - 3z = 0$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} -1 & 2 & -1 \\ 5 & -1 & -2 \\ 0 & -1 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & -2 \\ 1 & -1 & -3 \end{vmatrix}}$$

$$= \frac{-(-3-2)-2(-15)-(-5)}{(3-2)-2(-9+2)-(-3+1)}$$

$$= \frac{-1+30+5}{1+14+2} = \frac{34}{17} = 2$$

$$Y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 1 & -1 & -1 \\ 3 & 5 & -2 \\ 1 & 0 & -3 \end{vmatrix}}{17}$$

$$= \frac{(-15) + (-9 + 2) - (-5)}{17}$$

$$= \frac{-15 - 7 + 5 - 17}{17} = -1$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 5 \\ 1 & -1 & 0 \end{vmatrix}}{17}$$

$$= \frac{(5) - 2(-5) - (-3 + 1)}{17}$$

$$= \frac{5 + 10 + 2}{17} = \frac{17}{17} = 1$$

V.

(a) Expand  $(x + \frac{1}{x})^7$  using binomial theorem.

$$(a + b)^n = a^n + n c_1 a^{n-1} b + n c_2 a^{n-2} b^2 + \dots + n c_n b^n$$

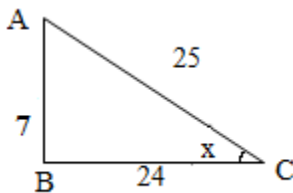
Now

$$(x + \frac{1}{x})^7 = x^7 + 7c_1 x^6 (\frac{1}{x}) + 7c_2 x^5 (\frac{1}{x})^2 + 7c_3 x^4 (\frac{1}{x})^3 + 7c_4 x^3 (\frac{1}{x})^4$$

$$+ 7c_5 x^2 (\frac{1}{x})^5 + 7c_6 x (\frac{1}{x})^6 + 7c_7 (\frac{1}{x})^7$$

$$= x^7 + 7x^5 + 21x^3 + 35x + 35x^{-1} + 21x^{-3} + 7x^{-5} + x^{-7}$$

(b) If  $\tan x = 7/24$  &  $x$  is in the 3<sup>rd</sup> quadrant. Find the value of  $3\sin x - 4\cos x$ .



$$\sin x = -7/25$$

$$[x \in 3^{\text{rd}} \text{ quadrant.}]$$

$$\cos x = -24/25$$

$$\begin{aligned} 3\sin x - 4\cos x &= 3 \times (-7/25) - 4 \times (-24/25) \\ &= (-21/25) + (96/25) = 75/25 = 3 \end{aligned}$$

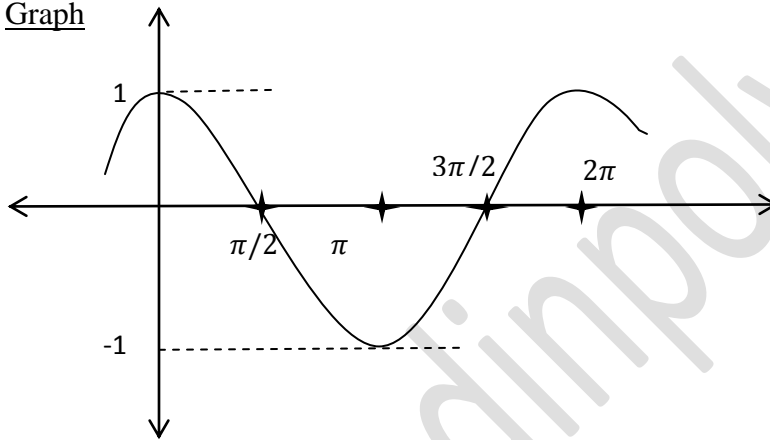
(c) Draw the graph of  $y = \cos 3x$

The graph of  $y = \cos x$

$$X : 0 \quad 90^\circ \quad 180^\circ \quad 270^\circ \quad 360^\circ$$

$$Y : 1 \quad 0 \quad -1 \quad 0 \quad 1$$

Graph



VI.

(a) Find the term independent of  $x$  in the expansion of  $(x + 3/x^3)^{10}$

$$T_{r+1} = n c_r a^{n-r} b^r,$$

$$T_{r+1} = 10 c_r (x)^{10-r} (3/x)^r$$

$$= 10 c_r 3^r x^{10-r} x^{-r}$$

$$= 10 c_r 3^r x^{10-2r}$$

$$10 - 2r = 0$$

$$\implies r = 5$$

$\therefore$  The term independent of  $x$

$$= 10 c_5 3^5 x^0$$

$$= 61236$$

(b) Write the sign of

(i)  $\cot(7\pi/4)$

(ii)  $\tan(500)$

(iii)  $\operatorname{cosec}(280)$ .

$$\begin{aligned} \text{(i) } \cot(7\pi/4) &= \cot(7 \times 45^\circ) = \cot(315) \\ &= \cot(3 \times 90 + 45) \\ &= -\tan 45 = -1 \end{aligned}$$

$\therefore$  Sign is negative.

$$\begin{aligned} \text{(ii) } \tan(500) &= \tan(5 \times 90 + 50) \\ &= -\cot 50 \end{aligned}$$

$\therefore$  Sign is negative.

(iii)  $\operatorname{Cosec}(280)$

$$= \operatorname{cosec}(3 \times 90 + 10) = \frac{1}{\sin(3 \times 90 + 10)} = \frac{1}{-\cos 10}$$

$\therefore$  Sign is negative.

(c) Prove that  $\frac{\tan 45 + \tan 30}{1 + \tan 45 \cdot \tan 30} = 2 - \sqrt{3}$

$$\begin{aligned} \frac{\tan 45 + \tan 30}{1 + \tan 45 \cdot \tan 30} &= \frac{1 + \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \\ &= \frac{(\sqrt{3} - 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{(\sqrt{3} - 1)^2}{\sqrt{3}^2 - 1^2} \\ &= \frac{3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3} \end{aligned}$$

VII.

(a) Prove the formula for  $\cos 3A$

$$\begin{aligned} \sin 3A &= \cos(2A + A) \\ &= \cos 2A \cos A - \sin 2A \sin A \\ &= (2\cos^2 A - 1)\cos A - 2\sin A \cdot \cos A \cdot \sin A \end{aligned}$$

$$\begin{aligned}
&= 2\cos^3 A - \cos A - 2\sin^2 A \cdot \cos A \\
&= 2\cos^3 A - \cos A - 2\sin^2 A \cdot \cos A \\
&= 2\cos^3 A - \cos A - 2(1 - \cos^2 A) \cdot \cos A \\
&= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A \\
&= 4\cos^3 A - 3\cos A
\end{aligned}$$

(b) If  $\sin 18 = (\sqrt{5} - 1)/4$ , find  $\cos 36$  and  $\sin 54$

Put  $\theta = 18^\circ$

$$\cos 36^\circ = \cos 2 \times (18) = \cos 2\theta = 1 - 2\sin^2 \theta$$

$$\begin{aligned}
1 - 2\sin^2 18 &= 1 - 2 \left[ \frac{\sqrt{5}-1}{4} \right]^2 \\
&= 1 - 2 \times 0.9549 \\
&= 0.8090
\end{aligned}$$

$$\sin 54^\circ = \sin 3 \times 18 = \sin 3\theta$$

$$= 3\sin \theta - 4\sin^3 \theta$$

$$\begin{aligned}
3\sin 18 - 4\sin^3 18 &= 3 \left[ \frac{\sqrt{5}-1}{4} \right] - 4 \left[ \frac{\sqrt{5}-1}{4} \right]^3 \\
&= 0.8090
\end{aligned}$$

$$\text{OR } \sin 54^\circ = \sin(90 - 36) = \cos 36 = 0.8090$$

(c) Prove that  $\cos \frac{\pi}{8} + \cos 3\frac{\pi}{8} + \cos 5\frac{\pi}{8} + \cos 7\frac{\pi}{8} = 0$

$$\cos \frac{\pi}{8} + \cos 3\frac{\pi}{8} + \cos 5\frac{\pi}{8} + \cos 7\frac{\pi}{8}$$

$$= \cos \frac{\pi}{8} + \cos 7\frac{\pi}{8} + \cos 3\frac{\pi}{8} + \cos 5\frac{\pi}{8}$$

$$= 2\cos \left( \frac{\frac{\pi}{8} + 7\pi}{2} \right) \cdot \cos \left( \frac{\frac{\pi}{8} - 7\pi}{2} \right) + 2\cos \left( \frac{3\pi + 5\pi}{2} \right) \cdot 2\cos \left( \frac{3\pi - 5\pi}{2} \right)$$

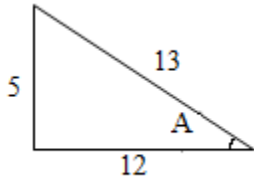
$$= 2\cos \frac{\pi}{2} \cdot \cos \left( -\frac{3\pi}{8} \right) + 2\cos \frac{\pi}{2} \cdot \cos \left( \frac{-\pi}{2} \right)$$

$$= 0 \quad \left[ \cos \frac{\pi}{2} = 0 \right]$$

VIII.

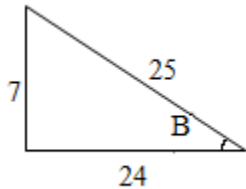
- (a) If  $\cos A = -12/13$ ,  $\cot B = 24/7$  and A is in the quadrant II and B is in quadrant I. find  $\cos(A - B)$

$$\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$



$$\text{Given } \cos A = -\frac{12}{13}$$

$$\therefore \sin A = \frac{5}{13} \quad [A \in 2^{\text{nd}} \text{ quadrant}]$$



$$\cot B = \frac{24}{7}$$

$$\therefore \sin B = \frac{7}{25} \quad [B \in 1^{\text{st}} \text{ quadrant}]$$

$$\cos B = \frac{24}{25}$$

$$\therefore \cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$= -\frac{12}{13} \times \frac{24}{25} + \frac{5}{13} \times \frac{7}{25}$$

$$= \frac{-288 + 35}{325} = \frac{-253}{325}$$

- (b) Prove that  $\cot A - \cot 2A = \operatorname{cosec} 2A$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\cot 2A = \frac{\cos 2A}{\sin 2A}$$

$$\cot A - \cot 2A = \frac{\cos A}{\sin A} - \frac{\cos 2A}{\sin 2A}$$

$$= \frac{\cos A \cdot \sin 2A - \cos 2A \cdot \sin A}{\sin A \cdot \sin 2A}$$

$$= \frac{\sin 2A \cdot \cos A - \cos 2A \cdot \sin A}{\sin A \cdot \sin 2A}$$

$$= \frac{\sin(2A - A)}{\sin A \cdot \sin 2A} = \frac{\sin A}{\sin A \cdot \sin 2A} = \operatorname{cosec} 2A$$

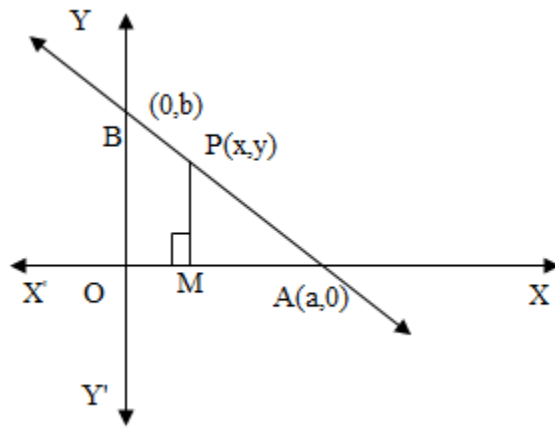
(c) Show that  $\left(\frac{a+b}{c}\right) \sin^2 c/2 = \cos\left(\frac{A+B}{C}\right)$

Question is wrong.

IX.

(a) Derive the equation of a straight line of the form  $\frac{x}{a} + \frac{y}{b} = 1$

The x intercept 'a' and y intercept 'b' are given, consider a straight line AB having x-intercept 'a' and y-intercept 'b'



Hence  $OA = a$  and  $OB = b$

Let  $(x, y)$  be any point on the line.

From the figure it is clear that  $\Delta AMP$  and  $\Delta AOB$  are similar.

$\therefore$  Their corresponding sides are proportional.

$$\frac{OA}{MA} = \frac{OB}{MP} \quad \text{--- (1)}$$

$$MA = OA - OM = a - x$$

$$MP = y$$

Substituting these in (1) we have

$$\frac{a}{a-x} = \frac{b}{y}$$

$$\implies ay = ab - bx$$

$$\implies bx + ay = ab$$

$$\frac{ay}{ba} + \frac{bx}{ab} = \frac{ba}{ab}$$



$$\implies \frac{x}{a} + \frac{y}{b} = 1$$

(b) Find the slope and intercept of the line  $5x - 3y + 15 = 0$

$$5x - 3y = -15$$

$$\text{Slope} = \frac{-a}{b} = \frac{-5}{-3} = \frac{5}{3}$$

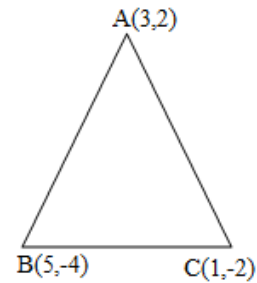
$$5x - 3y = -15$$

$$\frac{5x}{-15} + \frac{-3y}{-15} = 1$$

$$\implies \frac{x}{-3} + \frac{y}{5} = 1$$

$$\therefore \text{Intercept form} = \frac{x}{-3} + \frac{y}{5} = 1$$

(c) Find the angle of triangle having vertices  $(3, 2)$ ,  $(5, -4)$  and  $(1, -2)$



$$\text{Slope of AB} = m_1 = \frac{6}{-2} = -3$$

$$\text{Slope of AC} = m_2 = \frac{4}{2} = 2$$

$$\text{Slope of BC} = m_3 = \frac{2}{-4} = -\frac{1}{2}$$

$$\text{Angle between AB \& AC} = \theta = \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \tan^{-1} \left| \frac{-3 - 2}{1 + -6} \right| = \tan^{-1} \left| \frac{-5}{-5} \right|$$

$$= \tan^{-1}(1) = \frac{\pi}{4} = 45^\circ$$

$$\begin{aligned} \text{Angle between AC \& BC} = \theta &= \tan^{-1} \left| \frac{m_2 - m_3}{1 + m_2 m_3} \right| \\ &= \tan^{-1} \left| \frac{2 + \frac{1}{2}}{1 + 2 \times \frac{1}{2}} \right| = \tan^{-1} \left| \frac{\frac{5}{2}}{0} \right| \\ &= \tan^{-1}(\infty) = \frac{\pi}{2} = 90^\circ \end{aligned}$$

$$\text{Angle between AB \& AC} = 180 - (90 + 45) = 45^\circ$$

X.

- (a) Find the values of P if the lines  $(2P + 1)x - (5 - P)y = 8$  and  $(5P - 1)x - (P + 1)y = 3$  are parallel.

$$\begin{array}{l} (2P + 1)x - (5 - P)y = 8 \quad \text{--- (1)} \\ (5P - 1)x - (P + 1)y = 3 \quad \text{--- (2)} \end{array}$$

$$\text{Slope of (1) is } \frac{-(2P+1)}{-(5-P)}$$

$$\text{Slope of (2) is } \frac{-(5P-1)}{-(P+1)}$$

Since the lines are parallel, we have

$$\frac{-(2P+1)}{-(5-P)} = \frac{-(5P-1)}{-(P+1)}$$

$$(2P + 1)(P + 1) = (5P - 1)(5 - P)$$

$$2P^2 + 2P + P + 1 = 25P - 5P^2 - 5 + P$$

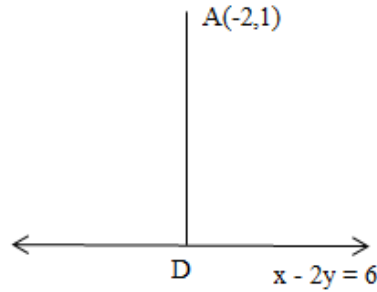
$$7P^2 - 23P + 6 = 0$$

$$P = \frac{23 \pm \sqrt{361}}{14} = \frac{23 \pm 19}{14}$$

$$= 3, 4/14$$

$$3 \text{ or } 2/7$$

(b) Find the foot of the perpendicular from  $(-2, 1)$  on the line  $x - 2y = 6$



Slope of  $x - 2y = 6$  is

$$m_1 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$$

Slope of AD = -2

Equation of AD:  $y - y_1 = m(x - x_1)$

$$y - 1 = -2(x + 2)$$

$$y - 1 = -2x - 4$$

$$2x + y = -3$$

Solving  $x - 2y = 6$  &  $2x + y = -3$

$$x = \frac{\begin{vmatrix} 6 & -2 \\ -3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}} = \frac{6-6}{1+4} = 0$$

Now  $2 \times 0 + y = -3 \implies y = -3$

$\therefore$  Foot of perpendicular is  $(0, -3)$

(c) Straight line cuts off on the axes of coordinates positive intercept whose sum is 5. Given that the line passes through  $(-4, 9)$ , find its equation.

Intercept form of a line is given by

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{--- (1)}$$

Given  $a + b = 5$

$$\implies b = 5 - a$$

Equation (1) passes through  $(-4, 9)$

$$\therefore \frac{-4}{a} + \frac{9}{5-a} = 1$$

$$\implies -4(5 - a) + 9a = a(5 - a)$$

$$\implies -20 + 4a + 9a = 5a - a^2$$

$$\implies a^2 + 8a - 20 = 0$$

$$a = \frac{-8 \pm \sqrt{64 + 80}}{2} = \frac{-8 \pm \sqrt{144}}{2} = \frac{-8 \pm 12}{2} = -10, 2$$

$$\therefore a = 2 \quad b = 3$$

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