TED (10)-1015 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- OCTOBER, 2011

TECHNICAL MATHEMATICS- II

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Evaluate $\lim_{\theta\to 0} \frac{\sin\theta}{6\theta}$

We have $\lim_{\theta\to 0} \frac{\sin m\theta}{n\theta} = \frac{m}{n}$

$\therefore \lim_{\theta\to 0} \frac{\sin\theta}{6\theta} = \frac{5}{6}$

(b) If y = $\sin^2 x$, find $\frac{d}{dx}$

y = $\sin^2 x$

$\therefore \frac{d}{dx} = 2 sinx x cosx$

(c) For what value of ‘x’ will be the tangent to the curve y = $\frac{x}{x^2+1}$

y = $\frac{x}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1) x 1 - x x 2x}{(x^2+1)^2}$ (Apply Quotient Rule)

$= \frac{(x^2+1)- 2x^2}{(x^2+1)^2}$

$\frac{\text{dy}}{\text{dx}} = \frac{1 - x^2}{(1+x^2)^2}$

Since the tangent is parallel to x – axis

$\therefore \frac{\text{d}}{\text{dx}} = 0$

$ie \frac{1 - x^2}{\left(1+x^2\right)^2} = 0$

$ie 1 - x^2 = 0$

$x^2 = 1$

ie x = ± 1

(d) Find $\int \cot x \, dx$

$\int \cot x \, dx = \int \frac{\text{cosx}}{sinx} \, dx$

Put sinx = u

$\frac{\text{du}}{\text{dx}} = \text{cosx}$

du = cosxdx

∴cosx dx = du

$= \int \frac{\text{du}}{\text{u}}$

$= \int \frac{1}{\text{u}} \, du$

= log u

= log sinx + c

(e) Solve $\int \frac{\text{d}\theta}{dt} = \frac{5}{2}\theta$

Apply variable – separable form

$\int \frac{\text{d}\theta}{\theta} = \frac{5}{2}\text{dt}$

Integrating on both sides

$$\int \frac{d\theta}{\theta} = \int \frac{5}{2}\, dt$$

$\log \theta = \frac{5}{2}t + c$

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Evaluate

i. $\lim_{x \to -2} \frac{x^3 + 8}{x+2}$

$$\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$$

Here n = 3, a= -2

$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \times a^{n-1}$

$\therefore \lim_{x \to -2} \frac{x^3 + 8}{x+2} = 3 \times (-2)^{3-1}$

$= 3 \times (-2)^2$

$= 3 \times 4$

$= 12$

ii. Examine whether the function given by $f(x) = \begin{cases} x^2 + 3, & if\ x \neq 0 \\ 1, & if\ x = 0 \end{cases}$ is continuous at x=0

$$f(x) = \begin{cases} x^2 + 3, & if\ x \neq 0 \\ 1, & if\ x = 0 \end{cases}$$

f(0) = 1

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 + 3$$

$= 0^2 + 3$

$= 3$

$$\therefore \lim_{x \to 0} f(x) \neq f(0)$$

∴ Function is not continents at x = 0

(b) Find $\frac{dy}{dx}$ if

i. $y = (1 - 2x + 7x^2)^{10}$

$y = (1 - 2x + 7x^2)^{10}$

$\frac{dy}{dx} = 10(1 - 2x + 7x^2)^9 x\ (-2 + 14x)$

ii. y = log (cosecx – cotx)

y = log (cosecx – cotx)

$\frac{dy}{dx} = \frac{1}{(cosecx - cotx)} x\ (-cosecx.cotx + cosec^2x)$

$= cosecx \frac{cosecx - cotx}{cosecx - cotx}$

=cosecx

(c) The distance‘s’ meters travelled by a particle is given by, $s = ae^{2t} + be^{-2t}$, where,‘t’ represents the time. Prove that the acceleration varies as the distance.

$$s = ae^{2t} + be^{-2t}$$

$\frac{ds}{dt} = ae^{2t}\ x\ 2 + be^{-2t}\ x - 2$

$= 2ae^{2t} - 2be^{-2t}$

$\frac{d^2s}{dt^2} = 4ae^{2t} + 4be^{-2t}$

$= 4\ (ae^{2t} + be^{-2t})$

= 4 x s

$\frac{d^2s}{dt^2}$= k x s , where k = s

Acceleration varies as the distance

(d) the being moment of a rod 10m long and weighing 40kg and resting on support supports at its ends at a distance of ‘x’ meters from one end is given by
$M = 2\ (10x – x^2)$. Find the maximum bending moment?

$M = 2\ (10x – x^2) = 20x – 2x^2$

$\frac{dM}{dx} = 20 – 4x$

$\frac{d^2m}{dx^2}$= - 4 < 0

$\therefore \frac{dm}{dx}$= 0 ==> 20 – 4x

20 = 4x

X = $\frac{20}{4}$ = 5

∴at x = 5, m is maximum

Maximum value, m = 20x – 2x^2

= 20 x 5 – 2 x 5^2

= 100 – 50

= 50

(e) Integrate with respect to x

i. $\frac{1+sinx}{\cos^2 x}$

$$\frac{1 + sinx}{\cos^2 x}$$

$\therefore \int \frac{1+sinx}{\cos^2 x}$dx

= $\int \frac{1}{\cos^2 x}$4 $\int \frac{sinx}{\cos^2 x}$dx

= $\int$ sec^2 $x$dx + $\int \frac{sinx}{\cos^2 x}$ X $\frac{1}{cosx}$dx

= $\int$ sec^2 $x$dx + $\int$ sec x $tanx$ dx

= tanx + secx + c

ii. $\cos^2 2x$

We have cos^{2}2x = $\frac{1+cos4x}{2}$

$\therefore \int cos^2 2x$dx = $\int \frac{1+cos4x}{2}$dx

= $\int \frac{1}{2}$dx +$\int \frac{cos4x}{2}$dx

= $\frac{1}{2}\int$ 1dx + — $\int \frac{cos4x}{2}$dx

$$= \frac{1}{2}x + \frac{1}{2}\frac{sin4x}{4} + c$$

(f) Find $\int_0^2 x^2\, logx$ dx

$\int_0^2 x^2\, logx\,dx = \int_0^2 logx\, x^2$ dx

U = logx

V = x^2

$\int uv dx = u\int v dx - \int\left(\frac{du}{dx}\int v dx\right)dx$

$\int_0^2 logx\, x^2 dx = logx\frac{x^3}{3} - \int\left(\frac{x^3}{3}.\frac{1}{x}\right)dx$

$= logx\frac{x^3}{3} - \int\frac{x^2}{3}dx$

$= \frac{x^3}{3}\ logx - \frac{x^3}{9} + c$

$\therefore \int_0^2 logx\, x^2 dx = \left[\frac{x^3}{3}\ logx - \frac{x^3}{9}\right]$

$= \frac{8}{3}log2 - \frac{8}{3} - 0 - 0$

$= \frac{8}{3}log2 - \frac{8}{9}$

(g) Solve $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

Dividing by $(1 + x^2)$, we get

$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

This is of the form $\frac{dy}{dx} + Py = Q$

Here $p = \frac{1}{1+x^2}$ $Q = \frac{e^{\tan^{-1}x}}{1+x^2}$

$\therefore$ I.F= $e^{\int p\,dx} = e^{\int\frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$

$\therefore$ Solution is Y x IF = $\int$(Q x IF) dx

$$Y \text{ x } e^{\tan^{-1}x} = \int (\frac{e^{\tan^{-1}x}}{1+x^2} \text{ x } e^{\tan^{-1}x})\, dx = \int u\,du \quad \text{where } u = e^{\tan^{-1}x}$$

$$= \frac{e^{2\tan^{-1}x}}{2}$$

$$= \log u$$

$$Y \text{ x } e^{\tan^{-1}x} = \log e^{\tan^{-1}x} + c$$

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0} \frac{(1+x)^{\frac{1}{2}}-1^{\frac{1}{2}}}{(x+1)-1}$$

$$= n\, a^{n-1}$$

$n = \frac{1}{2}$

a= 1

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \frac{1}{2} \text{ x } 1^{\frac{1}{2}-1}$$

$$= \frac{1}{2} \text{ x } 1$$

$$= \frac{1}{2}$$

(b) $Y = \frac{x\tan^{-1}x}{1+x^2}$ find $\frac{dy}{dx}$

$$Y = \frac{x\tan^{-1}x}{1+x^2}$$

$$\frac{dy}{dx} = \frac{(1+x^2)\left[x \text{ x } \frac{1}{1+x^2} + \tan^{-1}x \text{ x } 1\right] - x\tan^{-1}x \text{ x } 2x}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left[\frac{x}{1+x^2}+\tan^{-1}x\right]-2x^2 tan^{-1}x}{(1+x^2)^2}$$

(c) Using 1st principle find the derivatives of cosx

Y= cosx

f(x) = cosx

f(x+Δx) = cos(x+Δx)

$$\therefore \frac{dy}{dx} = \lim_{x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \lim_{x\to 0} \frac{\cos(x+\Delta x)-\cos x}{\Delta x}$$

$$= \lim_{x\to 0} \frac{-2\sin\left(\frac{x+\Delta x+x}{2}\right)\sin\left(\frac{x+\Delta x-x}{2}\right)}{\Delta x}$$

$$= \lim_{x\to 0} \frac{-2\sin\left(\frac{2x+\Delta x}{2}\right)\ \sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

$$= \lim_{x\to 0} \frac{\sin\left(x+\frac{\Delta x}{2}\right)\ \sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$= \lim_{x\to 0} \sin\left(x+\frac{\Delta x}{2}\right) \text{ x } 1$$

$$= -\sin(x+\frac{0}{2})$$

$$= -\sin x$$

(d) If y = $\sin^{-1}x$, prove that $(1 - x^2)y`` - xy` = 0$

$$Y` = \frac{1}{\sqrt{1-x^2}}$$

$$Y`\ \frac{1}{\sqrt{1-x^2}} = 1$$

Differentiating on both sides

$$y`\ \frac{1}{2\sqrt{1-x^2}} \text{ x } -2x + \sqrt{1-x^2} \text{ x } y`` = 0$$

$y` \times \frac{-x}{\sqrt{1-x^2}} + \sqrt{1-x^2} \times y`` = 0$

$\frac{-xy` + (1-x^2)y``}{\sqrt{1-x^2}} = 0$

$(1-x^2)y`` - xy` = 0$

IV.

(a) State the product rule and quotient rule of differentiation

<u>Product rule</u>

$\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

<u>Quotient rule</u>

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$

(b) Find $\frac{dy}{dx}$, if

i. $Y = \frac{1+cosx}{(x+sinx)^3}$

$Y = \frac{1+cosx}{(x+sinx)^3}$

$\frac{dy}{dx} = \frac{(x+sinx)^3 \times -sinx - (1+cosx) \times 3(x+sinx)^2(1+cosx)}{(x+sinx)^4}$

ii. $Y = \log(x+\sqrt{(1+x^2)})$

$Y = \log(x+\sqrt{(1+x^2)})$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{(1+x^2)}} \times \left(1 + \frac{1}{2\sqrt{(1+x^2)}} 2x\right)$

iii. $Y = sec^2(e^x)$

$Y = sec^2(e^x)$

$\frac{dy}{dx}$= 2sec(e^x) x sec(e^x)tan(e^x) x e^x

(c) If x = a($\theta + sin\theta$) = aθ + asinθ

y = a (1 – cosθ) = a – acosθ prove that $\frac{dy}{dx}$= tan $\frac{\theta}{2}$

$\frac{dx}{d\theta}$= a+acos θ

$\frac{dy}{d\theta}$ = asinθ

$$\therefore \frac{dx}{dy} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\text{asin}\theta}{\text{a+acos}\theta}$$

$$= \frac{\text{asin}\theta}{\text{a}(1+\cos\theta)}$$

$$= \frac{\sin\theta}{2\cos^2\left(\frac{\theta}{2}\right)}$$

$$= \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)}$$

$$= \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}$$

$$= \tan\left(\frac{\theta}{2}\right)$$

$\frac{dy}{dx}$= tan $\frac{\theta}{2}$

V.

(a) Find the equation of a tangent and normal to the curve y = 2logx at the point (1, 0)

Y = 2logx

$$\frac{dy}{dx} = \frac{2}{x}$$

$\therefore \frac{dy}{dx}$at (1, 0) = $\frac{2}{1}$= 2

$\therefore$ m = 2

$X_1 = 1$
$Y_1 = 0$
M = 2

Equation of tangent is $y - y_1 = m(x - x_1)$

$$Y - 0 = 2(x - 1)$$
$$Y = 2x - 2$$

Equation of normal is $y - y_1 = -\frac{1}{m}(x - x_1)$

$$Y - 0 = -\frac{1}{2}(x - 1)$$
$$Y = -\frac{1}{2}(x - 1)$$
$$Y = -\frac{1}{2}x + \frac{1}{2}$$

(b) A circular path of oil spreads out on water at the rate of 12 cm^2/min. how fast is the radius increasing when the radius is 2cms

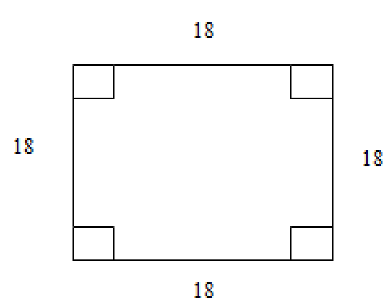
Area of the circle = πr^2

$$A = \pi r^2$$
$$\therefore \frac{dA}{dt} = \pi \times 2r \times \frac{dr}{dt}$$

But given $\frac{dA}{dt} = 12cm^2/min$

$$12 = \pi \times 2r \times \frac{dr}{dt}$$
$$12 = \pi \times 2 \times 2 \times \frac{dr}{dt}$$
$$12 = \pi \times 4 \times \frac{dr}{dt}$$
$$\frac{12}{4\pi} = \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{3}{\pi} cm/min$$

(c) An open box is to be made out a square sheet of side 18cms. By cutting off equal shares at each and turning up the sides. What size of the squares should be cut in order that the volume of the box may be maximum?



Length of the box = 18 – 2x

Breadth of the box = 18 – 2x

Height = x

$\therefore$ Volume v = (18 – 2x)(18 – 2x)x

$= (324 - 36x - 36x + 4x^2)\,x$

$= 324x - 72x^2 + 4x^3$

$V = 4x^3 - 72x^2 + 324x$

$\frac{dv}{dx} = 0$

$12x^2 - 72x + 324 = 0$

$12(x^2 - 12x + 27) = 0$

At a maxima or minima

$\frac{dv}{dx} = 0 \implies 12(x^2 - 12x + 27) = 0$

$ie\, x^2 - 12x + 27 = 0$

$ie\, (x - 9)(x - 3) = 0$

X = 3, or 9

But x = 9 is not possible

$\therefore x = 3$

VI.

(a) If $q = \frac{10p}{p-10}$, find the rate of change of q change with respect to 'p'

$$\frac{dq}{dp} = \frac{(p-10) \text{x} 10 - 10p \text{ x } 1}{(p-10)^2}$$

$$\frac{dq}{dp} = \frac{10p - 100 - 10p}{(p-10)^2}$$

$$\frac{dq}{dp} = \frac{-100}{(p-10)^2}$$

(b) Find the equation of the tangent to the curve x = cos2t, y = sin2t at $t = \pi/8$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{2cos2t}{-2sin2t}$$

$= -\cot 2t$

$\therefore m = -\cot 2\frac{\pi}{8} = -1$

Equation of tangent is

$(x_1,y_1) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

M = -1

$y - y_1 = m(x - x_1)$

$y - \frac{1}{\sqrt{2}} = -1(x - \frac{1}{\sqrt{2}})$

$y - \frac{1}{\sqrt{2}} \;=\; -x + \frac{1}{\sqrt{2}}$

(c) The displacement of a particle at time 't' iss $= \frac{1}{2}t^2 + \sqrt{t}$. Find the velocity at time t = 9 seconds.

$$s = \frac{1}{2}t^2 + \sqrt{t}$$

Velocity = $\frac{ds}{dt} = \frac{1}{2}2t + \frac{1}{2\sqrt{t}}$

$\frac{ds}{dt} = t + \frac{1}{2\sqrt{t}}$

At t = 9secs, $\frac{ds}{dt} = 9 + \frac{1}{2\sqrt{9}}$

$= 9 + \frac{1}{2 \text{ x } 3}$

$= 9 + \frac{1}{6}$

$\frac{ds}{dt} = \frac{55}{6}$

(d) A hollow cylindrical vessel to hold 100cc of water is to be made. So that the area of the metal used is minimum. Prove that the radius which will give minimum area is $\sqrt[3]{\left(\frac{100}{\pi}\right)}$ cms

V = $\pi r^2 h$

100 = $\pi r^2 h$

$\therefore$ h = $\frac{100}{\pi r^2}$ ____________(1)

Surface area S = $\pi r^2 + 2\pi r h$

From (1),

$$S = \pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right)$$

$$S = \pi r^2 + \frac{200}{r} \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$

If the surface area y minimum

1) $\frac{ds}{dr} = 0$

2) $\frac{d^2s}{dr^2} > 0$

$\frac{ds}{dr} = 0$ ===> $\frac{d}{dr}\left(\pi r^2 + \frac{200}{r^2}\right) = 0$

===> $2\pi r - \frac{200}{r^2} = 0$

===> $2\pi r = \frac{200}{r^2}$

===> $r^3 = \frac{200}{2\pi}$

===> $r^3 = \frac{100}{\pi}$

$\therefore r = \sqrt[3]{\left(\frac{100}{\pi}\right)}$

Check whether the surface area is minimum, at this value

$$\frac{d^2s}{dr^2} = \frac{d}{dr}\left(2\pi r + \frac{200}{r^2}\right)$$

$$= 2\pi r + \frac{400}{r^3} > 0$$

$\therefore$ The radius which will give minimum surface area is $\sqrt[3]{\left(\frac{100}{\pi}\right)}$cm

VII.

(a) $\int \frac{x^2}{1+x^6}dx$

Put $u = x^3$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3} = x^2\ dx$

$\int \frac{x^2}{1+x^6} dx = \int \frac{\frac{du}{3}}{1+u^2} dx$

$= \frac{1}{3} \int \frac{1}{1+u^2} dx$

$= \frac{1}{3} \tan^{-1} u$

$= \frac{1}{3} \tan^{-1} x^3 + c$

(b) $\int \frac{1}{1-7x}$

$\int \frac{1}{1-7x} = \frac{\log(1-7x)}{-7} + c$

(c) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{sin2x}{\sin^2 x} dx$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{sin2x}{\sin^2 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{2sinxcosx}{\sin^2 x} dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\frac{cosx}{sin\, x} dx$

$= 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cotx\, dx$

$= 2[log sinx]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= 2[\log \sin\left(\frac{\pi}{2}\right) - logsin(\frac{\pi}{4})]$

$= 2[\log 1 - \log \frac{1}{\sqrt{2}}]$

$= 2[0 - \log \frac{1}{\sqrt{2}}]$

$= 2[-[(\log 1) - \log\sqrt{2}]]$

$= 2[-(0 - \log\sqrt{2})]$

$= 2[\log\sqrt{2}]$

$= \log(\sqrt{2})^2$

$= \log 2$

(d) $\int x^2 e^{x^3}$ dx

U = x^3

$\frac{du}{dx} = 3x^2$

$x^2 dx = \frac{du}{3}$

$\therefore \int x^2 e^{x^3} dx = \int \frac{du}{3} e^u$

$= \frac{1}{3} \int e^u du$

$= \frac{1}{3} e^u$

$= \frac{1}{3} e^{x^3} + c$

(e) $\int \sin^{-1} x$ dx

$\int \sin^{-1} x dx = \int \sin^{-1} x \times 1 \, dx$

$= \sin^{-1} x \times x - \int x \times \frac{1}{\sqrt{1-x^2}} dx$

$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$ ____________(1)

Put u = $1 - x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2} = x dx$

$\int \sin^{-1} x dx = x\sin^{-1} x - \int \frac{-\frac{du}{2}}{\sqrt{u}}$

$= x\sin^{-1} x + \int \frac{1}{2} . \frac{1}{\sqrt{u}} du$

$= x\sin^{-1} x + \sqrt{u}$

$= x\sin^{-1} x + \sqrt{1 - x^2}$

VIII.

(a) Find $\int \frac{4x+2}{x^2+x+3}$dx

u = $x^2 + x + 3$

$\frac{du}{dx}$= 2x + 1

du = (2x + 1) dx

$\int \frac{4x+2}{x^2+x+3}$dx = $\int \frac{2(2x+1)}{(x^2+x+3)}$dx

= $\int \frac{2du}{u}$dx

= 2$\int \frac{1}{u}$du

= 2logu

= 2 log($x^2 + x + 3$)

= log$(x^2 + x + 3)^2$

(b) $\int \frac{\tan^{-1} 5x}{1+25x^2}$dx

u = $\tan^{-1} 5x$

$\frac{du}{dx} = \frac{1}{1+25x^2}$x 5

$\frac{du}{5} = \frac{1}{1+25x^2}$ dx

= $\int u \quad \frac{du}{5}$

= $\frac{1}{5}\frac{u^2}{2}$

= $\frac{1}{5}\frac{(\tan^{-1} 5x)^2}{2}$+ c

(c) $\int \sqrt{4x-3}$dx

= $\int (4x-3)^{\frac{1}{2}}\, dx$

= $\frac{\frac{(4x-3)^{\frac{3}{2}}}{\frac{1}{2}+1}}{4}$

=

= $(4x-3)^{\frac{3}{2}}$ x $\frac{1}{6}$ +c

(d) $\int_0^1 x^2(x+1)^2$dx

$\int_0^1 x^2(x+1)^2$dx= $\int_0^1 x^2(x^2+2x+1)$ dx

= $\int_0^1(x^4+2x^3+x^2)$dx

=$[\frac{x^5}{5}+\frac{2x^4}{4}+\frac{x^3}{3}]_0^1$

=$\frac{1}{5}+\frac{2}{4}+\frac{1}{3}$

=$\frac{1}{5}+\frac{1}{2}+\frac{1}{3}$

=$\frac{1}{5}+\frac{5}{6}$

$=\frac{6+25}{30}=\frac{31}{30}$

(e) $\int xcosx\ dx$

$\int xcosx\ dx$ = x sinx – $\int sinx$ x 1 dx

= x sinx + $\int sinx\ dx$

= x sinx + cosx + c

IX.

(a) Find the area enclosed between one arch of the curve sin3x and the x axis

Y = sin3x x = 0, $\frac{\pi}{3}$

∴ Area A = $\int_a^b y\ dx$

A = $\int_0^{\frac{\pi}{3}} sin3x\ dx$

= $[-\frac{cos3x}{3}]_0^{\frac{\pi}{3}}$

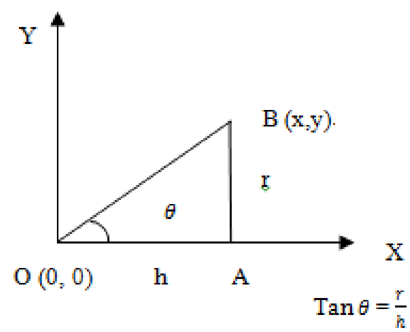
= $(-\frac{cos\pi}{3}+\frac{cos0}{3})$

= $(-\frac{-1}{3}+\frac{1}{3})$

$= (\frac{1}{3} + \frac{1}{3})$

$= \left(\frac{2}{3}\right)$square units

(b) Show by integration, the volume of a right circular cone of height h & base radius is $\frac{1}{3}\pi r^2 h$



Y= mx

Y = tan θ x

Y = $\frac{r}{h}$ x

$\therefore$ V=$\pi \int_0^h y^2 dx$

$= \pi \int_0^h (\frac{r}{h}\text{ x})^2 dx$

$= \pi \int_0^h \frac{r^2}{h^2} \text{x}^2 dx$

$= \frac{\pi r^2}{h^2} [\frac{x^3}{3}]_0^h$

$= \frac{\pi r^2}{h^2} \frac{h^3}{3}$

$= \frac{1}{3}\pi r^2 h$

(c) Solve x$\frac{dy}{dx}$+ y = x^2

Dividing both sides by x

$$\frac{dy}{dx} + \frac{y}{x} = \frac{x^2}{x}$$

$$\frac{dy}{dx} + \frac{1}{x}y = x$$

This is the form $\frac{dy}{dx} + \text{Py} = \text{Q}$

P = $\frac{1}{x}$ Q = x

I.F = $e^{\int P\,dx}$

$= e^{\int \frac{1}{x} dx} = e^{log x} = x$

∴ Solution is

Y x IF = $\int (Q \text{ x IF}) dx$

Y x x= $\int (x \text{ x } x) dx$

Yx = $\int x^2 dx$

xy = $\frac{x^3}{3}$+ c

X.

(a) Find the area bounded by the curve y = x^2 and y = 3x

f(x) = x^2

g(x) = 3x

x^2 = 3x

x^2 – 3x = 0

x(x – 3) = 0

x = 0 or x = 3

∴ Area = $\int_a^b f(x) - g(x)$ dx

$= \int_0^3 (x^2 - 3x) dx$

$= [\frac{x^3}{3} - \frac{3x^2}{2}]_0^3$

Area = $\frac{3^3}{3} - \frac{3 \text{ x } 3^2}{2} - (\frac{0^3}{3} - \frac{3x0^2}{2})$

$= \frac{27}{3} - \frac{27}{2} - 0$

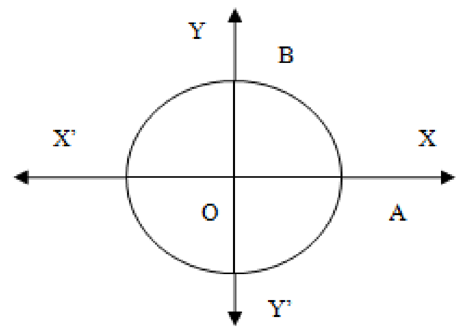
$= \frac{54-81}{6}$

$= -\frac{27}{6}$

$= -\frac{9}{2}$

Area = $\frac{9}{2}$square units (area cannot -ve)

(b) Find the volume of a sphere of radius r



We have V = $\pi \int_a^b y^2\, dx$

∴ Volume of the solid generated by rotation of Quadrant OAB

$$= \pi \int_a^b y^2\, dx$$

$$= \pi \int_a^b r^2 - x^2\, dx$$

$$= \pi \int_0^r r^2 - x^2\, dx$$

$$= \pi[r^2 x - \frac{x^3}{3}]_0^r$$

$$= \pi[r^3 - \frac{r^3}{3}]$$

$$= \pi[\frac{2r^3}{3}]$$

$$= [\frac{2\pi r^3}{3}]$$

$$= \frac{2}{3}\pi r^3$$

∴Volume of sphere = 2 x volume of OAB

$$= 2 \text{ x} \frac{2}{3}\pi r^3$$

$$= \frac{4}{3}\pi r^3$$

(c) solve $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\frac{dy}{\sqrt{1-y^2}} = -\frac{-dx}{\sqrt{1-x^2}}$$

Integrating on both sides we get

$$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = \sin^{-1} x$$

$$\sin^{-1} y + \sin^{-1} x = c$$