TED (10)-1002 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2011

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) If A = $\begin{bmatrix} 4 & 3 \\ 1 & 7 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$ find A – 3B

A – 3B = $\begin{bmatrix} 4 & 3 \\ 1 & 7 \end{bmatrix}$ - 3$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

= $\begin{bmatrix} 4 & 3 \\ 1 & 7 \end{bmatrix}$ - $\begin{bmatrix} 3 & 0 \\ 9 & 6 \end{bmatrix}$ = $\begin{bmatrix} 1 & 3 \\ -8 & 1 \end{bmatrix}$

(b) If $nc_7 = nc_2$ find the value of n?

$nc_r = nc_s$ ===>r + s = n or r = s

∴ n = 7 + 2 = 9

(c) Find the value of $\sin30^o.\cos60^o + \cos30^o.\sin60^o$

sinA.cosB + cosA.sinB = sin(A + B)

$\sin30^o.\cos60^o + \cos30^o.\sin60^o$ = sin(30 +60) = sin90 = 1

(d) Evaluate $\frac{2\tan15^o}{1+\tan^2 15^o}$

$\frac{2\tan\theta}{1+\tan^2\theta}$ = sin2θ =

$\therefore \frac{2\tan 15^o}{1+\tan^2 15^o}$ = sin30° = ½

(e) Write down the equation of the line having slope ½ and y intercept -1

Slope-intercept form of a line is y = mx + c

∴Required equation is y =½ x-1

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Solve using determinants:

x + y – z = 4

3x – y + z = 4

2x – 7y + 3z = -6

AX = B

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 2 & -7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ -6 \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 4 & 1 & -1 \\ 4 & -1 & 1 \\ -6 & -7 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 2 & -7 & 3 \end{vmatrix}}$$

$$= \frac{4(-3+7) - (12+6) - (-28-6)}{(-3+7) - (9-2) - (-21+2)} \qquad = \frac{16-18+34}{4-7+19} = 32/16 = 2$$

$$y = \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix} 1 & 4 & -1 \\ 3 & 4 & 1 \\ 2 & -6 & 3 \end{vmatrix}}{16}$$

$$= \frac{(12+6) - 4(9-2) - (-18-8)}{16} = \frac{18-28+26}{16} = 16/16 = 1$$

$$z = \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 1 & 1 & 4 \\ 3 & -1 & 4 \\ 2 & -7 & 6 \end{vmatrix}}{16}$$

$$= \frac{(6+28) - (-18-8) - 4(-21+2)}{16} = -16/16 = -1$$

(b) If $A = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 2 & 3 \\ 2 & -5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 4 & 1 \\ 3 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$ be two matrices, find AB & BA

$$AB = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 2 & 3 \\ 2 & -5 & 7 \end{bmatrix} \begin{bmatrix} -2 & 4 & 1 \\ 3 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 13 & 11 \\ 20 & -3 & 12 \\ 9 & 20 & 13 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 4 & 1 \\ 3 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ -1 & 2 & 3 \\ 2 & -5 & 7 \end{bmatrix} = \begin{bmatrix} -8 & 1 & 15 \\ 14 & -9 & 17 \\ 17 & -9 & 32 \end{bmatrix}$$

(c) Find the middle term in the expression of $(2x - {}^{3}/_{x})^{11}$

n = 11 n + 1 = 12

so 6^{th} and 7^{th} terms are middle terms

$T_{r+1} = nc_r x^{n-r} y^r$

$T_6 = 11c_5 (2x)^6 (3/x)^5$

$= 11c_5 2^6 x^6 3^5 x^{-5}$

$= 7185024x$

$T_7 = 11c_6 (2x)^5 (3/x)^6$

$= 11c_6 2^5 x^5 3^6 x^{-6}$

$= 10777536x^{-1}$

(d) Prove that

(i) $(\cot A - 1)^2 + (\cot A + 1)^2 = 2\mathrm{cosec}^2 A$

$(\cot A - 1)^2 = \cot^2 A - 2\cot A + 1$

$(\cot A + 1)^2 = \cot^2 A + 2\cot A + 1$

$\therefore (\cot A - 1)^2 + (\cot A + 1)^2 = 2\cot^2 A + 2 = 2(\cot^2 + 1) = 2\mathrm{cosec}^2 A$

(ii) $\frac{1+\sin A}{\cos A} = \frac{\cos A}{1-\sin A}$

$$\frac{1+\sin A}{\cos A}\frac{(1-\sin A)}{(1-\sin A)} = \frac{1^2-\sin^2 A}{\cos A(1-\sin A)} = \frac{\cos^2 A}{\cos A(1-\sin A)} = \frac{\cos A}{1-\sin A}$$

(e) Evaluate tan22½^o without using tables

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

put θ = 22 ½^o

then $\tan 2(22\tfrac{1}{2}^o) = \frac{2\tan 22\tfrac{1}{2}^o}{1-\tan^2 22\tfrac{1}{2}^o}$

$\tan 45^o = \frac{2\tan\theta}{1-\tan^2\theta}$

$1 \quad = \frac{2\tan\theta}{1-\tan^2\theta}$

$2\tan\theta = 1 - \tan^2\theta$ ——— (1)

===> $\tan^2\theta + 2\tan\theta - 1 = 0$

Put $\tan\theta = \tan 22\tfrac{1}{2}^o = x$

Then (1) becomes $x^2 + 2x - 1 = 0$

===> $x = \frac{-2\pm\sqrt{4}+4}{2} = \frac{-2\pm\sqrt{8}}{2}$

x = tan22 ½^o = $-1+\sqrt{2}$ is admissible

(f) Prove that $\frac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A} = \tan 3A$

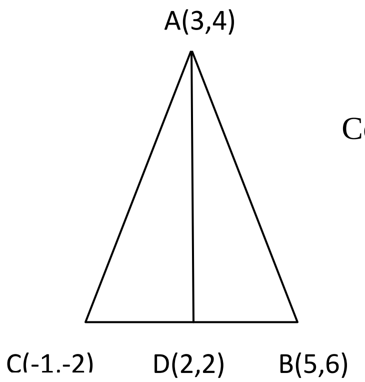
$$\frac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A} = \frac{\sin A+\sin 5A+\sin 3A}{\cos A+\cos 5A+\cos 3A}$$

$$= \frac{2\sin 3A.\cos^{-} 2A+\sin 3A}{2\cos 3A.\cos^{-} 2A+\cos 3A}$$

$$=\frac{\sin 3A(2\cos 2A+1)}{\cos 3A(2\cos 2A+1)}$$

$$=\tan 3A$$

(g) The vertices of a triangle are A(3,4), B(5,6) and C(-1,-2). Find the equation to the median through A



Coordinates of

$$D = \left(\frac{5-1}{2},\frac{6\pm2}{2}\right)$$
$$= (2,2)$$

∴equation of AD = $\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$

===> $\frac{y-4}{2-4} = \frac{x-3}{2-3}$

$\frac{y-4}{-2} = \frac{x-3}{-1}$

===> $\frac{y-4}{-2} = \frac{x-3}{-1}$

===> y – 4 = 2x – 6
===> 2x – y = 2

PART –C
(maximum mark : 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If $\begin{vmatrix} 2 & 1 & x \\ 3 & -1 & 2 \\ 1 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 4 & x \\ 3 & 2 \end{vmatrix}$ find x

$\begin{vmatrix} 2 & 1 & x \\ 3 & -1 & 2 \\ 1 & 1 & 6 \end{vmatrix}$ = 2(-6 - 2) – (18 – 2) + x(3 + 1)

= -16 -16 + 4x

= 4x – 32

$\begin{vmatrix} 4 & x \\ 3 & 2 \end{vmatrix}$ = 8 – 3x

∴ 4x – 32 = 8 – 3x

===> 7x = 40

===> x = 40/7

(b) If A is a square matrix, show that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric.

$(A + A^T)^T = A^T + (A^T)^T = A^T + A$ $\quad [(A + B)^T = A^T + B^T]$

∴ $A + A^T$ is symmetric

$(A - A^T)^T = A^T - (A^T)^T = A^T – A = -(A – A^T)$

∴ $A - A^T$ is skew symmetric

(c) Solve the following system of equations using the inverse of coefficient matrix

x - y + z = 4

2x + y - 3z = 0

x + y + z = 2

AX = B

$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

First we find the inverse of A

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 10$$

<u>Minors</u>

$m_{11} = \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 4$

$m_{12} = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 5$

$m_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$m_{21} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$

$m_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$m_{23} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$

$m_{31} = \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 2$

$m_{32} = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -5$

$m_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3$

Minor matrix = $\begin{bmatrix} 4 & \mathbf{5} & 1 \\ \mathbf{-2} & 0 & \mathbf{2} \\ 2 & \mathbf{-5} & 3 \end{bmatrix}$

Cofactor matrix = $\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$

Adjoint matrix = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

So inverse matrix = $\dfrac{Adj.A}{|A|} = \dfrac{\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}}{10}$

∴ Solution in $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}}{-17}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{\begin{bmatrix} \frac{20}{10} \\ -\frac{10}{10} \\ \frac{10}{10} \end{bmatrix}}{-17} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

x = 2 y = -1 z = 1

IV.

(a) Solve for x if $\begin{vmatrix} 3 & 1 & 9 \\ 2x & 2 & 6 \\ x^2 & 3 & 3 \end{vmatrix} = 0$

$$\begin{vmatrix} 3 & 1 & 9 \\ 2x & 2 & 6 \\ x^2 & 3 & 3 \end{vmatrix} = 3(6 - 18) - (6x - 6x^2) + 9(6x - 2x^2) = 0$$

$= -36 - 6x + 6x^2 + 54x - 18x^2 = 0$

$= -12x^2 + 48x - 36 = 0$

$= 12(-x^2 + 4x - 3) = 0$

$= 12(x^2 - 4x + 3) = 0$

$= x^2 - 4x + 3 = 0$

$$\Longrightarrow x = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} = 3,1$$

(b) If $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix}$

Show that (A + B)C = AC + BC

$$(A + B)C = \left(\begin{bmatrix} 0 & 1 & -1 \\ 2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \end{bmatrix} \right) \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 58 & 12 \\ 3 & 58 & 12 \end{bmatrix} \quad \text{—— (1)}$$

$$AC + BC = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1\ 1 & 1 \\ 6 & 710 & 15 \end{bmatrix} + \begin{bmatrix} 4 & 4\ \ 7 & 11 \\ -3 & -2-2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 58 & 12 \\ 3 & 58 & 12 \end{bmatrix} \quad \text{——— (2)}$$

From (1) and (2) we have (A + B)C = AC + BC

(c) Find the inverse of $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} = 3\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix}$$

= 3(2 - 3) + 2(4 + 4) + 3(-6 -4)

= -3 + 16 -30 = -17

|A| = – 17

<u>Minors</u>

$m_{11} = \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix}$ = -1

$m_{12} = \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix}$ = 8

$m_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix}$ = -10

$m_{21} = \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix}$ = 5

$m_{22} = \begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix}$ = -8

$m_{23} = \begin{vmatrix} 3 & -2 \\ 4 & -3 \end{vmatrix}$ = -1

$m_{31} = \begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix}$ = -1

$m_{32} = \begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix}$ = -7

$m_{33} = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix}$ = 7

Minor matrix = $\begin{bmatrix} -1 & \mathbf{8} & -10 \\ \mathbf{5} & -8 & \mathbf{-1} \\ -1 & \mathbf{-7} & 7 \end{bmatrix}$

Cofactor matrix = $\begin{bmatrix} -1 & -8 & -10 \\ -5 & -8 & 1 \\ -1 & 7 & 7 \end{bmatrix}$

Adjoint matrix = $\begin{bmatrix} -1 & -5 & -1 \\ -8 & -8 & 7 \\ -10 & 1 & 7 \end{bmatrix}$

So inverse matrix = $\frac{\text{Adj.A}}{|A|} = \frac{\begin{bmatrix} -1 & -5 & -1 \\ -8 & -8 & 7 \\ -10 & 1 & 7 \end{bmatrix}}{-17}$

$$= \begin{bmatrix} \frac{1}{17} & \frac{5}{17} & \frac{1}{17} \\ \frac{8}{17} & \frac{6}{17} & -\frac{9}{17} \\ \frac{10}{17} & -\frac{1}{17} & -\frac{7}{17} \end{bmatrix}$$

V.

(a) If $nC_{n-2} = 210$ find the value of 'n'

$nC_{n-2} = 210$

we have $nC_{n-r} = nC_r$

$\Longrightarrow nC_{2} = 210$

$\Longrightarrow n(n-1)/2 = 210$

$\Longrightarrow n^2 - n = 420$

$\Longrightarrow n^2 - n - 420 = 0$ $\qquad n = \frac{1 \pm \sqrt{1-1680}}{2} = \frac{1 \pm 41}{2} = 21, -20$

n = 20 is admissible

$\therefore$ n = 21

(b) Find the term independent of x in the expansion of $(2x^2 + 1/x^2)^{15}$

$T_{r+1} = nc_r\, a^{n-r} b^r$

$= 15c_r\, (2x^2)^{15-r}\, (1/x)^r$

$= 15c_r\, (2^{15-r})(x^{30-2r})\, x^{-r} = 15c_r\, (2^{15-r}\, (x^{30-3r}) = 15c_r 2^{15-r}\, x^{30-3r}$

Then $30 - 3r = 0$

$\Longrightarrow 3r = 30$

$\Longrightarrow r = 10$

$\therefore T_{11} = 15c_{10} 2^5 x^0$

Term independent of x = T_{11} = 96096

(c) Prove that $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\text{cosec}A$

$$\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)\sin A} = \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{(1+\cos A)\sin A}$$

$$= \frac{1 + 1 + 2\cos A}{(1+\cos A)\sin A} = \frac{2(1+\cos A)}{(1+\cos A)\sin A} = 2\text{cosec}A$$

VI.

(a) Expand $(x^3 – 1/x^2)^5$ binomially

We know $(a + b)^n = a^n + nc_1\, a^{n-1}\, b + nc_2\, a^{n-2}\, b^2 + \ldots\ldots\ldots + b^n$

Thus $(x^3 – \frac{1}{x^2})^5 = (x^3)^5 + 5c_1(x^3)^4 (\frac{1}{x^2}) + 5c_2(x^3)^3 (\frac{1}{x^2})^2$

$- 5c_3 (x^3)^2 (\frac{1}{x^2})^3 + 5c_4 (x^3)^1 (\frac{1}{x^2})^4 + (\frac{1}{x^2})^5$

$= x^{15} – 5 \times x^{12} \times \frac{1}{x^2} + 10x^9\, x^{-4} – 10x^6\, x^{-6} + 5x^3\, x^{-8} – x^{-10}$

$= x^{15} – 5x^{10} + 10x^5 – 10 + 5x^{-5} – x^{-10}$

(b) Find the coefficient of x^{10} in the expansion of $(2x^2 – 3/x)^{11}$

$T_{r+1} = nc_r\, a^{n-r} b^r$

$T_{r+1} = 11c_r\, (2x^2)^{11-r}\, (-3/x)^r$

$= 11c_r 2^{11-r}\, x^{22-2r}(-3)^r x^{-r} = 11c_r 2^{11-r}(-3)^r x^{22-3r}$

Now $22 – 3r = 10$

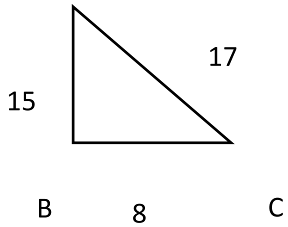
$-3r = -12$

$r = 4$

$T_5 = 11c_4 2^7 (-3)^4\, x^{10} = 3421440x^{10}$

Coefficient of x^{10} is 3421440

(c) If $\cot A = -\frac{15}{8}$, $A \in 4^{rd}$ quadrant. Find all other trigonometric functions.



$\cot A = -\frac{15}{8}$

$\tan A = -\frac{8}{15}$ [A ∈ 4^{rd} quadrant.]

$\sin A = -\frac{8}{17}$ $\operatorname{cosec} A = -\frac{17}{8}$

$\cos A = \frac{15}{17}$ $\sec A = \frac{17}{15}$

VII.

(a) Show that $\tan 15^o + \cot 15^o = 4$ without using tables

$\tan 15^o = \tan(45 - 30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 . \tan 30}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$

$\cot 15^o = \frac{1}{\tan 15} = \frac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$

$\therefore \tan 15^o + \cot 15^o = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

(b) Show that $\sin 10^o . \sin 50^o . \sin 70^o = 1/8$

$\sin 10 (\sin 50 . \sin 70) = \sin 10 \text{ x } -\frac{1}{2}[\cos 120 - \cos(-20)]$

$= -\frac{1}{2} \sin 10 [\cos 120 - \cos 20]$

$= -\frac{1}{2} \sin 10 [-\cos 60 - \cos 20]$

$= -\frac{1}{2} \sin 10 [-1/2 - \cos 20]$

$= \frac{1}{4} \sin 10 + \frac{1}{2} \sin 10 . \cos 20$

$= \frac{1}{4} \sin 10 + \frac{1}{2} . \frac{1}{2} [\sin 30 + \sin(-10)]$

$= \frac{1}{4} \sin 10 + \frac{1}{4} \text{ x } \frac{1}{2} - \frac{1}{4} \sin 10$

$= \frac{1}{8}$

(c) State and prove Napier's formulae

<u>Statement</u>

in any ΔABC, $\tan\left(\frac{B-C}{2}\right) = \left(\frac{B-C}{B+C}\right).\cot A/2$

<u>Proof:</u>

Consider$\left(\frac{B-C}{B+C}\right)\cot A/2 = \frac{2R\sin B - 2R\sin C}{2R\sin B + 2R\sin C}.\cot A/2$

$= \frac{2R(\sin B - \sin C)}{2R(\sin B + \sin C)}.\cot A/2 = \frac{(\sin B - \sin C)}{2R(\sin B + \sin C)}.\cot A/2$

$= \frac{2\cos\frac{B-C}{2}.\ \sin\left(\frac{B-C}{2}\right)}{2\sin\left(\frac{B-C}{2}\right).\ \cos\left(\frac{B-C}{2}\right)}.\cot A/2$

$= \cot\left(\frac{B-C}{2}\right).\tan\left(\frac{B-C}{2}\right).\cot A/2$

$= \tan\left(\frac{B-C}{2}\right).\cot(90 - \frac{A}{2}).\cot A/2$

$= \tan\left(\frac{B-C}{2}\right).\tan A/2.\cot A/2$

$= \tan\left(\frac{B-C}{2}\right)$

VIII.

(a) Express √3cosx + sinx in the form Rsin(x+2)

$\sqrt{3}\cos x + \sin x = R.\sin(x + \alpha)$

$= R.\sin x.\cos\alpha + R\cos x.\sin\alpha$

Equating the similar terms on both sides,

$\sqrt{3}\cos x = R\sin\alpha.\cos\alpha$

$\text{Sin}x = R\sin x.\cos\alpha$

$\Longrightarrow \sqrt{3} = R\sin\alpha$ —— (1)

$\Longrightarrow 1 = R\cos\alpha$ —— (2)

Squaring and adding (1) & (2)

$3 + 1 = R^2\sin^2\alpha + \cos^2\alpha$

$4 = R^2 \quad \Longrightarrow R = \pm 2$

(1) (2)

÷ $\Longrightarrow \sqrt{3} = \frac{\sin\alpha}{\cos\alpha} \Longrightarrow \tan\alpha = \sqrt{3}$

$\Longrightarrow \alpha = \tan^{-1}(\sqrt{3})$

$\Longrightarrow \alpha = 60^o$

(b) Derive expression for sin3A and cos3A

(i) $\text{Sin}3A = \sin(2A + A)$

$= \sin 2A\cos A + \cos 2A\sin A$

$= 2\sin A\cos A.\cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A\cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A\,(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

(ii) $\text{Cos}3A = \cos(2A + A)$

$= \cos 2A.\cos A - \sin 2A.\sin A$

$= (2\cos^2 A - 1)\cos A - 2\sin A.\cos A.\sin A$

$= 2\cos^3 A - \cos A - 2\sin^2 A.\cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$

(c) Prove that bc.cosA + ca..cosB + .ab.cosC $= \frac{a^2+b^2+c^2}{2}$

Using cosine rule,

$a^2 = b^2 + c^2 - 2bc.\cos A$ ——— (1)

$b^2 = a^2 + c^2 - 2ac.\cos B$ ——— (2)

$c^2 = a^2 + b^2 - 2ab.\cos C$ ——— (3)

Adding (1), (2) and (3) we get

$a^2 + b^2 + c^2 = 2(b^2 + c^2 + a^2) - 2(bc.\cos A + ac.\cos B + ab.\cos C)$

$\Longrightarrow 2(b^2 + c^2 + a^2) - (a^2 + b^2 + c^2) = 2(bc.\cos A + ac.\cos b + ab.\cos C)$

===> 2(bc.cosA + ca.cosB + ab.cosC)

∴ bc.cosA + ca..cosB + .ab.cosC $= \frac{a^2+b^2+c^2}{2}$

IX.

(a) Solve the Δ with a = 4cm, b = 5cm and c = 7cm

A = $\cos^{-1}(\frac{b^2+c^2-a^2}{2bc}) = \cos^{-1}(\frac{5^2+7^2-4^2}{2\times5\times7})$

$= \cos^{-1}(\frac{58}{70})$

$= \cos^{-1}(0.8286) = 34^o03'$

B = $\cos^{-1}(\frac{a^2+c^2-b^2}{2ac}) = \cos^{-1}(\frac{4^2+7^2-5^2}{2\times4\times7}) = \cos^{-1}(\frac{40}{56})$

$= \cos^{-1}(0.7143) = 44^o25'$

C = $180^o - (A + B) = 180^o - (34^o03' + 44^o25') = 101^o32'$

(b) The x intercept of a line is 3 times its y- intercept. The line passes through (-6,3). Find its equation.

The intercept form of a line is

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{3b} + \frac{y}{b} = 1$ ——— (1)

Given that a = 3b

(1) Passes through (-6,3) then,

$\frac{-6}{3b} + \frac{3}{b} = 1$

$\frac{-2}{b} + \frac{3}{b} = 1$

===> 1/b = 1 ===> b = 1

∴ Equation of the line is $\frac{x}{3} + \frac{y}{1} = 1$ or x + 3y = 1

(c) Find the value of 'k' for which the line :

3x + y – 2 = 0
kx + 2y – 3 = 0
2x – y – 3 = 0 is concurrent

It is concurrent

$\therefore \begin{vmatrix} 3 & 1 & -2 \\ k & 2 & -3 \\ 2 & -1 & -3 \end{vmatrix} = 0$

$= 3\begin{vmatrix} 2 & -3 \\ -1 & -3 \end{vmatrix} - 1\begin{vmatrix} k & -3 \\ 2 & -3 \end{vmatrix} - 2\begin{vmatrix} k & 2 \\ 2 & -1 \end{vmatrix} = 0$

= 3(-6 – 3) – 1(-3k + 6) – 2(-k – 4) = 0

= -27 + 3k – 6 + 2k + 8 = 0

5k – 25 = 0

5k = 25

$K = \frac{25}{5} = 5$

X.

(a) Solve ΔABC given a = 87cm , b = 53cm and C = 70°

$\tan(\frac{A-B}{2}) = \frac{a-b}{a+b} \cot \frac{C}{2}$

$A - B = 2\tan^{-1}[\frac{a-b}{a+b} \cot \frac{C}{2}]$

$A - B = 2\tan^{-1}[\frac{87-53}{87+53} \cot 35]$

$= 2\tan^{-1}[\frac{34}{140} \cot 35^o]$

$= 2\tan^{-1}[0.3469] = 2 \times 19^o08' = 38^o16'$

$A + B = 180 - 70 = 110^o$

$A = 148^o16'/2 = 74^o08'$

$B = 110 - 74^o08' = 35^o52'$

Now we have to find ‘c’, we have

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{87}{\sin 74^o08'} = \frac{c}{\sin 70^o}$$

$$===> c = \frac{\sin 70\ 87}{\sin 74^o 08'} = \frac{0.9397 \text{ x } 87}{0.9619} = 84.99\text{cm}$$

(b) Find the equation to the straight line passing through the point of intersection of the lines 2x – y – 3 = 0 and x – 2y + 1 = 0 and

(i) Parallel and

(ii) Perpendicular to the line x – y = 5

2x – y – 3 = 0

x – 2y + 1 = 0

$$x = \frac{\begin{vmatrix} 3 & -1 \\ -1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}} = \frac{-6-1}{-3} = -\frac{7}{-3} = \frac{7}{3}$$

$$y = \frac{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}}{-3} = -\frac{5}{-3} = \frac{5}{3}$$

Case I

Equation of a line Parallel to the line x – y = 5 is x – y + k = 0

x – y + k passes through $(\frac{7}{3}, \frac{5}{3})$

We have $\frac{7}{3} - \frac{5}{3} + k = 0$

$===> k = -2/3$

∴ Required equation is

$x - y - \frac{2}{3} = 0$

ie, 3x – 3y = 2

Case II: perpendicular to the line x – y = 15

Equation of the line perpendicular to x – y = 5 is –x – y + k = 0 which passing through $(\frac{7}{3}, \frac{5}{3})$

We have $\frac{-7}{3} - \frac{5}{3} + k = 0$

$===> k = 4$

∴ Required equation is

x – y + 4 = 0

or x + y = 4

(c) Find the point of intersection of the line 2x – 3y = 11 and 3x + 4y = 8

2x – 3y = 11 ——— (1)

3x + 4y = 8 ——— (2)

$$x = \frac{\begin{vmatrix} 11 & -3 \\ 8 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 8 & 4 \end{vmatrix}} = \frac{44+24}{8+9} = \frac{68}{17} = 4$$

$$y = \frac{\begin{vmatrix} 2 & 11 \\ 3 & 8 \end{vmatrix}}{17} = \frac{16-33}{17} = \frac{-17}{17} = -1$$