TED (10)-1002 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2012

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks:100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) If A = $\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$ find 2A + B

2A + B = $2\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 6 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 11 & 2 \end{bmatrix}$

(b) If $nc_{n-2} = 28$ what is the value of n?

$nc_{n-2} = nc_2$ ===> $\frac{n(n-1)}{2}$

$\frac{n(n-1)}{2} = 28$

$n^2 - n = 56$

$n^2 - n - 56 = 0$

n = $\frac{1 \pm \sqrt{1+224}}{2} = \frac{1 \pm \sqrt{225}}{2}$

= 16/2 or -14/2

= 8 or -7

n = 8 is admissible.

(c) If $\sin\theta = \frac{3}{5}$, find $\cos\theta$ and $\tan\theta$

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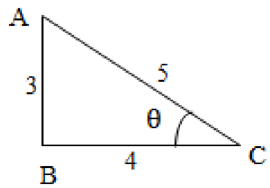

sinθ = $\frac{3}{5}$ BC = $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$

cosθ = $\frac{4}{5}$

tanθ = $\frac{3}{4}$

(d) If sinA = 0.8, A is acute find cos2A

Cos2A = $1 - 2\sin^2 A = 1 - 2(0.8)^2$

$= 1 - 2 \times 0.64 = -0.28$

(e) Find the slope of the line joins the point (7,4) & (5,-2)

slope = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-4}{5-7} = \frac{-6}{-2} = 3$

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PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Find k, if the following system of equation are consistent
x + y + 1 = 0, x + 2y + 1 = 0, 2x+3y + k = 0

If the system is consistent then,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & k \end{vmatrix} = 0$$

$1\begin{vmatrix} 2 & 1 \\ 3 & k \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & k \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 0$

1(2k – 3) – (k – 2) + (3 - 4) = 0

2k – 3 – k + 2 – 1 = 0

k – 2 = 0

k = 2

(b) If $A = \begin{bmatrix} 1 & 4 & 3 \\ -4 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 5 \\ -1 & 1 \end{bmatrix}$ find AB & BA and show that AB ≠ BA

$AB = \begin{bmatrix} 1 & 4 & 3 \\ -4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 5 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 25 \\ -5 & -7 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 2 \\ 0 & 5 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 3 \\ -4 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -7 & 4 & 5 \\ -20 & 0 & 5 \\ -5 & -4 & -2 \end{bmatrix}$

Clearly AB ≠ BA

(c) Find the 4th term in the expansion of $(x^2 - \frac{1}{x})^9$

$T_{r+1} = (-1)^r nc_r a^{n-r} b^r$

$T_{r+1} = (-1)^3\, 9c_3 (x^2)^6 (\frac{1}{x})^3$

$= 9c_3 x^{12} (-1)^3\, x^{-3}$

$= -9c_3\, x^9$

(d) Prove that $\frac{\text{cosecA}}{\text{cosecA}-1} + \frac{\text{cosecA}}{\text{cosecA}+1} = 2\sec^2 A$

L.H.S $= \frac{\text{cosecA}(\text{cosecA}+1) + \text{cosecA}(\text{cosecA}-1)}{(\text{cosecA}-1)(\text{cosecA}+1)}$

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$$= \frac{\text{cosec}^2\text{A} + \text{cosecA} + \text{cosec}^2\text{A} - \text{cosecA}}{\text{cosec}^2\text{A} - 1}$$

$$= \frac{2\text{cosec}^2\text{A}}{\cot^2 \text{A}} = \frac{2\frac{1}{\sin^2 \text{A}}}{\frac{\cos^2 \text{A}}{\sin^2 \text{A}}} = 2\sec^2\text{A}$$

(e) Prove that $\frac{\tan60^o - \tan45^o}{1+\tan60^o.\tan45^o} = 2 - \sqrt{3}$

$$\text{L.H.S} = \frac{\tan60^o - \tan45^o}{1+\tan60^o.\tan45^o} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{3-1} = \frac{3+1-2\sqrt{3}}{2} = 2 - \sqrt{3}$$

(f) Show that sin10. sin50. sin70 = 1/8

$$\sin10\ (\sin50.\ \sin70) = \sin10 \times \frac{-1}{2}[\cos120 - \cos(-20)\ ]$$

$$= \frac{-1}{2}\sin10\ [\cos120 - \cos20]$$

$$= \frac{-1}{2}\sin10\ [-\cos60 - \cos20]$$

$$= \frac{-1}{2}\sin10\ [\frac{-1}{2} - \cos20]$$

$$= \frac{1}{4}\sin10 + \frac{1}{2}\sin10\ .\ \cos20$$

$$= \frac{1}{4}\sin10 + \frac{1}{2}.\frac{1}{2}[\sin30 + \sin(-10)]$$

$$= \frac{1}{4}\sin10 + \frac{1}{4} \times \frac{1}{2} - \frac{1}{4}\sin10$$

$$= \frac{1}{8}$$

(g) Find the angle between two lines with slope $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$

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$$\text{Tan}\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \text{ x } \frac{1}{\sqrt{3}}}\right| = \left|\frac{\frac{3-1}{\sqrt{3}}}{1+1}\right| = \left|\frac{\frac{2}{\sqrt{3}}}{2}\right| = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^{o} = \frac{\pi^{c}}{6}$$

PART –C

Answer four full questions. Each question carries 15 marks.

III.

(a) Solve for z if $\begin{vmatrix} 2 & 3 & 5 \\ 2 & z & 5 \\ 3 & -1 & 2 \end{vmatrix} = 0$

$\begin{vmatrix} 2 & 3 & 5 \\ 2 & z & 5 \\ 3 & -1 & 2 \end{vmatrix} = 0$ ===>2(2z + 5) – 3(4 - 15) + 5(-2 - 3z) = 0

===> 4z + 10 – 12 + 45 – 10 – 152 = 0

===> -11z = -33

$\therefore$z = 3

(b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 1 & 3 & 2 \end{bmatrix}$ compute $A + A^T$ and show that $A + A^T$ is symmetric.

$$A + A^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 1 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 4 \\ 2 & -4 & 9 \\ 4 & 9 & 4 \end{bmatrix}$$

Clearly $(A + A^T)^T = \begin{bmatrix} 2 & 2 & 4 \\ 2 & -4 & 9 \\ 4 & 9 & 4 \end{bmatrix}$

$\therefore A + A^T$is symmetric.

(c) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 4 & 1 \end{bmatrix}$

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Cofactors

$m_{11} = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = 1 - 8 = -7$

$m_{12} = \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = -4$

$m_{13} = \begin{vmatrix} 0 & 1 \\ 2 & 4 \end{vmatrix} = -2$

$m_{21} = \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = -10$

$m_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1 - 6 = -5$

$m_{23} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0$

$m_{31} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$

$m_{32} = \begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = 2$

$m_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$

Cofactor matrix = $\begin{bmatrix} -7 & 4 & -2 \\ 10 & -5 & 0 \\ 1 & -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj.A}}{|A|} = \frac{\begin{bmatrix} -7 & 10 & 1 \\ 4 & -5 & -2 \\ -2 & 0 & 1 \end{bmatrix}}{|A|} = \frac{\begin{bmatrix} -7 & 10 & 1 \\ 4 & -5 & -2 \\ -2 & 0 & 1 \end{bmatrix}}{-5}$

IV.

(a) Solve using determinants

2a- 3b + c = -1
a + 4b – 2c = 3
4a – b +3c = 11

2a- 3b + c = -1
a + 4b – 2c = 3
4a – b +3c = 11

AX = B

$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 11 \end{bmatrix}$

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$$a = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} -1 & -3 & 1 \\ 3 & 4 & -2 \\ 11 & -1 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ 4 & -1 & 3 \end{vmatrix}}$$

$$= \frac{-(12-2) + 3\,(9+22) + 1(-3-44)}{2(12-2)+3(3+8)+(-1-16)}$$

$$= \frac{-10+93-47}{20+33-17}$$

$$= \frac{36}{36} = 1$$

$$b = \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 4 & 11 & 3 \end{vmatrix}}{36}$$

$$= \frac{2(9+22)+1(3+8)+1(11-12)}{36}$$

$$= \frac{62+11-1}{36}$$

$$= \frac{72}{36} = 2$$

$$c = \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 2 & -3 & -1 \\ 1 & 4 & 3 \\ 4 & -1 & 11 \end{vmatrix}}{36}$$

$$= \frac{2(44+3)+3(11-12)-(-1-16)}{36}$$

$$= \frac{94-3+17}{36}$$

$$= \frac{108}{36} = 3$$

(b) If $A = \begin{bmatrix} 4 & 1 & 0 \\ 2 & 1 & 5 \\ -1 & 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ Show that A(B + C) = AB + AC

$$A\,(B + C) = \begin{bmatrix} 4 & 1 & 0 \\ 2 & 1 & 5 \\ -1 & 1 & 0 \end{bmatrix}\left(\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}\right)$$

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$$= \begin{bmatrix} 4 & 1 & 0 \\ 2 & 1 & 5 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 & 6 \\ 0 & 3 & 0 \\ 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 3 & 24 \\ 28 & -2 & 37 \\ -4 & 3 & -6 \end{bmatrix} \quad \text{——— (1)}$$

$$AB + AC = \begin{bmatrix} 4 & 1 & 0 \\ 2 & 1 & 5 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 0 \\ 2 & 1 & 5 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -2 & 13 \\ 15 & -10 & 32 \\ -1 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 7 & 5 & 11 \\ 13 & 8 & 5 \\ -3 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 3 & 24 \\ 28 & -2 & 37 \\ -4 & 3 & -6 \end{bmatrix} \quad \text{——— (2)}$$

∴ A(B + C) = AB + AC

(c) Solve the following system of equations by finding the inverse of their coefficient matrix.

3x + y – z = 3

x + y + z = 1

x + y + z = 3

$$\begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

∴ Inverse does not exist.

V.

(a) Prove that $nc_r = n-1_{c_{r-1}} + n-1_{c_r}$

$$\text{R.H.S} = n-1_{c_{r-1}} + n-1_{c_r} = \frac{(n-1)!}{(r-1)!(n-1-(r-1))!} + \frac{(n-1)!}{r!(n-1-r)!}$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r!(n-1-r)!}$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r)(n-r-1)!} + \frac{(n-1)!}{r(r-1)!(n-r-1)!}$$

$$= \frac{(n-1)!}{(r-1)!(n-r-1)!}\left[\frac{1}{n-r} + \frac{1}{r}\right]$$

$$= \frac{(n-1)!}{(r-1)!(n-r-1)!}\left[\frac{r+n-r}{r(n-r)}\right]$$

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$$= \frac{(n-1)!}{(r-1)!(n-r-1)!}\left[\frac{n}{r(n-r)}\right]$$

$$= \frac{(n-1)!}{r(r-1)!(n-r)(n-r-1)!} = \frac{n!}{r!(n-r)!} \quad = n_{c_r}$$

(b) Find the middle term of $\left(2a + \frac{b}{3}\right)^{10}$

n = 10 n + 1 = 11, odd

∴ 6^{th} term is the middle term.

$T_{r+1} = n_{c_r} a^{n-r} b^r$

$T_6 = 10_{c_5}\ (2a)^5 (\frac{b}{3})^5$

$= 10_{c_5} 2^5 a^5 \frac{b^5}{3^5}$

$= 10_{c_5} 2^5 \text{ x } \frac{1}{3^5} \text{x } a^5 b^5$

$= \frac{8064}{243} a^5 b^5$

(c) Prove that $\frac{1+\sin A}{\cos A} = \frac{\cos A}{1+\sin A}$

L.H.S = $\frac{1+\sin A}{\cos A} = \frac{1+\sin A(1-\sin A)}{\cos A(1-\sin A)}$

$= \frac{1+\sin^2 A}{\cos A(1-\sin A)}$

$= \frac{\cos^2 A}{\cos A(1-\sin A)} = \frac{\cos A}{1-\sin A}$

VI.

(a) Expand $\left(x^2 + \frac{1}{x^2}\right)^7$ binomially.

$(a + b)^n = a^n + nc_1\ a^{n-1}\ b + nc_2\ a^{n-2}\ b^2 + \dots\dots\dots + b^n$

$\left(x^2 + \frac{1}{x^2}\right)^7 = (x^2)^7 + 7c_1(x^2)^6(\frac{1}{x^2})^1 + 7c_2(x^2)^5(\frac{1}{x^2})^2$

$+ 7c_3(x^2)^4(\frac{1}{x^2})^3 + 7c_4(x^2)^3(\frac{1}{x^2})^4 + 7c_5(x^2)^2(\frac{1}{x^2})^5$

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$+7c_6(x^2)(\frac{1}{x^2})^6 + 7c_7(\frac{1}{x^2})^7$

$= x^{14} + 7x^{12}\times\frac{1}{x^2} + 21x^{10}\times\frac{1}{x^4} + 35x^8\times\frac{1}{x^6} +$

$35x^6\times\frac{1}{x^8} + 21x^4\times\frac{1}{x^{10}} + 7x^2\times\frac{1}{x^{12}} + \frac{1}{x^{14}}$

$= x^{14} + 7x^{10} + 21x^6 + 35x^2 + 35x^{-2} + 21x^{-6} + 7x^{-10} + x^{-14}$

(b) Find the constant term in the expansion of $(\sqrt{x} + \frac{2}{x^2})^{10}$

$T_{r+1} = nc_r\, a^{n-r}b^r$

$= 10c_r\, (\sqrt{x})^{10-r}(\frac{2}{x^2})^r$

$= 10c_r x^{\frac{10-r}{2}} 2^r \times x^{-2r} = 10c_r x^{\frac{10-r}{2}-2r} 2^r$

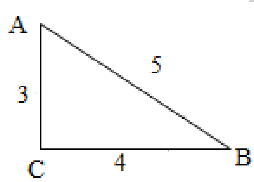
$\frac{10-r}{2} - 2r = 0$

$$\frac{10-r}{2} = 2r$$

$10 - r = 4r$ ===> $10 = 5r$ ===> $r = 2$

So constant term is $T_3 = 10c_2 \times 2^2 = 180$

(c) If sinB = 3/5, B lies in second quadrant, find all other t-functions.



$BC = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$

$\sin B = \frac{3}{5}$

$\cos B = -\frac{4}{5}$ [$B \in 2^{nd}$ quadrant]

$\tan B = -\frac{3}{4}$

$\cot B = -\frac{4}{3}$

$\sec B = -\frac{5}{4}$

$\text{cosec}B = \frac{5}{3}$

VII.

(a) If A + B = 45^{O} , show that (1 + tanA)(1 + tanB) = 2

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$

$\tan 45^{o} = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$

$1 = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$

$===> 1 - \tan A.\tan B = \tan A + \tan B$

$===> \tan A.\tan B + \tan A + \tan B = 1$

Adding 1 on both sides

$1 + \tan A.\tan B + \tan A + \tan B = 2$

===>(1 + tanA)(1 + tanB) = 2

(b) Show that $\sin 18^{o} = \frac{\sqrt{5}-1}{4}$

Let $\theta = 18^{o}$ $\quad 2\theta = 36^{o}$

Sin2θ = sin36 = sin(90 - 54) = cos54 = cos3θ

ie, sin2θ = cos3θ

$2\sin 2\theta.\cos\theta = 4\cos^{3}\theta - 3\cos\theta$

$2\sin\theta = 4\cos^{2}\theta - 3$

$2\sin\theta = 4(1 - \sin^{2}\theta) - 3$

$2\sin\theta = 4 - 4\sin^{2}\theta - 3$

$2\sin\theta = (1 - 4\sin^{2}\theta)$

$===> 4\sin^{2}\theta + 2\sin\theta - 1 = 0$

Let sinθ = x then we have

$4x^{2} + 2x - 1 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4 \text{ x } 4 \text{ x} -1}}{8} = \frac{-2 \pm \sqrt{20}}{8}$$

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$= \frac{-1\pm\sqrt{5}}{4} = \frac{\sqrt{5}-1}{4}$ or $\frac{-1-\sqrt{5}}{4}$

Since 18 is an acute angle, we have

$$\sin 18 = \frac{\sqrt{5}-1}{4} \; (+\text{ve value})$$

(c) State and prove sine rule

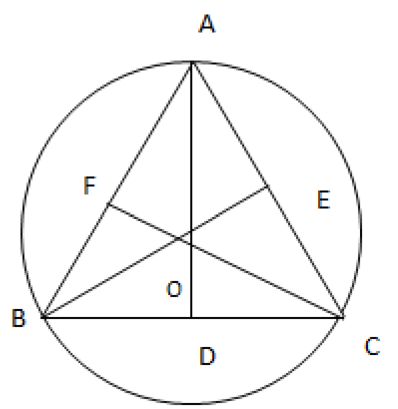
Statement

In any ΔABC $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

Proof

Consider the circumcircle of ΔABC.

The perpendicular bisectors of the sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$ intersect at 'O'. Therefore 'O' is the circumcentre such that OA = OB = OC = R



We have $< BOC = 2< BAC = 2A$

So $< BOD = < COD = A$

In ΔODB , sin<BOD = sinA = BD/OB = $\frac{\frac{a}{2}}{R}$

$\sin A = \frac{a}{2R}$ ==> a = 2RsinA

Similarly,

$\sin B = \frac{b}{2R}$ ==>b = 2RsinB

siCB = $\frac{c}{2R}$ ==>c = 2RsinC

It is clear that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

VIII.

(a) Prove that $\frac{\cos A - \sin A}{\cos A + \sin A} = \tan(45 - A)$

$$\frac{\cos A - \sin A}{\cos A + \sin A} = \frac{\frac{\cos A}{\cos A} - \frac{\sin A}{\cos A}}{\frac{\cos A}{\cos A} + \frac{\sin A}{\cos A}}$$

$$= \frac{1 - \tan A}{1 + \tan A}$$

$$= \frac{\tan 45 - \tan A}{1 + \tan 45 . \tan A}$$

$$= \tan(45 - A)$$

(b) Prove that $\frac{\sin x + \sin 2x}{\cos x + \cos 2x} = \tan\frac{3x}{2}$

We have $\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

$$\text{L.H.S} = \frac{\sin x + \sin 2x}{\cos x + \cos 2x} = \frac{2\sin\frac{1}{2}(x+2x)\cos\frac{1}{2}(x-2x)}{2\cos\frac{1}{2}(x+2x)\cos\frac{1}{2}(x-2x)}$$

$$= \frac{\sin\frac{3x}{2}}{\cos\frac{3x}{2}} = \tan\frac{3x}{2}$$

(c) In a ΔABC, $R(a^2 + b^2 + c^2) = abc(\cot A + \cot B + \cot C)$

$$\text{R.H.S} = abc\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right)$$

$$= abc\frac{\cos A}{\sin A} + abc\frac{\cos B}{\sin B} + abc\frac{\cos C}{\sin C}$$

$$= \frac{a}{\sin A}bc.\cos A + \frac{b}{\sin B}ac.\cos B + \frac{c}{\sin C}ab.\cos C$$

$$= 2R.bc.\cos A + 2R.ac.\cos B + 2R.ab.\cos C$$

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$$= 2R\left[bc\,.\frac{b^2+c^2-a^2}{2bc} + ac.\frac{a^2+c^2-b^2}{2ac} + ab.\frac{a^2+b^2-c^2}{2ab}\right]$$

$$= 2R\left[\frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2}\right]$$

$$= 2R\left[\frac{a^2+b^2+c^2}{2}\right]$$

$= R(a^2 + b^2 + c^2) =$ L.H.S

IX.

(a) Solve the ΔABC given a = 24.5 b = 18.6 c = 26.4

$$\tan A/2 = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \qquad \text{where } s = \frac{a+b+c}{2} = \frac{24.5+18.6+26.4}{2} = 34.75$$

$$= \sqrt{\frac{(34.75-18.6)(34.75-26.4)}{34.75 \text{ x } 10.25}}$$

$$= \sqrt{\frac{16.15 \text{ x } 8.35}{34.75 \text{ x } 10.25}}$$

$\tan A/2 = 0.6153 ===> A/2 = \tan^{-1}(0.6153) = 31^o36'$

and $A = 2 \text{ x } 31^o36' = 63^o12'$

$$\tan B/2 = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$= \sqrt{\frac{(34.75-24.5)(34.75-26.4)}{34.75\ (34.75-18.6)}}$$

$$= \sqrt{\frac{10.25 \text{ x } 8.35}{34.75 \text{ x } 16.15}} = 0.3905$$

$\tan B/2 = 0.6153 ===> B/2 = \tan^{-1}(0.3905) = 21^o20'$

and $B = 2 \text{ x } 21^o20' = 42^o40'$

$C = 180 – (A + B) = 18 – (63^o12' + 42^o40') = 74^o08'$

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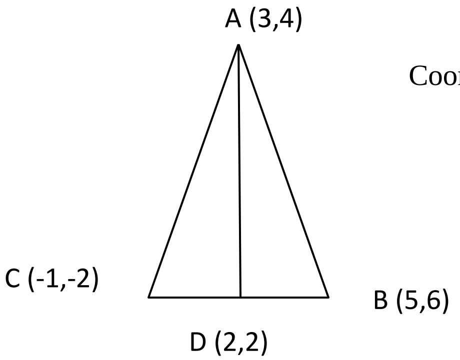
(b) The vertices of a triangle are A(3,4), B(5,6) and C(-1,-2). Find the equation to the median through A.



Coordinates of D = $\left(\frac{-1+5}{2}, \frac{-2+6}{2}\right)$ = (2, 2)

Equation of AD = $\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$

===> $\frac{y-4}{2-4} = \frac{x-3}{2-3}$

===> $\frac{y-4}{-2} = \frac{x-3}{-1}$ ===> 2x – y = 2

(c) Prove that the lines 2x – 3y – 7 = 0
3x + 4y – 10 = 0 are concurrent.
8x + 11y – 5 = 0

$$\therefore \begin{vmatrix} 2 & -3 & -7 \\ 3 & -4 & -10 \\ 8 & 11 & -5 \end{vmatrix}$$

= $2\begin{vmatrix} -4 & -10 \\ 11 & -5 \end{vmatrix} + 3\begin{vmatrix} 3 & -10 \\ 8 & -5 \end{vmatrix} - 7\begin{vmatrix} 3 & -4 \\ 8 & 11 \end{vmatrix}$

= 2(20 + 110) + 3(-15 + 80) – 7(33 + 32)

= 2 x 130 + 3 x 65 – 7 x 65

= 260 + 195 – 455

= 455 – 455

= 0

∴ The lines are concurrent.

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X.

(a) Solve ΔABC, given a = 87cm, b = 53cm and C = 110^o

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b} \cdot \cot\frac{C}{2}$$

$$A - B = 2\tan^{-1}\left[\left(\frac{a-b}{a+b}\right) . \cot\left(\frac{C}{2}\right)\right]$$

$$= 2\tan^{-1}\left[\left(\frac{87-53}{87+53}\right) . \cot(\frac{110}{2})\right.$$

$$= 2\tan^{-1}\left[\left(\frac{34}{140}\right) . \cot 55\right]$$

$$A - B = 2\tan^{-1}[0.24285 \text{ x } 0.70020]$$

$$= 2\tan^{-1}[0.170043] = 19.30087 = 19^o18'$$

$A + B = 180 - 110 = 70^o$ ——— (1)

$A - B = 19^o18'$ ——— (2)

Solving (1) and (2) we get

$2A = 89^o18'$

$A = 44.650435$

$= 44^o39'$

$B = 70^o - 44^o39'$

$= 25^o21'$

Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$C = \frac{\sin 110^o \text{ x } 87}{\sin 44^o 39'} = 116.32\text{cm}$$

(b) Find the value of ‘q’ for which the straight line 8qx + (2 – 3q)y + 1 = 0 and qx + 8y = 7 = 0 are perpendicular.

$8qx + (2 - 3q)y + 1 = 0$ ——— (1)

$qx + 8y = 7 = 0$ ——— (2)

Slope of (1) is $\frac{-a}{b} = \frac{-8q}{2-3q}$

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Slope of (2) is $\frac{-a}{b} = \frac{-q}{8}$

If (1) and (2) are perpendicular then

Slope of (1) x slope of (2) = -1

$\frac{-8q}{2-3q} \times \frac{-q}{8} = -1$

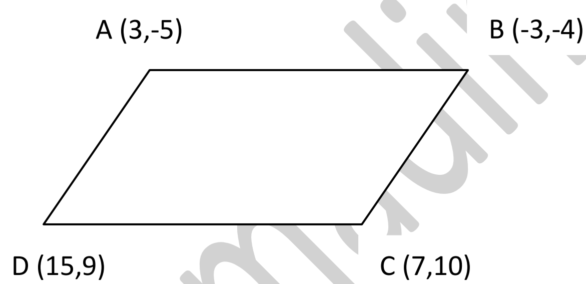
$\frac{q^2}{2-3q} = -1$

$q^2 = 3q - 2$

$q^2 - 3q + 2 = 0$

$q = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = 2,1$

(c) Prove that the points(3,-5), (-5,-4), (7,10) and (15,9) then in order are the vertices of a parallelogram



Slope of AB = $\frac{-4+5}{-5-3} = \frac{1}{-8} = \frac{-1}{8}$

Slope of CD = $\frac{10-9}{7-15} = \frac{1}{-8} = \frac{-1}{8}$

$\therefore$ AB & CD are parallel

Slope of AD = $\frac{-5-9}{3-15} = \frac{-14}{-12} = \frac{7}{6}$

Slope of BC = $\frac{-4-10}{-5-7} = \frac{-14}{12} = \frac{7}{6}$

$\therefore$ AD & BC are parallel

Hence the result

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