TED (10)-1002 Reg. No. ………………………….

(REVISION-2010) Signature …….……………………

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- OCTOBER, 2010

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Find the values of a, b, c and d given that,

$$\begin{bmatrix} 2a & 3b \\ 5-c & d \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 7 & 0 \end{bmatrix}$$

Here 2a = 4 ===> a=4/2=2

a+3b=11 ===>2+3b=11 ===> 3b=9 ===> b=9/3=3

5-c = 7 ===> c = 5-7 = -2

(b) Evaluate $\begin{vmatrix} sinA & cosA \\ -cosA & sinA \end{vmatrix}$

$\begin{vmatrix} sinA & cosA \\ -cosA & sinA \end{vmatrix}$= sinA x sinA - -cosA x cosA

= $\sin^2A + \cos^2A$

= 1

[$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$= a×d-b×c

(c) If nc_{12} = nc_{13} what is the value of n?

$nc_{12} = nc_{13}$ ===> n = 12 + 13 = 25 $\left[nc_r = nc_s \text{===> } r + s = n \text{ or } r = s \right]$

(d) Prove that tanA + cotA = 2cosecA

$$\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = \frac{1}{\sin A \cos A} = \frac{2}{2\sin A \cos A}$$

$$= \frac{2}{\sin 2A} = 2\text{cosec}\, 2A$$

(e) Find the angle of inclination of the line joins the point (5,3) & (-8,3)

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{-8-5} = \frac{0}{-13} = 0$$

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) If A = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ find AB & BA and show that AB ≠ BA

$$AB = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 12 & 11 \\ -1 & 7 & 8 \\ -2 & -1 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 9 & 13 \\ -1 & 2 & 4 \\ -2 & 2 & 4 \end{bmatrix}$$

Clearly $AB \neq BA$

(b) Solve the following equations using cramer's rule.

$2x + 3y - z = 5$
$x - 2y + 3z = 6$
$3x - y + 2z = 7$

Here

$$\Delta x = B$$

$x = \frac{\Delta 1}{\Delta}$
$y = \frac{\Delta 2}{\Delta}$
$z = \frac{\Delta 3}{\Delta}$

$$x = \frac{\begin{vmatrix} 5 & 3 & -1 \\ 6 & -2 & 3 \\ 7 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & -1 \\ 1 & -2 & 3 \\ 3 & -1 & 2 \end{vmatrix}} = 14/14 = 1$$

$$y = \frac{\begin{vmatrix} 2 & 5 & -1 \\ 1 & 6 & 3 \\ 3 & 7 & 2 \end{vmatrix}}{14} = \frac{28}{14} = 2$$

$$z = \frac{\begin{vmatrix} 2 & 3 & 5 \\ 1 & -2 & 6 \\ 3 & -1 & 7 \end{vmatrix}}{14} = \frac{42}{14} = 3$$

(c) Obtain the coefficient of x^{12} in the expansion $(x^2 - \frac{1}{x^2})^{10}$

$T_{r+1} = (-1)^r nc_r a^{n-r}\, b^r$

$T_{r+1} = (-1)^r\, 10c_r\, (x^2)^{10-r}\, (1/x^2)^r$

$= (-1)^r 10c_r x^{20-2r}\, (1)^r / x^{2r}$

$= (-1)^r 10c_r x^{20-4r}$

But we have 20 – 4r = 12 ===> -4r = -8 ===> r = 2

Then $T_3 = (-1)^2 10c_{2\,x}{}^{12}$

So coefficient of x^{12} is $10c_2$

(d) Prove that $\sin^2 A - \cos^2 A = 2\sin^2 A - 1 = 1 - 2\cos^2 A$

$\sin^2 A - \cos^2 A =$

$= \sin^2 A - (1-\sin^2 A)$

$= \sin^2 A = \sin^2 A - 1$

$= 2\sin^2 A - 1$

Also

$\sin^2 A - \cos^2 A =$

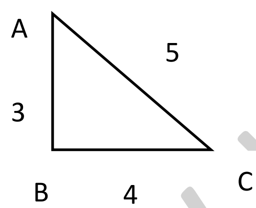
$= (1-\cos^2 A) - \cos^2 A$

$= 1-\cos^2 A-\cos^2 A$

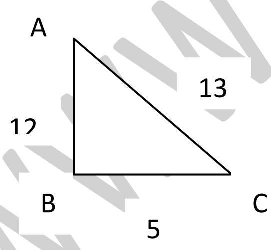
$= 1 - 2\cos^2 A$

Hence the results are same.

(e) If sinA=4/5 and sinB = 12/13, A and B are acute angles, find cos(A – B)



sinA=4/5 cosA=3/5



sinB=12/13 cosB=15/13

cos(A – B) = cosA.cosB + sinA.sinB

= 3/5 × 5/13 + 4/5 × 12/13

= 15/65 + 48/65 = 63/65s

(f) Prove that $\sin(A+B).\sin(A-B) = \sin^2 A - \sin^2 B$

$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$

$\sin(A-B) = \sin A.\cos B - \cos A.\sin B$

$\sin(A+B).\sin(A-B) = (\sin A.\cos B + \cos A.\sin B)\ (\sin A.\cos B - \cos A.\sin B)$

$= \sin^2 A.\cos^2 B - \sin A.\cos A \sin B.\cos B$

$+ \sin A \cos A.\sin B \cos B - \cos^2 A.\sin^2 B$

$= \sin^2 A.\cos^2 B - \cos^2 A.\mathrm{sins}^2 B$

$= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 A.\sin^2 B - \sin^2 B - \sin^2 A.\sin^2 B$

$= \sin^2 A - \sin^2 B$

(g) Express the equation $3x + 4y - 12 = 0$ in (i) slope-intercept form and (ii) intercept form. Hence find the slope and intercept made on the axes.

$3x + 4y = 12$ —— (1)

slope intercept form ➔ $y = mx + c$

$\frac{3x}{12} + \frac{4y}{12} = \frac{12}{12}$

intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{4} + \frac{y}{3} = 1$ —— (2)

$4y = 12 - 3x$

$y = \frac{-3x}{4} + \frac{12}{4}$

$y = \frac{-3x}{4} + 3$ —— (3)

Slope intercept form: $y = \frac{-3x}{4} + 3$

Intercept form : $\frac{x}{4} + \frac{y}{3} = 1$

PART –C

Answer four full questions. Each question carries 15 marks.

III.

(a) If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ find $A^2 - 3A + 5I$

$$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix}$$

$$5I = 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 - 3A + 5I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & -1 \\ 3 & 0 & -4 \\ -3 & 2 & 3 \end{bmatrix}$$

(b) Express the matrix $\begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix}$ as the sum of a symmetric & skew symmetric matrices

$$\frac{A+A^T}{2} = \frac{\begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix}}{2} = \frac{\begin{bmatrix} 2 & -2 & 8 \\ -2 & 2 & 8 \\ 8 & 8 & 14 \end{bmatrix}}{2} = \begin{bmatrix} 1 & -1 & 4 \\ -1 & 1 & 4 \\ 4 & 4 & 7 \end{bmatrix}$$

This is a symmetric matrix.

$$\frac{A-A^T}{2} = \frac{\begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix}}{2} = \frac{\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 4 \\ -2 & -4 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix}$$

This is a skew symmetric matrix.

$$\frac{A+A^T}{2} + \frac{A+A^T}{2} = \begin{bmatrix} 1 & -1 & 4 \\ -1 & 1 & 4 \\ 4 & 4 & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix}$$

Clearly $A = \frac{A+A^T}{2} + \frac{A - A^T}{2}$

That A can be expressed as a sum of symmetric and skew symmetric matrices.

(c) Solve the following system of equations using the inverse of coefficient matrix

3x - 2y + 3z = 4
2x + y - z = 2
4x – 3y + 2z = 3

AX = B

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

$X = A^{-1}B$

<u>Calculation for A^{-1}</u>

$m_{11} = \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = -1$

$m_{12} = \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} = 8$

$m_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix} = -10$

$m_{21} = \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = 5$

$m_{22} = \begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} = -6$

$m_{23} = \begin{vmatrix} 3 & -2 \\ 4 & -3 \end{vmatrix} = -1$

$m_{31} = \begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix} = -1$

$m_{32} = \begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} = -9$

$m_{33} = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 7$

Minor matrix = $\begin{bmatrix} -1 & \mathbf{8} & -10 \\ \mathbf{5} & -6 & \mathbf{-1} \\ -1 & \mathbf{-9} & 7 \end{bmatrix}$

Cofactor matrix of A = $\begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$

Adjoint matrix = $\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix}$

$= 3\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix}$

= 3x -1 + 2 x 8 + 3 x -10

= -3 + 16 – 30 = -17

So inverse matrix = $\dfrac{\text{Adj.A}}{|A|} = \dfrac{\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}}{-17}$

$X = A^{-1}B$

$= \dfrac{\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}}{-17} = \dfrac{\begin{bmatrix} -17 \\ -17 \\ -17 \end{bmatrix}}{-17} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

x = 1 y = 1 z = 1

IV.

(a) If $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ verify that $(A + B)^T = A^T + B^T$

$A + B = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 2 \end{bmatrix}$

$(A + B)^T = \begin{bmatrix} 5 & 2 \\ 7 & 2 \end{bmatrix}$ ——— (1)

$A^T = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 7 & 2 \end{bmatrix}$ ——— (2)

Clearly from (1) (2) $(A + B)^T = A^T + B^T$

(b) Solve for x if $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$(5x – 16) – 2(10 – 12) + 3(8 – 3x) = 0$

$5x – 16 + 4 + 24 -9x = 0$

$-4x = -12$

$x = 3$

(c) If $P = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$ and $Q = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$ verify that $(PQ)^{-1} = Q^{-1}P^{-1}$

$P = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$

$Q = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$

$PQ = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 11 \\ 70 & 49 \end{bmatrix}$

<u>To find $(PQ)^{-1}$</u>

$PQ = \begin{bmatrix} 16 & 11 \\ 70 & 49 \end{bmatrix}$

<u>Minors</u>

$m_{11} = 49$

$m_{21} = 11$

$m_{12} = 70$

$m_{22} = 16$

Minor matrix = $\begin{bmatrix} 49 & \mathbf{70} \\ \mathbf{11} & 16 \end{bmatrix}$

Cofactor matrix = $\begin{bmatrix} 49 & -70 \\ -11 & 16 \end{bmatrix}$

Adj (PQ) = $\begin{bmatrix} 49 & -11 \\ -70 & 16 \end{bmatrix}$

$= 784 – 770$

$= 14$

$(PQ)^{-1} = \frac{Adj.PQ}{|PQ|} = \frac{\begin{bmatrix} 49 & -11 \\ -70 & 16 \end{bmatrix}}{14}$

<u>To find Q^{-1}</u>

$Q = \begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$

<u>Minors</u>

$m_{11} = 5$

$m_{21} = 1$

$m_{12} = 6$

$m_{22} = 4$

Minor matrix $= \begin{bmatrix} 5 & \mathbf{6} \\ \mathbf{1} & 4 \end{bmatrix}$

Cofactor matrix $= \begin{bmatrix} 5 & -6 \\ -1 & 4 \end{bmatrix}$

Adj Q $= \begin{bmatrix} 5 & -1 \\ -6 & 4 \end{bmatrix}$

$|Q| = \begin{vmatrix} 4 & 1 \\ 6 & 5 \end{vmatrix}$

$= 20 - 6 = 14$

$Q^{-1} = \frac{Adj.Q}{|Q|} = \frac{\begin{bmatrix} 5 & -1 \\ -6 & 4 \end{bmatrix}}{14}$

<u>To find P^{-1}</u>

$P = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$

<u>Minors</u>

$m_{11} = 9$

$m_{21} = 2$

$m_{12} = 4$

$m_{22} = 1$

Minor matrix $= \begin{bmatrix} 9 & \mathbf{4} \\ \mathbf{2} & 1 \end{bmatrix}$

Cofactor matrix = $\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}$

Adj P = $\begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$

$|P| = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix} = 1$

$P^{-1} = \frac{\text{Adj.P}}{|P|} = \frac{\begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}}{14}$

$Q^{-1} \times P^{-1} = \frac{\begin{bmatrix} 5 & -1 \\ -6 & 4 \end{bmatrix}}{14} \times \frac{\begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}}{14} = \frac{\begin{bmatrix} 49 & 11 \\ -70 & 16 \end{bmatrix}}{14}$

$\therefore (PQ)^{-1} = Q^{-1}P^{-1}$

V.

(a) Find the middle terms in the expression of $(x^2 + \frac{2}{x})^7$

n = 7   n + 1 = 8, even , $T_{r+1} = nc_r a^{n-r} b^r$

So $\frac{8}{2}^{th}$ and $(\frac{8}{2} + 1)^{th}$ are the middle terms

ie, 4^{th} & 5^{th} are middle terms

$T_4 = 7c_3 (x^2)^4 (\frac{2}{x})^3 = 7c_3\, x^8 \times \frac{2^3}{x^3} = 7c_3 \times 8 \times x^5 = 280x^5$

$T_5 = 7c_4 (x^2)^3 (\frac{2}{x})^4 = 7c_4\, x^6 \times \frac{2^4}{x^4} = 7c_4 \times 2^4 \times x^2 = 560x^5$

(b) Expand $(3x + 2y)^5$

We know $(a + b)^n = a^n + nc_1\, a^{n-1}\, b + nc_2\, a^{n-2}\, b^2 + \ldots\ldots. + nc_n b^n$

Thus $(3x + 2y)^5 = (3x)^5 + 5c_1(3x)^4(2y) + 5c_2(3x)^3(2y)^2$

$+ 5c_3(3x)^2(2y)^3 + 5c_4(3x)^1(2y)^4 + (2y)^5$

$= 243x^5 + 5 \times 81 \times 2x^4y + 10 \times 27x^3 \times 4y^2 +$

$10 \times 9x^2 \times 8 \times y^3 + 5 \times 3x \times 16y^4 + 32y^5$

$= 243x^5 + 810x^4y + 1080\, x3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$

(c) Show that $\sin 60^o \cos 30^o - \cos 60^o \sin 30^o = 1/2$

So $\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

VI.

(a) Find the constant term in the expansion of $(x^3 + 3/x^2)^{15}$

$T_{r+1} = nc_r\, a^{n-r} b^r$

$= 15c_r\, (x^3)^{15-r}\, (3/x^2)^r$

$= 15c_r\, x^{45-3r}\, 3^r/x^{2r} = 15c_r\, x^{45-3r-2r}\, 3^r = 15c_r\, x^{45-5r}\, 3^r$

Now $45 - 5r = 0$

$45 = 5r \implies r = 9$

So $T_{10} = 15c_9\, (x)^0 3^9 = 15c_9 3^9$

So constant term $= 3^9 \times 15c_9 = 98513415$

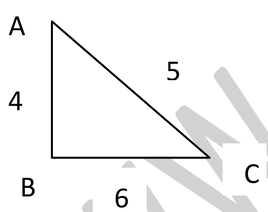
(b) Evaluate $\tan^2 60^o + 3\tan^2 45^o$

$\tan 60 = \sqrt{3}$ $\tan^2 60 = \sqrt{3}^2 = 3$

$\tan^2 45^o = (\tan 45)^2 = (1)^2 = 1$

so $\tan^2 60^o + 3\tan^2 45^o = 3 + 3 \times 1 = 6$

(c) If $\sin\theta = -4/5$, $\theta \in 3^{rd}$ quadrant. Find all other trigonometric functions of θ



$\sin\theta = -4/5$ BC $= \sqrt{5^2 - 4^2} = \sqrt{9} = 3$

So $\cos\theta = -3/5$ ($\theta \in 3^{rd}$ quadrant)

$\tan\theta = 4/3$ $\cot\theta = 3/4$

$\text{cosec}\,\theta = {}^1/_{\sin\theta} = -5/4$

$\sec\theta = {}^1/_{\cos\theta} = -5/3$

VII.

(a) Prove that $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 4\cos\theta.\cos 2\theta.\sin 4\theta$

$\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta$ $\left[\sin C + \sin D = 2\sin {}^{C+D}/_2 . \cos {}^{C-D}/_2\right]$

$= (\sin\theta + \sin 7\theta) + (\sin 3\theta + \sin 5\theta)$

$= 2\sin 4\theta.\cos 3\theta + 2\sin 4\theta.\cos\theta$

$= 2\sin4\theta(\cos3\theta + \cos\theta)$ $\left[\cos C + \cos D = 2^{\cos(C + D/2).\cos(C - D/2)}\right]$

$= 2\sin4\theta \,.\, 2(\cos2\theta \,.\, \cos\theta)$

$= 2\sin4\theta \,.\, \cos2\theta \,.\, \cos\theta$, hence the result.

(b) Prove that $2\tan10^o + \tan40^o = \tan50^o$

Consider $\tan(50 - 40) = \frac{\tan50 - \tan40}{1 + \tan50.\tan40}$

$\tan10^o = \frac{\tan50 - \tan40}{1 + \tan50.\tan(90 - 40)}$

$= \frac{\tan50 - \tan40}{1 + \tan50.\cot50}$

$= \frac{\tan50 - \tan40}{1 + 1}$

$\implies 2\tan10^o = \tan50 - \tan40$

$\implies 2\tan10^o + \tan40^o = \tan50^o$

(c) In any ABC, prove that $a^2 + bc = b^2 + c^2$, if $A = 60^o$

By cosine rule we have $a^2 = b^2 + c^2 - 2bc.\cos A$

$= b^2 + c^2 - 2bc.\cos60^o$

$= b^2 + c^2 - 2bc.1/2$

$= b^2 + c^2 - bc \implies a^2 + bc = b^2 + c^2$

VIII.

(a) Prove that $\cos20^o.\cos40^o.\cos80^o = 1/8$

$\cos20^o.\cos40^o.\cos80^o = \cos20 (\cos40.\cos80)$

$= \cos20 [½ (\cos120 + \cos(-40))]$

$= ½ . \cos20 [\cos120 + \cos40]$

$= ½ . \cos20 [-½ + \cos40]$

$= -¼ \cos20 + ½ \cos20.\cos40$

$= -¼ \cos20 + ¼ \cos60 + ¼ \cos20$

= ¼ × ½ =1/8

(b) If A + B = 45^{O} , show that (1 + tanA)(1 + tanB) = 2

tan(A + B) = tan(A+ tanB) = $\frac{tanA+tanB}{1-tanA.tanB}$

tan45^{o} = $\frac{tanA+tanB}{1-tanA.tanB}$

1 = $\frac{tanA+tanB}{1-tanA.tanB}$

===>$1 - tanA.tanB = tanA + tanB$

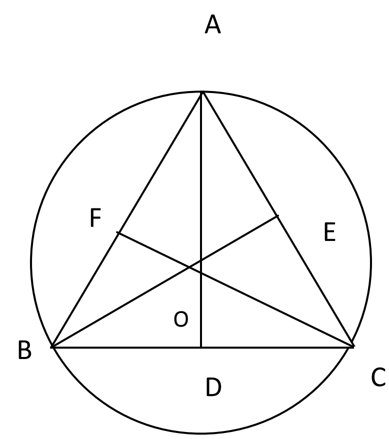
===>$tanA.tanB + tanA + tanB = 1$

Adding 1 on both sides

$1 + tanA.tanB + tanA + tanB = 2$

===>(1 + tanA)(1 + tanB) = 2

(c) State and prove sine rule



Statement

In any ΔABC $^{a}/_{sinA} = {}^{b}/_{sinB} = {}^{c}/_{sinC}$ = 2R

Proof

Consider the circumcircle of ΔABC.

The perpendicular bisectors of the sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$ intersect at 'O'. Therefore 'O' is the circumcentre such that OA = OB = OC = R

We have $< BOC = 2 < BAC$ = 2A

So $< BOD = < COD$ = A

In ΔODB, sin<BOD = sinA = BD/OB = $\frac{\frac{a}{2}}{R}$

sinA = $\frac{a}{2R}$ ==> a = 2RsinA

Similarly,

sinB = $\frac{b}{2R}$ ==>b = 2RsinB

siCB = $\frac{c}{2R}$ ==>c = 2RsinC

It is clear that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

IX.

(a) Solve the Δ with a = 2cm, b = 3cm and c = 4cm

$A = \cos^{-1}(\frac{b^2+c^2-a^2}{2bc}) = \cos^{-1}(\frac{3^2+4^2-2^2}{2\times 3\times 4})$

$= \cos^{-1}(\frac{25-4}{24}) = \cos^{-1}(\frac{21}{24})$

$= \cos^{-1}(0.875) = (28.96)^o = 28^o57'$

$B = \cos^{-1}(\frac{a^2+c^2-b^2}{2ac}) = \cos^{-1}(\frac{2^2+4^2-3}{2\times 2\times 4}) = \cos^{-1}(\frac{4+16-9}{16}) = \cos^{-1}(\frac{11}{16})$

$= \cos^{-1}(0.6875) = 46^o34'$

$C = 180^o - (A + B) = 180^o - (28^o57' + 46^o34') = 104^o29'$

(b) Find the acute angle between the lines 2x – y + 3 = 0 and 3x – 3y + 4 = 0

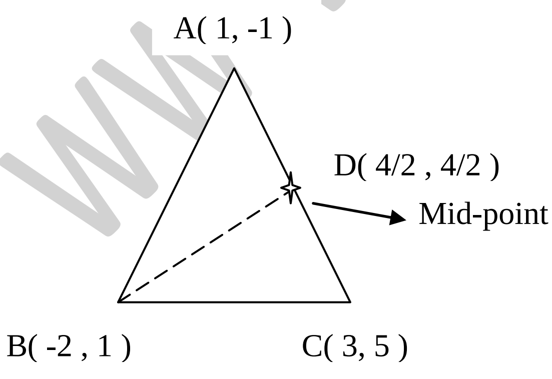
$2x - y = -3$ $\quad\quad m_1 = {}^{-2}/_{-1} = 2$

$3x - 3y = -4$ $\quad\quad m_2 = {}^{-3}/_{-3} = 1$

$\tan\theta = \left|\frac{m1-m2}{1+m1m2}\right| = \left|\frac{2-1}{1+2\times 1}\right| = \left|\frac{1}{3}\right|$

$\theta = \tan^{-1}(1/3) = 18^o26'$

(c) If A(1, -1) , B(-2 , 1) and C(3, 5) are the vertices of a triangle , then find the equation ofmedian through B



Slope of AB $= \frac{y2-y1}{x2-x1}$

$= \frac{1- -1}{-2-1} = 2/3$, (m_1)

Slope of AC $= \frac{5- -1}{3-1} = 6/2 = 3$, (m_2)

Slope of BC $= \frac{5\ -1}{3--2} = 4/5$, (m_3)

Equation of BD

$$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$$

$$\frac{y-y1}{2-1} = \frac{x--2}{2--2}$$

$$\frac{y-1}{1} = \frac{x+2}{4}$$

$y - 1/_1 = x + 2/_4$

4(y – 1) = x + 2

4y – 4 = x + 2

x – 4y = -6

x – 4y + 6 = 0

X.

(a) Using Napier’s formula, find the values of the angles A , B inΔABC , if a = 5cm, b = 8cm and C = 30^o

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$\frac{A-B}{2} = \tan^{-1}\left[\frac{a-b}{a+b} \cot \frac{C}{2}\right]$$

$$\frac{A-B}{2} = \tan^{-1}\left[\frac{5-8}{13} \cot \frac{30}{2}\right]$$

$$\frac{A-B}{2} = \tan^{-1}\left[\frac{-3}{13} \cot 15^o\right]$$

$$\frac{A-B}{2} = \tan^{-1}[-0.8612\ ] = -40.736$$

A – B = -81.473 —— (1)

A + B = 180 – 30 =150 —— (2)

Solving (1) + (2)

A = 34.2635 = 34^O16’ B = 150 - 34.2635 = 115^O44’

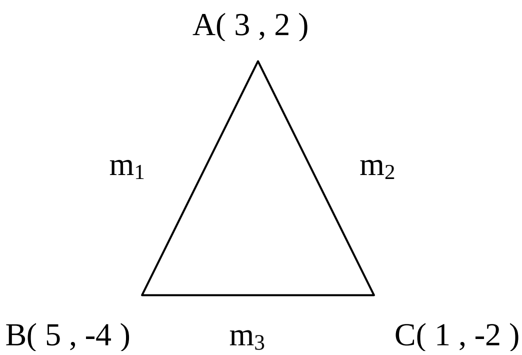
Now we have to find ‘c’

We have $a/\sin A = c/\sin C$

$5/\sin 34^{o}16' = c/\sin 30^{o}$

$c = 5/0.5629996 \times \sin 30^{o} = 4.44\text{cm}$

(b) Find the angles of the triangle having vertices (3 , 2) , (5 , -4) and (1 , -2)



m_1= slope of AB = $(y2 - y1)/(x2 - x1)$

$= (-4 - 2)/(5 - 3)$

$= -6/-2 \quad = -3$

m_2 = slope of AC = $(y2 - y1)/(x2 - x1)$

$= (-2 - 2)/(1 - 3)$

$= -4/-2 = 2$

m_1 = slope of BC = $(y2 - y1)/(x2 - x1)$

$= (-2 — 4)/(1 - 5)$

$= (-2 + 4)/-4$

$= {}^{2}/_{-4} = {}^{-1}/_{2}$

$\tan A = \left|\frac{m1-m2}{1+m1m2}\right| = \left|\frac{-3-2}{1+-3\times 2}\right| = \left|\frac{-5}{1-6}\right| = \left|\frac{-5}{-5}\right| = 1$

$\tan B = \left|\frac{m1-m2}{1+m1m2}\right| = \left|\frac{-3- -{}^{1}/_{2}}{1+3\times{}^{1}/_{2}}\right| = \left|\frac{-3+{}^{1}/_{2}}{1+{}^{3}/_{2}}\right| = \left|\frac{-{}^{5}/_{2}}{{}^{5}/_{2}}\right| = 1$

tanB = 1 ===> $B = \tan^{-1}(1) = 45^o$ tanA = 1 implies $A = \tan^{-1}(1) = 45^0$

so $C = 180 - (A + B) = 180^o - 90^o = 90^o$

(c) Show that the three lines are concurrent

5x + 2y – 4 = 0

2x – 5y + 11 = 0

3x – 4y -18 = 0

$$\begin{vmatrix} 5 & 2 & -4 \\ 2 & 5 & 11 \\ 3 & -4 & -18 \end{vmatrix} =$$

$$= 5\begin{vmatrix} 5 & 11 \\ -4 & -18 \end{vmatrix} - 2\begin{vmatrix} 2 & 11 \\ 3 & -18 \end{vmatrix} - 4\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix}$$

= 5(-90 + 44) – 2(-36 – 33) – 4(-8 – 15)

= 5 x -46 – 2 x -69 – 4 x 23

= -230 + 138 + 92

= -230 + 230

= 0

∴ So the given lines are concurrent