TED (10)-1015 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2012

TECHNICAL MATHEMATICS- II

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Evaluate $\lim_{x\to\infty} \frac{7-x}{3x+1}$

$\lim_{x\to\infty} \frac{7-x}{3x+1} = \lim_{x\to\infty} \frac{x\left(\frac{7}{x}-1\right)}{x\left(3+\frac{1}{x}\right)}$

$= \frac{\left(\frac{7}{\infty}-1\right)}{\left(3+\frac{1}{\infty}\right)} = \frac{0-1}{3+0}$

$= \frac{-1}{3}$

(b) Find $\frac{dy}{dx}$, if y = $\frac{1}{sec\sqrt{x}}$

y = $\frac{1}{sec\sqrt{x}}$

y = $cos\sqrt{x}$

$\frac{dy}{dx}$= - $sin\sqrt{x}\,\frac{1}{2\sqrt{x}}$

(c) Find the slope if the curve y = $\frac{3}{x^2}$ at the point (1, 3)

Slope = $\frac{dy}{dx}$

$=\frac{-6}{x^3}$

∴ At (1, 3), slope = $\frac{-6}{1^3}=\frac{-6}{1}$= -6

(d) Evaluate $\int_0^3 x(x^2+1)dx$

$\int_0^3 x(x^2+1)dx = \int_0^3 (x^3+x)dx$

$$= \left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^3$$

$$=\frac{3^4}{4}+\frac{3^2}{2}-\frac{0^4}{4}+\frac{0^2}{2}$$

$$=\frac{81}{4}+\frac{9}{2}-0$$

$$=\frac{81+18}{4}$$

$$=\frac{99}{4}$$

(e) Solve $\frac{d^2y}{dx^2}$= sinx

$\frac{d^2y}{dx^2}$= sinx

Integrating both sides w.r.to x

$\frac{dy}{dx}$= -cosx

Again integrating both sides w.r.to x

Y = -sinx + c

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a)

i. Evaluate $\lim_{\theta\to 0} \frac{\sin 3\theta \cos\theta}{\theta}$

$$\lim_{\theta\to 0} \frac{\sin 3\theta \cos\theta}{\theta} = \lim_{\theta\to 0} \frac{\sin 3\theta}{\theta} \text{ x } \lim_{\theta\to 0} \cos\theta$$

= 3 xcosθ

= 3 x 1

= 3

= 1

ii. Find ‘a’ if $f(x) = \begin{cases} \frac{sinx}{x}, & x \neq 0 \\ a, & x = 0 \end{cases}$ is consistent at x = 0.

f(0)= a

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{sinx}{x}$

= 1

∴ Function is continuous at x = 0

∴ $\lim_{x\to 0} f(x) = f(0)$

ie a = 1

(b) If $y = x^2 sinx$, prove that $x^2y'' - 4xy' + (x^2 + 6)y = 0$

$$y = x^2 sinx$$

$$y' = x^2 cosx + sinx2x$$

$$y'' = x^2x - sinx + cosx2x + sinx\ x\ 2 + 2x\ x\ cosx$$

$$y'' = -x^2 sinx + 2xcosx + 2sinx + 2xcosx$$

$$y'' = -x^2 sinx + 4xcosx + 2sinx$$

$\therefore x^2y'' - 4xy' + (x^2 + 6)y = \therefore x^2y'' - 4xy' + (x^2y + 6y)$ -

$= x^2(-x^2 sinx + 4xcosx + 2sinx)$

$-4x(x^2 cosx + sinx2x) + x^2x^2 sinx + 6x^2 sinx$

$= -x^4 sinx + 4x^3 cosx + 2x^2 sinx - 4x^3 cosx - x^2 sinx$

$$+x^4 sinx + 6x^2 sinx$$

$= 0$

(c) A balloon is spherical in shape. Gas is escaping from it at the rate of 10 cc/sec. How fast is the surface area shrinking when the radius is 15cms?

$\frac{dr}{dt} = 0$

R = 15cm

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4}{3}\pi 3r^2 \text{x} \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$10 = 4\pi 15^2 \frac{dr}{dt}$

$10 = 4\pi 225 \frac{dr}{dt}$

$10 = 900\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{10}{900\pi} = \frac{1}{90\pi}$

Surface area $S = 4\pi r^2$

$\frac{ds}{dt} = 4\pi\ 2r\ \frac{dr}{dt}$

$\frac{ds}{dt} = 8\pi r\ \frac{dr}{dt}$

$\frac{ds}{dt} = 8\pi r\ \frac{1}{90\pi}$

$\frac{ds}{dt} = 8 \text{x } 15 \text{ x} \frac{1}{90}$

$= \frac{120}{90}$

$= \frac{4}{3}$

∴ Surface area shrinking at the rate of $\frac{4}{3}$ cm^2/sec

(d) Find the maximum value of y, if $y = 2x^3 - 9x^2 + 12x$

$y = 2x^3 - 9x^2 + 12x$

$$\frac{dy}{dx} = 6x^2 - 18x + 12$$

$$\frac{d^2y}{dx^2} = 12x - 18$$

$\frac{dy}{dx} = 0$

$6x^2 - 18x + 12 = 0$

$6(x^2 - 3x + 2) = 0$

$x^2 - 3x + 2 = 0$

(x – 2) (x – 1) = 0

X = 1, 2

$$\frac{d^2y}{dx^2} = 12x - 18$$

At x = 1, $\frac{d^2y}{dx^2} = 12 - 18 = -6 < 0$

∴at x = 1, y is maximum

And maximum value of $y = 2x^3 - 9x^2 + 12x$ is

$y = 2 \text{x } 1^3 - 9 \text{x } 1^2 + 12 \text{ x } 1$

$y = 2 - 9 + 12$

$y = 14 - 9$

$y = 5$

(e) Evaluate

i. $\int \sqrt{1 + \sin 2x}\ dx$

ii. $\int \frac{\sin^{-1} 2x}{\sqrt{1-4x^2}} dx$

i. $\int \sqrt{1 + \sin 2x}\ dx = \int \sqrt{\sin^2 x + \cos^2 x + 2sinxcosx}\ dx$

$= \int \sqrt{(sinx + cosx)^2}\ dx$

$= \int sinx + cosx\ \ dx$

$= -cosx + sinx$

$= sinx - cosx + C$

ii. $\int \frac{\sin^{-1} 2x}{\sqrt{1-4x^2}}dx = \int u \frac{du}{2}$

Let u = $\sin^{-1} 2x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-4x^2}}$ x 2

$\frac{du}{2} = \frac{1}{\sqrt{1-4x^2}}$ dx

$= \frac{1}{2} \int u\, du$

$= \frac{1}{2} [\frac{u^2}{2}]$

$= \frac{u^2}{4}$

$= \frac{(sin^{-1}2x)^{\ \ 2}}{4} + C$

(f) Find $\int (logx)^2$dx

$\int (logx)^2$x 1dx

=(logx)^2 x x - $\int x$ x 2(logx) $\frac{1}{x}$dx

= (logx)^2 x x - 2 $\int logx$ dx

= (logx)^2 x x - 2[$\int (logx$ x 1)dx]

= (logx)^2 x x - 2[$logx$ x $x - \int x$ x$\frac{1}{x}$dx]

= x (logx)^2 - 2 log x x x + 2 $\int$ 1dx

= x (logx)^2 - 2x log x + 2x + C

(g) Solve $\frac{dy}{dx} + \frac{x\sqrt{1+y^2}}{y\sqrt{(1+x^2)}} = 0$

$\frac{dy}{dx} = -\frac{x\sqrt{1+y^2}}{y\sqrt{1+x^2}}$

$\frac{ydy}{\sqrt{1+y^2}} = -\frac{x}{\sqrt{1+x^2}}dx$

Integrating on both sides

$\int \frac{ydy}{\sqrt{1+y^2}} = -\int \frac{x}{\sqrt{1+x^2}}dx$

$\int \frac{1}{\sqrt{v}}\frac{dv}{2} = -\int \frac{1}{\sqrt{u}}\frac{du}{2}$

$\frac{1}{2}[\frac{\sqrt{v}}{\frac{3}{2}}] = -\frac{1}{2}[\frac{\sqrt{v}}{\frac{3}{2}}]$

$\sqrt{v} = -\sqrt{u}$

$\sqrt{1+y^2} = \sqrt{1+x^2}$

$\sqrt{1+y^2}+\sqrt{1+x^2} = C$

Put $1 + x^2 = u$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$1 + y^2 = v$

$\frac{dv}{dy} = 2y$

$\frac{dv}{2} = y\,dy$

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Evaluate $\lim_{x\to 4}\frac{x-4}{x^2-16}$

$\lim_{x\to 4}\frac{x-4}{x^2-16} = \frac{ma^{m-1}}{na^{n}-1}$

$= \frac{1 \text{ x } 4^{1-1}}{2\text{x}4^{2-1}}$

$= \frac{1 \text{ x } 4^{0}}{2\text{x}4^{1}}$

$= \frac{1 \text{ x } 1}{2\text{x } 4}$

m = 1

n = 1

a = 4

$= \frac{1}{8}$

(b) Find $\frac{dy}{dx}$ if:

i. $y = \frac{e^{2x} logx}{x^2}$

ii. $y = \frac{\tan^{-1} x}{(1+x^2)^2}$

i. $y = \frac{e^{2x} logx}{x^2}$

$$\frac{dy}{dx} = \frac{x^2\left[e^{2x}\frac{1}{x} + logx\ e^{2x} \text{x } 2\right] - e^{2x} logx \text{ x } 2x}{(x^2)^2}$$

$$\frac{dy}{dx} = \frac{\left[x^2 e^{2x} + 2x^2 logx\ e^{2x} - 2x\ e^{2x} logx\right]}{x^4}$$

ii. $y = \frac{\tan^{-1} x}{(1+x^2)^2}$

$$\frac{dy}{dx} = \frac{(1+x^2)^2 \text{x} \frac{1}{(1+x^2)} - \tan^{-1} x\ 2(1+x^2)2x}{(1+x^2)^4}$$

$$= \frac{(1+x^2)^2 [1 - 4\text{x}\tan^{-1}\text{x}]}{(1+x^2)^4}$$

$$= \frac{1 - 4x\ tan^{-1} x}{(1+x^2)^3}$$

(c) find $\frac{dy}{dx}$, if x and y are connected by:

$x = 3sin\theta - \sin^3\theta$

$y = 3cos\theta - \cos^3\theta$

$\frac{dx}{d\theta} = 3cos\theta - 3\sin^3\theta\, cos\theta$

$\frac{dy}{d\theta} = -3sin\theta - 3\cos^2\theta \text{ x} - sin\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \frac{-3sin\theta + 3sin\theta\cos^2\theta}{3cos\theta - 3\sin^2\theta cos\theta}$

$= \frac{3sin\theta(\cos^2\theta - 1)}{3cos\theta(1 - \sin^2\theta)}$

$= \tan\theta \text{ x } \frac{-\sin^2\theta}{\cos^2\theta}$

$= -\tan^3\theta$

(d) If $2x^2 + 3xy + 5y^2 = 0$, find $\frac{dy}{dx}$

$$4x + 3x\frac{dy}{dx} + 3y + 10y\frac{dy}{dx} = 0$$

$\frac{dy}{dx}$(3x+10y) = -4x – 3y

$\therefore \frac{dy}{dx} = \frac{-4x - 3y}{3x + 10y}$

IV.

(a) Using 1st principles, find the derivative of sinx

y = sinx

f(x) = sinx

f(x+Δx) = sin(x+Δx)

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{sin(x+\Delta x) - \sin x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2\cos\left(\frac{x+\Delta x+x}{2}\right) sin\left(\frac{x+\Delta x-x}{2}\right)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2\cos\left(x+\frac{\Delta x}{2}\right) sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}\times 2}$

$= \cos(x + \frac{0}{2})$x 1

= cosx

(b) If $y = a\cos(logx) + bsin(logx)$, prove that $x^2y'' + xy' + y = 0$

$$y' = -a\sin(logx)\,x\frac{1}{x} + bcos(logx)\frac{1}{x}$$

$$xy' = -a\sin(logx) + bcos(logx)$$

$xy'' + y' = -a\cos(logx)\frac{1}{x} + bsin(logx)\frac{1}{x}$ {Differentiating on both sides

$$x(xy'' + y') = -(a\ cos\ logx + bsin(logx))$$

$$x^2y'' + xy' = -y$$

$ie, x^2y'' + xy' + y = 0$

(c) If p = $\frac{x}{x+R}$ find $\frac{d^2p}{dx^2}$

$\frac{dp}{dx} = \frac{(x+R)-x}{(x+R)^2}$

$= \frac{x+R-x}{(x+R)^2}$

$\frac{dp}{dx} = \frac{R}{(x+R)^2}$

$\therefore \frac{d^2p}{dx^2} = \frac{-2R}{(x+R)^3}$

(d) If x and y are connected by $ylogx = x - y$. Prove that $\frac{dy}{dx} = \frac{logx}{(1+logx)^2}$

$$ylogx = x - y$$

Differentiating on both sides we get

$y + \frac{1}{x} + \log x \frac{dy}{dx} = 1 - \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\frac{dy}{dx}\left(logx + 1\right) = 1 - \frac{y}{x}$

$\frac{dy}{dx} = \frac{\frac{x-y}{x}}{1+logx}$

$\frac{dy}{dx} = \frac{\frac{ylogx}{x}}{1+logx}$

$\frac{dy}{dx} = \frac{logx}{(1+logx)^2}$ $\{ \therefore \frac{y}{x} = \frac{1}{1+logx}$

V.

(a) Find the equation of the tangent and normal to the rectangular hyperbola x = ct, y = $\frac{c}{t}$ at (ct, $\frac{c}{t}$)

x = ct y = $\frac{c}{t}$ (Question is wrong)

$\frac{dx}{dt} = c$ $\frac{dy}{dt} = -\frac{c}{t^2}$

$\therefore \frac{dy}{dx} = \frac{\frac{-c}{t^2}}{c} = -\frac{1}{t^2}$

∴Slope m = $\frac{-1}{t^2}$

(b) Find the rate of change of volume of a cone with respect to radius when radius is equal to height

V = $\frac{1}{3}\pi r^2 h$

Given that r = h

∴ V = $\frac{1}{3}\pi r^2$ x r

V = $\frac{1}{3}\pi r^3$

$\therefore \frac{dv}{dr} = \frac{1}{3}\ 3\pi r^2$

$\frac{dv}{dr} = \pi r^2$

(c) An open tank is to be constructed with a square base and vertical side to hold a given quantity of water , show that the expense of lining it with be the least, if the depth is be half the depth

Let base width = b

Height = h

∴ Surface area $S = b^2 + 4bh$

Given that $h = \frac{b}{2}$

$S = b^2 + 4b \times \frac{b}{2}$

$S = b^2 + 2b^2$

$S = 3b^2$

$\therefore \frac{ds}{db} = 6b$

$\frac{d^2s}{db^2} = 6b$

$\frac{d^2s}{db^2} > 0$

∴ S is minimum when $h = \frac{b}{2}$

VI.

(a) Prove that the function $x^3 + 6x^2 + 12x - 9$ is an increasing function for all values of x

$$y = x^3 + 6x^2 + 12x - 9$$

$\frac{dy}{dx} = 3x^2 + 12x + 12$

$\frac{dy}{dx} = 3(x^2 + 4x + 4)$

$\frac{dy}{dx} = 3(x^2 + 2)^2 > 0$ for all values of x

(b) A circular plate contracts when cooled. Find the rate of decrease in area if the radius is decreases at the rate of 0.1cm/min: when the radius is 4 cm.

Area of the circle A = πr^2

$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$

$= \pi$ x 2 x 4 x 0.1

$= 8\pi$ x 0.1

$\frac{dA}{dt}$ = .08π cm^2/min

(c) A particle is projected vertically upwards at its height 'h' feet at time 't' is given by $h = 60t - 16t^2$

$$h = 60t - 16t^2$$

$\frac{dh}{dt}$= 60 – 32t

$\frac{d^2h}{dt^2}$= -32 < 0

∴h is maximum

$\frac{dh}{dt}$= 0 ===> 60 – 32t

t = $\frac{60}{32} = \frac{15}{8}$

∴ h = $60t - 16^2$

h = 60x $\frac{15}{8} - 16$x $\left(\frac{15}{8}\right)^2$

= $\frac{15x\,15}{2}$– 16 x $\frac{225}{64}$

= $\frac{225}{2} - \frac{225}{4}$

$= \frac{450-225}{4}$

$= \frac{225}{4}$

VII.

(a) Evaluate $\int \frac{2x^4}{1+x^{10}}\, dx$

$\int \frac{2x^4}{1+x^{10}}\, dx$

U =x^5

$\frac{du}{dx} = 5x^4$

$\frac{du}{5}$ = x^4 dx

$= \int \frac{2x^4}{1+(x^5)^2}\, dx$

=

$= \frac{2}{5} \int \frac{1}{1+u^2}$du

$=\frac{2}{5}\tan^{-1}u$

$=\frac{2}{5}\tan^{-1}x^5 + C$

(b) Evaluate $\int_0^{\sqrt{3}} x\sqrt{(1+x^2)}dx$

$\int_0^{\sqrt{3}} x\sqrt{(1+x^2)}dx = \int_1^4 \sqrt{u}\frac{du}{2}$

$= \frac{1}{2}\int_1^4 u^{\frac{1}{2}}du$

$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_1^4$

$= \frac{1}{3}(4^{\frac{3}{2}} - 1^{\frac{3}{2}})$

$= \frac{1}{3}(8-1)$

$= \frac{7}{3}$

u = $1 + x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2}$ = x dx

When x= 0, u = 1

X =$\sqrt{3}$, u = 1+ $\left(\sqrt{3}\right)^2$

= 1 + 3 = 4

(c) Evaluate $\int(tanx + cotx)^2\, dx$

$\int(tanx + cotx)^2\, dx = \int(\tan^2 x + \cot^2 x + 2tanxcox)dx$

$= \int(\sec^2 x - 1 + \text{cosec}^2\, x - 1 + 2)dx$

$=\int(\sec^2 x + \mathrm{cosec}^2 x)dx$

= tanx – cotx +C

(d) Evaluate $\int\sqrt{(1+sin2x)}\,dx$

$\int\sqrt{(1+sin2x)}\,dx = \int\sqrt{\sin^2 x + \cos^2 x + 2sinxcosx}\ \ dx$

$= \int\sqrt{(sinx + cosx)^2}\,dx$

$= \int(sinx + cosx)\,dx$

= -cosx + sinx

= sinx – cosx + C

(e) Evaluate $\int xe^{-x}\,dx$

$\int xe^{-x}\,dx = \frac{xe^{-x}}{-1} - \int\frac{e^{-x}}{-1}dx$

$= -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{-x} - e^{-x} + C$

VIII.

(a) Evaluate $\int\frac{x^2}{(8+x^3)^4}dx$

$u = 8 + x^3$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3} = x^2\,dx$

$= \int\frac{1}{u^4}\frac{du}{3}$

$= \frac{1}{3}\int u^{-4}\,du$

$= \frac{1}{3}\frac{u^{-3}}{-3}$

$= \frac{u^{-3}}{-9}$

$= \frac{(8+x^3)^{-3}}{-9} + C$

(b) Evaluate $\int \frac{cos2x}{\cos^2 x sin^2 x} dx$

$$\int \frac{cos2x}{\cos^2 x sin^2 x} dx = \int \frac{cos^2 x - \sin^2 x}{\cos^2 x sin^2 x} dx$$

$$= \int \frac{cos^2 x}{\cos^2 x \, sin^2 x} dx - \int \frac{\sin^2 x}{\cos^2 x sin^2 x} dx$$

$$= \int \frac{1}{sin^2 x} dx - \int \frac{1}{cos^2 x} dx$$

$$= \int cosec^2 x \, dx - \int \sec^2 x dx$$

$$= -cotx - tanx + \text{C}$$

(c) Evaluate $\int_0^{\pi} \frac{1}{1+sinx} dx$

$$= \int_0^{\pi} \frac{1}{1+sinx} \text{ x } \frac{1-sinx}{1-sinx} dx$$

$$= \int_0^{\pi} \frac{1-sinx}{1-\sin^2 x} dx$$

$$= \int_0^{\pi} \frac{1-sinx}{\cos^2 x} dx$$

$$= \int_0^{\pi} \frac{1}{\cos^2 x} dx - \int_0^{\pi} \frac{sinx}{\cos^2 x} dx$$

$$= \int_0^{\pi} \sec^2 x \, dx - \int_0^{\pi} secxtanx \, dx$$

$$= [tanx - secx]_0^{\pi}$$

$$= tan\pi - sec\pi - (tan0 - sec0)$$

= 0- -1 – (0 – 1)

= 0 + 1 + 1

= 2

(d) Evaluate $\int_0^2 x^2 logx\, dx$

$\int_0^2 x^2 logx\, dx = \int_0^2 logx\; x^2\, dx$

$= \left[logx \text{ x } \frac{x^3}{3}\right]_0^2 - \int_0^2 \frac{x^3}{3} \text{ x } \frac{1}{x} dx$

$= \left[\frac{x^3}{3} logx\right]_0^2 - \frac{1}{3}\int_0^2 x^2 dx$

$= \left[\frac{x^3}{3} logx\right]_0^2 - \frac{1}{3} \text{ x } \frac{x^3}{3}$

$= \left(\frac{2^3}{3} log2 - \frac{0^3}{3} log0\right) - \left(\frac{2^3}{9} - \frac{0^3}{9}\right)$

$= \frac{8}{2}\log 2 - \frac{8}{9}$

IX.

(a) Find the area enclosed between the curve $y = x^2 - x - 2$ at the x – axis

$$y = x^2 - x - 2$$

$\therefore x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$$x = 2, x = -1$$

$\therefore$ Area A = $\int_a^b y\, dx$

$= \int_{-1}^{2} (x^2 - x - 2)\, dx$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{2}$

$= \left(\frac{2^3}{3} - \frac{2^2}{2} - 2\text{x}2\right) - \left(\frac{(-1)^3}{3} - \frac{(-1)^2}{2} - 2\text{x} - 1\right)$

$= \left(\frac{8}{3} - \frac{4}{2} - 4\right) - \left(\frac{-1}{3} - \frac{1}{2} + 2\right)$

$= \left(\frac{8}{3} - 6\right) + \frac{5}{6} - 2$

$= \frac{8}{3} + \frac{5}{6} - 6 - 2$

$= \frac{16+5}{6} - 8$

$= \frac{21}{6} - 8$

$= \frac{7}{2} - 8$

$= -\frac{9}{2}$

$\therefore$ Area $= \frac{9}{2}$ unit^2

(b) Find the volume generated when the point of parabola $y^2 = 4x$ between x = 0, and x = 4 revolves about the x – axis.

$Y^2 = 4x$

a = 0, b= 4

$V = \pi \int_a^b y^2 dx$

$V = \pi \int_0^4 4x\, dx$

$V = \pi \left[\frac{4x^2}{2}\right]_0^4$

$V = \pi[2 \text{ x } 4^2 - 2 \text{ x } 0^2]_0^4$

$V = \pi[2 \text{ x } 16 - 0]$

$V = 32\pi$ unit^3

(c) Solve $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

$\frac{dy}{dx} = e^x e^y + x^2 e^y$

$\frac{dy}{dx} = e^y(e^x + x^2)$

$\frac{dy}{e^y} = (e^x + x^2)dx$

$e^{-y}dy = (e^x + x^2)dx$

Integrating on both sides, we get

$-e^{-y} = e^{x} + \frac{x^3}{3} + c$

X.

(a) Find the area enclosed between the curves $y = x^2$ and $2x + y - 3 = 0$

$y = x^2$ = f(x)

y = $3 - 2x$ = g(x)

$$\therefore x^2 = 3x - 2$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

x = -3, x = 1

$\therefore$ A = $\int_a^b (\text{f(x)} - \text{g(x)})\, dx$

= $\int_{-3}^{1} (x^2 - (3 - 2x))\, dx$

= $\int_{-3}^{1} x^2 + 2x - 3\, dx$

=$\left[\frac{x^3}{3} + x^2 - 3x\right]_{-3}^{1}$

= $\left(\frac{1^3}{3} + 1^2 - 3 \text{ x } 1\right) - \left(\frac{(-3)^3}{3} + (-3)^2 - 3 \text{ x } (-3)\right)$

=$\frac{1}{3} + 1 - 3 - (\frac{-27}{3} + 9 + 9)$

=$\frac{1}{3} - 2 - (-9 + 9 + 9)$

=$\frac{1}{3} - 2 - 9$

=$\frac{1}{3} - 11$

= $-\frac{32}{3}$

Area cannot be –ve.

$\therefore$ Area = $\frac{32}{3}$unit^2

(b) Find the volume of the solid obtained b rotating one arch of the curve $y = sinx$ about the x – axis arch of the curve $y = sinx$ about the x – axis.

y = sinx, x = 0,π

Volume V = $\int_a^b y^2\,dx$

V = $\pi\int_0^{\pi}\sin^2 x\,dx$

V = $\pi\int_0^{\pi}\frac{1-cos2x}{2}$ dx

= $\frac{\pi}{2}\left[x-\frac{sin2x}{2}\right]_0^{\pi}$

=$\frac{\pi}{2}\left[\pi-\frac{sin2\pi}{2}-\ 0+\frac{sin2\text{ x }0}{2}\right]$

=$\frac{\pi}{2}\left[\pi-0+0+0\right]$

= $\frac{\pi^2}{2}$unit^3

(c) Solve $x(1+y^2)dx+y(1+x^2)dy=0$

$x(1+y^2)dx=-y(1+x^2)dy$

$\frac{dy}{dx}=\frac{-x(1+y^2)}{y(1+x^2)}$

$\frac{ydy}{(1+y^2)}=\frac{-x}{(1+x^2)}$dx

Integrating on both sides we get

$\int\frac{y}{1+y^2}$dy = $-\int\frac{x}{1+x^2}$dx

$\int\frac{1}{u}\frac{du}{2}=-\int\frac{1}{v}\frac{dv}{2}$

$\frac{1}{2}\int\frac{1}{u}$du= $-\frac{1}{2}\int\frac{1}{v}$dv

$\frac{1}{2}$logu= $-\frac{1}{2}$log v

$\frac{1}{2}$log $(1+y^2)$= $-\frac{1}{2}$log $(1+x^2)$

$\frac{1}{2}$log $(1+y^2)$= $-\frac{1}{2}$log $(1+x^2)$

Put $1+y^2=u$

$\frac{du}{dx}$= 2y

$\frac{du}{2}$=y dy

V = $1+x^2$

$\frac{dV}{dx}$= 2x

$\frac{dV}{2}$=x dx