TED (10)-1015 Reg. No. ………………………….

(REVISION-2010) Signature ……..……………………

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- OCTOBER, 2013

TECHNICAL MATHEMATICS- II

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Find the derivatives of $y = 3cosx - 4tanx$

$$y = 3cosx - 4tanx$$

$\frac{dy}{dx} =$ 3 x – sinx– 4sec^{2}x

= -3sinx – 4sec^{2}x

(b) Evaluate $\lim_{x\to 0} \frac{2x-3}{3x+4}$

$$\lim_{x\to 0} \frac{2x-3}{3x+4} = \frac{2 \text{ x } 0-3}{3 \text{ x } 0+4} = \frac{-3}{4}$$

(c) Check whether the function $x^2 - 3x + 2$ is decreasing at x = 1

$$y = x^2 - 3x + 2$$

$\frac{dy}{dx}$ = 2x – 3

At x = 1

$\frac{dy}{dx}$ = 2 x 1 – 3 = 2 – 3 = -1 < 0

$\therefore \frac{dy}{dx}$< 0 at x = 1

∴Function is decreasing at x = 1

(d) Find $\int (2x+1)^2\, dx$

$\int (2x+1)^2\, dx = \int 4x^2 + 1 + 4x\, dx$

$= 4\int x^2 dx + \int 1\, dx + \int 4x\, dx$

$= 4\frac{x^3}{3} + x + 4\frac{x^2}{2} + C$

(e) Solve $\frac{d^2y}{dx^2} = \sec^2 x$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \sec^2 x$

Integrating on both sides we get,

$\frac{dy}{dx} = \tan x + C_1$

Again integrating on both sides we get,

$y = \log\sec x + C_1x + C_2$

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a)

i. Find the differential coefficient of 'tanx' using quotient rule.

$Y = \tan x = \frac{\sin x}{\cos x}$

We know by quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \text{ x } \frac{du}{dx} - u \text{ x } \frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(tanx) = \frac{cosx \text{ x } \frac{d(sinx)}{dx} - sinx \text{ x } \frac{d(cosx)}{dx}}{cos^2x}$$

$$= \frac{cosx \text{ x} cosx - sinx \text{ x} -sinx}{cos^2x}$$

$$= \frac{\cos^2 x + \sin^2 x}{cos^2x}$$

$$= \frac{1}{cos^2x} = \sec^2 x$$

ii. Evaluate $\lim_{x\to\infty} \frac{x^2-2x+8}{4x^3-3}$

$$\lim_{x\to\infty} \frac{x^2-2x+8}{4x^3-3}$$

$$= \lim_{x\to\infty} \frac{x^2[1-\frac{2x}{x^2}+\frac{8}{x^3}]}{x^3[4-\frac{3}{x^3}]}$$

$$= \lim_{x\to\infty} \frac{1}{x} \frac{[1-\frac{2x}{x^2}+\frac{8}{x^3}]}{[4-\frac{3}{x^3}]} \qquad [\lim_{x\to\infty} \frac{1}{x} = 0]$$

$$= 0 \frac{(1-\frac{2}{\infty}+\frac{8}{\infty})}{(4-\frac{3}{\infty})}$$

$$= 0$$

(b) If $y = \sin^{-1} x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

$$y = \sin^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$\frac{dy}{dx}\sqrt{1-x^2} = 1$

Differentiating on both sides,

$\frac{dy}{dx}$ x $\frac{1}{2\sqrt{1-x^2}}$x -2x + $\sqrt{1-x^2}\,\frac{d^2y}{dx^2}$= 0

$\frac{-xdy}{dx}$ x $\frac{1}{\sqrt{1-x^2}}$+ $\sqrt{1-x^2}\,\frac{d^2y}{dx^2}$= 0

Multiplication on both sides by $\sqrt{1-x^2}$we get,

$\frac{-xdy}{dx}$+$(1-x^2)\frac{d^2y}{dx^2}$= 0

Or $(1-x^2)\frac{d^2y}{dx^2} - \frac{xdy}{dx}$= 0

(c) For what value of xtangent to the curve y = $\frac{x}{(1-x)^2}$will be parallel to:

i. X axis ii. Y axis

i. Parallel to x axis

Y = $\frac{x}{(1-x)^2}$

$\frac{dy}{dx}$=$\frac{(1-x)^2 \text{x } 1 - \text{x} \times 2(1-x) \times -1}{((1-x)^2)^2}$

If y = $\frac{x}{(1-x)^2}$is parallel to x axis then,

$\frac{dy}{dx}$= 0 ===>$\frac{(1-x)^2 + 2x(1-x)}{(1-x)^4}$

$\frac{1-x^2}{(1-x)^4}$= 0

$\frac{(1+x)(1-x)}{(1-x)^4}$= 0

$\frac{(1-x)}{(1-x)^3}$= 0

$$x = -1$$

ii. Parallel to y axis

If y = $\frac{x}{(1-x)^2}$ is parallel to y axis then

Slop = ∞

Ie, $\frac{dy}{dx} = \infty$ ===> $\frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} = 0$

===> $(1-x)^4 = 0$

===> x = 1

(d) Find the minimum value of $y = 4x^3 + 9x^2 - 12x + 2$

$$y = 4x^3 + 9x^2 - 12x + 2$$

$\frac{dy}{dx} = 12x^2 + 18x - 12 = 0$

$\frac{dy}{dx} = 0$ ===> $12x^2 + 18x - 12 = 0$

$6(2x^2 + 3x - 2) = 0$

x = $\frac{-3\pm\sqrt{9+16}}{4} = \frac{-3\pm5}{4}$

= -2, ½

$\frac{d^2y}{dx^2} = 24x + 18$

At minimum

$\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} > 0$

At x = -2

$\frac{d^2y}{dx^2}$ = 24 x -2 +18 = -48 + 18 = -3 < 0

At x = ½

$\frac{d^2y}{dx^2}$= 24 x ½+18 = 12 + 18 = 30 > 0

At x = ½ y is minimum

∴ Minimum value y = 4 x $\left(\frac{1}{2}\right)^3 + 9\left(\frac{1}{2}\right)^2 - 12$ x $\frac{1}{2} + 2$

$= -\frac{5}{4}$

(e) Find

i. $\int \frac{x^2+3x-2}{x}dx$ ii. $\int sin3xcosx\ dx$

i. $\int \frac{x^2+3x-2}{x}dx = \int \frac{x^2}{x}dx + \int \frac{3x}{x}dx - \int \frac{2}{x}dx$

$= \int x dx + \int 3dx - 2\int \frac{1}{x}dx$

$= \frac{x^2}{2} + 3x - 2\log x + C$

ii. $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\sin 3x \cos x = \frac{1}{2}[\sin 4x + \sin 2x]$

$\int \sin 3x \cos x\ dx = \frac{1}{2}\int(\sin 4x + \sin 2x)dx$

$= \frac{1}{2}\int sin4x\ dx + \frac{1}{2}\int sin2x\ dx$

$= \frac{1}{2}\left(\frac{-cos4x}{4}\right) + \frac{1}{2}\left(\frac{-cos2x}{2}\right)$

$= \frac{-cos4x}{8} + \frac{-cos2x}{4} + C$

(f) Find $\int \tan^{-1} x \; dx$

We have $\int \tan^{-1} x \; dx = \int \tan^{-1} x \text{ x } 1 \; dx$

$$= \tan^{-1}x \int 1\, dx - \int \frac{d}{dx}((\tan^{-1} x) \int 1dx)\, dx$$

$$= \tan^{-1} x \text{ x } x - \int \frac{1}{1+x^2} x\, dx$$

$$= x tan^{-1}x - \frac{1}{2}\log(1 + x^2) + C$$

(g) Solve $\frac{dy}{dx} = \frac{xy^2+x}{yx^2+x}$

$$\frac{dy}{dx} = \frac{x(y^2 + 1)}{y(x^2 + 1)}$$

$$dy. y(x^2 + 1) = dx. x(y^2 + 1)$$

$\frac{ydy}{y^2+1} = \frac{xdx}{x^2+1}$ Integrating both sides

$$\int \frac{ydy}{y^2+1} = \int \frac{xdx}{x^2+1} \_\_\_\_\_\_\_\_\_\_ (1)$$

Consider $\int \frac{x}{x^2+1}dx$ put $u = x^2 + 1$

$\frac{du}{dx} = 2x$ $du = 2x\, dx$ $\frac{du}{2} = x\, dx$

$$\int \frac{x}{x^2+1}dx = \int \frac{du}{2u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\log u$$

$$= \frac{1}{2}\log(x^2 + 1) + C_1$$

Similarly $\int \frac{ydy}{y^2+1} = \log\frac{(y^2+1)}{2}$+ C_2

(1) Becomes $\frac{1}{2}\log(y^2 + 1)$ + C_2 = $\frac{1}{2}\log(x^2 + 1)$ + C_1

$\frac{1}{2}\log\left(y^2 + 1\right) - \frac{1}{2}\log(x^2 + 1)$ + C_1 = C [take $C_1 - C_2$ = C]

===>$\frac{1}{2}\log\frac{1+y^2}{1+x^2}$= C

===>$\log\frac{1+y^2}{1+x^2}$= 2C

===>$1 + y^2 = e^{2c}(1 + x^2)$

===>$1 + y^2 = k(1 + x^2)$

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Using 1st principles, find the derivative of sinx

By 1st principle we have

$\frac{dy}{dx}$= $\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ ________________ (1)

$f(x) = sinx$

$f(x + \Delta x) = \sin(x + \Delta x)$

Then (1) becomes

$\frac{dy}{dx}$= $\lim_{\Delta x\to 0} \frac{\sin(x+\Delta x)-sinx}{\Delta x}$

= $\lim_{\Delta x\to 0} \frac{2\left[\frac{\cos(2x+\Delta x)}{2}\sin\left(\frac{\Delta x}{2}\right)\right]}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{2\left[\frac{\cos(2x+\Delta x)}{2} \sin\left(\frac{\Delta x}{2}\right)\right]}{\frac{\Delta x}{2} \text{x } 2}$$

$$= \cos\left(\frac{2x+0}{2}\right) = cosx$$

(b) Find $\frac{dy}{dx}$if:

i. $y = e^x tanx$

$y = e^x tanx$ $\left[\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}\right]$

$$\frac{dy}{dx} = e^x \sec^2 x + tanx\, e^x$$

ii. $y = \log(secx + tanx)$

$$\frac{dy}{dx} = \frac{dy}{dx} \log(secx + tanx)$$

$$= \frac{1}{secx+tanx} (secxtanx + \sec^2 x)$$

$$= \frac{secx(tanx+secx)}{(secx+tanx)} = \text{secx}$$

(c) If $ax^2 + 2hxy + by^2 = 0$ find $\frac{dy}{dx}$

$$ax^2 + 2hxy + by^2 = 0$$

Differentiating on both sides

$$2ax + 2h\left(x\frac{dy}{dx} + y \text{ x } 1\right) + 2by\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2hx + 2by) + 2ax + 2hy = 0$$

$$\therefore \frac{dy}{dx} = \frac{-(ax+hy)}{(hx+by)}$$

IV.

(a) If $x = a(\theta + sin\theta)$, $y = a(1 + cos\theta)$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\frac{dy}{d\theta} = a \text{ x} - \sin\theta = -2\,a\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

$$\frac{dx}{d\theta} = a(1 + cos\theta) = 2\,a\cos^2\frac{\theta}{2}$$

(1) Becomes

$$\frac{dy}{dx} = \frac{-2\,a\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\,a\cos^2\frac{\theta}{2}}$$

$$= -\tan\left(\frac{\theta}{2}\right)$$

(b) If $y = ae^x + be^{2x}$ show that $y'' - 3y' + 2y = 0$

$$y = ae^x + be^{2x}$$

$$y' = ae^x + 2be^{2x}$$

$$y'' = ae^x + 4be^{2x}$$

$$y'' - 3y' + 2y = (ae^x + 4be^{2x}) - 3(ae^x + 2be^{2x}) + 2(ae^x + be^x)$$

$$= ae^x + 4be^{2x} - 3ae^x - 6be^{2x} + 2ae^x + 2be^x$$

$$= 0$$

(c) Find $\frac{d^2y}{dx^2}$, if $y = sinxcosx$

$$y = sinxcosx$$

$$\frac{dy}{dt} = sinx \text{ x } \frac{d}{dt}cosx + cosx \text{ x} \frac{d}{dt}\text{sinx}$$

$$= sinx \text{ x} - sinx + cosx \text{ cosx}$$

$$= -\sin^2 x + \cos^2 x$$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

V.

(a) If the displacement of a body is given by $s = 2t^3 - 3t^2 - 12t + 6$, find when the body attains the greatest height and also find the acceleration then.

$$s = 2t^3 - 3t^2 - 12t + 6$$

At maximum

$\frac{ds}{dt} = 0, \frac{d^2s}{dt^2} < 0$

$\frac{ds}{dt} = 0 \Longrightarrow 6t^2 - 6t - 12 = 0$

$6(t^2 - t - 2) = 0$

$t = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = 2, -1$

$\frac{d^2s}{dt^2} = 12t - 6$

At t = 2

$\frac{d^2s}{dt^2} = 12 \times 2 - 6$

$= 24 - 6 = 18 > 0$

At t = -1

$\frac{d^2s}{dt^2} = 12 \times -1 - 6$

$= -12 - 6 = -18 < 0$

At t = -1

Height $s = 2(-1)^3 - 3(-1)^2 - 12 \times -1 + 6$

$= -2 - 3 + 12 + 6 = 18 - 5 = 13$

Acceleration is $\frac{d^2s}{dt^2} = -18$

(b) A stone is dropped into still water. The radius of the outermost ripple then formed increases at the rate of 6cm/sec. how fast is the area increasing when the radius is 16cms.

Area of circle A = πr^2

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ ________________ (1)

$\frac{dr}{dt} = 6$ cm/sec

Then (1) becomes

$\frac{dA}{dt} = 2\pi \times 16 \times 6$

$= 2\pi \times 96$

$= 192\pi\ cm^2/sec$

∴ Area is increasing at the rate of $192\pi\ cm^2/sec$ when radius is 16cm.

(c) Show that a rectangle of fixed perimeter has its maximum area when it becomes a square

Let x and y be the length and breadth of the rectangle of fixed perimeter 'p'

Ie, 2x +2y = p

2y = p – 2x

$y = \frac{p-2x}{2}$ ________________ (1)

Area of rectangle = x x y

A = x x y

$= \frac{p-2x}{2} \times x$

$= \frac{1}{2}(px - 2x^2)$

At maximum or minimum $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = \frac{d}{dx}\frac{1}{2}(px - 2x^2) = \frac{1}{2}(p - 4x) = 0$

$\Longrightarrow x = \frac{p}{4}$

$\therefore y = \frac{p-2x}{2} = \frac{p-2\frac{p}{4}}{2} = \frac{p-\frac{p}{2}}{2} = \frac{p}{4}$

$$x = \frac{p}{4}$$

$$y = \frac{p}{4}$$

$\frac{d^2A}{dx^2} = \frac{d}{dx}\frac{1}{2}(p - 4x) = \frac{1}{2}$x -4 < 0

Since $\frac{d^2A}{dx^2}$< 0, A is maximum at x =$\frac{p}{4}$, y =$\frac{p}{4}$

VI.

(a) Find the range of values of x for which the function $x^2 - 3x + 4$ is:

i. Increasing ii. Decreasing

i. $y = x^2 - 3x + 4$

$\frac{dy}{dx}$= 2x – 3

If the function is increasing $\frac{dy}{dx}$> 0

Ie, 2x – 3 > 0

2x > 3 ===> x >$\frac{3}{2}$

ii. If the function is decreasing $\frac{dy}{dx}$< 0

2x – 3 < 0 ===> 2x < 3

x <$\frac{3}{2}$

(b) Show that the maximum value of function $M = 2x^3 - 9x^2 + 12x$is 5

$$M = 2x^3 - 9x^2 + 12x$$

At maximum $\frac{dM}{dx}$= 0 &$\frac{d^2M}{dx^2}$< 0

$\frac{dM}{dx}$= 0 ===>$6x^2 - 18x + 12 = 0$

===> $6(x^2 - 3x + 2)$

$$X = \frac{3 \pm \sqrt{9-8}}{2}$$

$$= \frac{3 \pm 1}{2} = 2, 1$$

At x = 1

$$\frac{d^2M}{dx^2} = 12x - 18 = 12 \times 1 - 18 = -6 < 0$$

At x = 2

$$\frac{d^2M}{dx^2} = 12x - 18 = 12 \times 2 - 18 = 24 - 18 = 6 > 0$$

∴ M is maximum at x = 1

And maximum value M = $2(1)^3 - 9(1)^2 + 12 \times 1$

$= 2 - 9 + 12 = 5$

(c) Water is running out of a conical funnel at the rate of 1 cubic inch per second. If the radius of the funnel is 4 inches and altitude is 8 inches. Find the rate at which the water level is dropping when its depth is 6 inches.

Let v be the volume of the water in the cone.

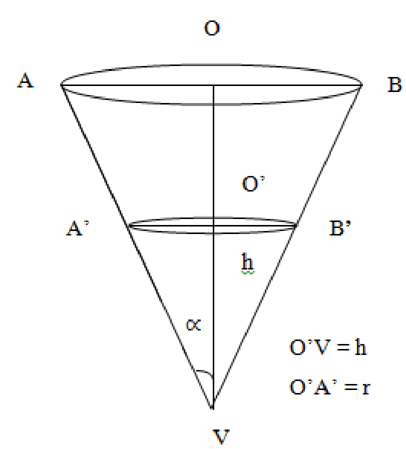
Let $\propto$ be the semi vertical angle of the cone.

Then

$$\tan \propto = \frac{OA}{OV} = \frac{4}{8} = \frac{1}{2}$$

Also $\tan \propto = \frac{O'A'}{O'V'} = \frac{r}{h}$

$$\therefore \frac{r}{h} = \frac{1}{2} \text{ ===> } r = \frac{h}{2}$$



Then $V = \frac{1}{3}\pi r^2 h = V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$$= V = \frac{1}{3}\pi \frac{h^3}{4}$$

$= \frac{\pi h^3}{12}$m

$\frac{dv}{dt} = -\frac{1}{12}$ x π x $3h^2 \frac{dh}{dt}$

[-ve sign is due to decreasing v]

∴ $1 = -\frac{1}{12}$ x π x $3h^2 \frac{dh}{dt}$ [Given $\frac{dv}{dt} = 1$]

∴ $\frac{dh}{dt} = \frac{-12}{\pi \text{ x } 3h^2} = \frac{-4}{\pi h^2}$

$= \frac{-4}{\pi 6^2} = \frac{-4}{36\pi}$inch/sec

∴ The water level is dropped at the rate of $\frac{-4}{36\pi}$inch/sec

VII. Evaluate

(a) $\int (2x^3 - 3sinx + 5x)dx$

$$\int (2x^3 - 3sinx + 5x)dx$$

$= \int 2x^3 dx - \int 3sinx\, dx + \int 5x\, dx$

$= \frac{2x^4}{4}$ – 3x – cosx + $\frac{5x^2}{2}$ + C

$= \frac{x^4}{2}$+ 3cosx + $\frac{5x^2}{2}$ + C

(b) $\int \frac{3x^2}{\sqrt{1-x^6}}$ dx

Put $u = x^3$

$Du = 3x^2dx$

$\int \frac{3x^2}{\sqrt{1-x^6}}dx = \int \frac{3x^2}{\sqrt{1-(x^3)^2}}$ dx

$= \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$

$= \sin^{-1} x^3 + C$

(c) $\int x^2 e^x\, dx$

[Use ILATE Rule]

Taking x^2 as 1^{st} function, we have

$\int x^2 e^x\, dx = x^2 \int e^x dx - \int (\frac{d}{dx} x^2 \int e^x dx)\, dx$

$= x^2 e^x - \int 2x \text{ x } e^x dx$ ____________ (1)

$\int x \quad e^x\, dx = x \int e^x\, dx - \int \frac{d}{dx} x \int e^x\, dx\, dx$

$= xe^x - \int e^x\, dx$

$= xe^x - e^x + \text{C}$ ____________ (2)

Putting (2) in (1) we have

$\int x^2 e^x\, dx = x^2 e^x - 2[xe^x - e^x + \text{C}]$

$= x^2 e^x - 2xe^x + 2e^x + 2\text{C}$

$= x^2 e^x - 2xe^x + 2e^x + C_1$

(d) $\int_0^{\frac{\pi}{2}} \frac{cosx}{1+sinx} dx$

Put u = 1 + sinx

du = cosx dx

$\int_0^{\frac{\pi}{2}} \frac{cosx}{1+sinx} dx$

$= \int_0^{\frac{\pi}{2}} \frac{du}{u}$ = log u = log (1 + sinx) = f(x)

$F(\frac{\pi}{2}) = \log(1+\sin\frac{\pi}{2}) = \log 2$

F(0) = log(1 + sin 0) = log1 = 0

$\int_0^{\frac{\pi}{2}} \frac{cosx}{1+sinx} dx = f(\frac{\pi}{2}) - f(0) = \log 2$

VIII. Find

(a) $\int(4\sec^2 x + 3sinx + e^x)dx$

$$\int(4\sec^2 x + 3sinx + e^x)dx$$

$= 4tanx + 3\text{ x} - \text{cosx} + \text{e}^{\text{x}} + C$

$= 4tanx - 3cosx + e^x + C$

(b) $\int_1^e logx\ dx$

$$\int logx\ dx = \int logx \text{ x } 1\ dx$$

$= \text{log}x \text{ x} \int 1\ dx - \int \frac{d}{dx} logx \text{ x} \int 1\ dx\ dx$

$= \text{log}x\text{x } x \text{ dx} - \int \frac{1}{x} \text{x } x\ dx$

$= \text{xlogx} - \int 1\ dx = \text{xlogx} - \text{x}$

$\int logx\ dx = \text{xlogx} - \text{x} = \text{f(x)}$

$f(e) = \text{eloge} - \text{e} = 0$

$f(1) = \text{1log1} - 1 = -1$

$\therefore \int_1^e logx\ dx = \text{f(e)} - \text{f(1)} = 1$

(c) $\int_0^{\frac{\pi}{2}} \cos^2 x\ dx$

$\int \cos^2 x\ dx = \int \frac{1+cos2x}{2}\text{dx}$

$= \int \frac{1}{2}\text{dx} + \int \frac{cos2x}{2}\text{dx}$

$= \frac{x}{2} + \frac{sin2x}{4} = \text{f(x)}$

$F(\frac{\pi}{2}) = \frac{\frac{\pi}{2}}{2} + \frac{sin2\frac{\pi}{2}}{4} = \frac{\pi}{4}$

F(0) = 0

$\therefore \int_0^{\frac{\pi}{2}} \cos^2 x \, dx = F(\frac{\pi}{2}) - f(0) = \frac{\pi}{4}$

(d) $\int (1 + e^{tanx}) \sec^2 x \, dx$

Put u = tanx

$\int (1 + e^t$ du = $\sec^2 x dx$

$= \int 1du + \int e^u du$

$= u + e^u + C$

$= \tan x + e^{\tan x} + C$

IX.

(a) Find the area enclosed between the curve $y = x^2 - x + 1$. The x axis and the ordinate x = 1 and x = 3

Area $= \left| \int_a^b f(x) - g(x) dx \right|$

$F(x) = x^2 - x + 1$ g(x) = 0

a = 1 b = 3

Required area = $\left| \int_1^3 x^2 - x + 1 \, dx \right|$

$\int x^2 - x + 1 \, dx = \int x^2 dx - \int x \, dx + \int 1 \, dx$

$= \frac{x^3}{3} - \frac{x^2}{2} + x = f(x)$

$F(3) = \frac{3^3}{3} - \frac{3^2}{2} + 3 = \frac{27}{3} - \frac{9}{2} + 3$

$= \frac{54-27}{6} + 34$

$= \frac{27}{6} + 3 = \frac{27+18}{6}$

$= \frac{45}{6}$

F(1) = $\frac{1^3}{3} - \frac{1^2}{2}$+ 1

$= \frac{x^3}{3} - \frac{1}{2}$+ 1 = $\frac{2-3}{6}$+ 1 = $\frac{-1}{6}$+ 1

$= \frac{-1+6}{6} = \frac{5}{6}$

Required Area = f(3) – f(1)

$$= \frac{45}{6} - \frac{5}{6} = \frac{20}{3}\text{unit}^2$$

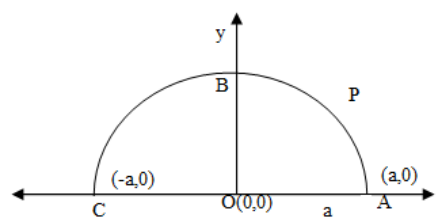
(b) Find the volume of sphere obtained by rotating the circle $x^2 + y^2 = a^2$ about the x axis is $\frac{4}{3}\pi a^3$

Equation of circle: $x^2 + y^2 = a^2$

Volume = $\pi \int_{-a}^{a} y^2 dx$

$= \pi\left[\int_{-a}^{a} (a^2 - x^2)\, dx\right]$

$= \int (a^2 - x^2)\, dx = a^2 x - \frac{x^3}{3}$ = f(x)



F(a) = a^2 x a $- \frac{a^3}{3}$ = a^3 $- \frac{a^3}{3} = \frac{2a^3}{3}$

F(-a) = a^2 x – a $- \frac{(-a^3)}{3} = -$a^3 $+ \frac{a^3}{3} = -\frac{2a^3}{3}$

F(a) – f(-a) = $\frac{4}{3}a^3$

Volume = $\pi \int_{-a}^{a} y^2 dx = \pi\left[\text{F(a)} - \text{f}(-\text{a})\right]$

$= \pi$ x $\frac{4}{3}a^3$

(c) Solve $\frac{dy}{dx} + \sqrt{\left(\frac{1-y^2}{1-x^2}\right)} = 0$

$$\frac{dy}{dx} + \sqrt{\left(\frac{1-y^2}{1-x^2}\right)} = 0$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{-dx}{\sqrt{1-x^2}}$$

Integrating on both sides,

$$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = -\sin^{-1} x + \mathrm{C}$$

===> $\sin^{-1} y + \sin^{-1} x + \mathrm{C}$

X.

(a) Find the area enclosed between one arch of the curve $y = sinx$ and the x axis

Required area

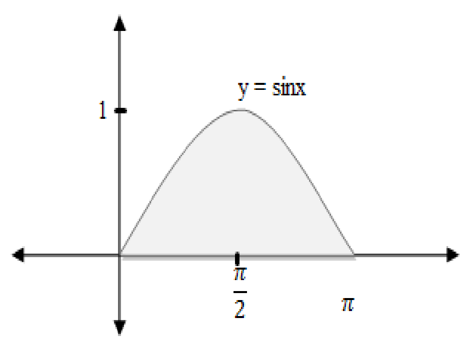
$= \int_a^b f(x)dx$

$= \int_0^\pi sinx\ dx$

$= [-cosx]_0^\pi$

$= [-cos\pi + cos0]$

$= 2$ unit^2



(b) Find the volume of the solid obtained by rotating the area consider the parabola $y^2 = 4x$ between the ordinates at x = 0, x = 2 and the x axis

$\mathrm{V} = \pi \int_a^b y^2 dx$

$= \pi \int_0^2 4x \, dx$

$= \pi \left[\frac{4x^2}{2}\right]_0^2$

$= \pi [2x^2]_0^2$

$= \pi [2 \text{ x } 2^2]$

$= 8\pi$ cubic unit.

(c) Solve $\frac{dy}{dx}$ + ycotx = cosecx

Integrating factor = I.F = $e^{\int cotx \, dx}$

$= e^{logsinx}$

= sinx

Solution: y x IF = $\int cosecx$ x IF dx

ysinx = $\int dx$

ysinx = x + C