TED (10)-1002 Reg. No. …………………………….

(REVISION-2010) Signature ……..……………………

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2013

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time*: 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) If $\begin{vmatrix} 3x & 7 \\ 2 & 8 \end{vmatrix} = 0$ find the value of x.

$\begin{vmatrix} 3x & 7 \\ 2 & 8 \end{vmatrix} = 0$

===> 24x – 14 = 0

===> 24x = 14

===> $x = \frac{14}{24} = \frac{7}{12}$

(b) If $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -3 \\ 0 & 1 \end{bmatrix}$ find 2A – 3B

$2A - 3B = 2\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} + 3\begin{bmatrix} 3 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 9 & -9 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -7 & 13 \\ 4 & -5 \end{bmatrix}$

(c) $n_{c_{10}} = n_{c_{15}}$ find n

$n_{c_r} = n_{c_s}$ ===> r = s or r + s = n

Then n = r + s

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$n = 10 + 15 = 25$

(d) Evaluate cosθ and tanθ if sinθ = ½

$\text{Sin}^2\theta + \cos^2\theta = 1$

$\text{Cos}^2\theta = 1 - (1/2)^2$

$\text{Cos}^2\theta = 1 - (1/4)$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{1}{\sqrt{3}}$

(e) Find the slope of the line whose inclination to the x axis is 45^o

Slope = $\tan\theta = \tan 45^o = 1$

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) If $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ show that AA^T is symmetric.

$A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}$

$AA^T = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}$

Clearly $(AA^T)^T = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}^T = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}$

$\therefore AA^T$ is symmetric.

(b) Solve using determinants:

$3x + y - z = 3$
$-x + y + z = 1$
$x + y + z = 3$

$AX = B$

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{3(0)-1(1-3)-(1-3)}{3(0)-(-1-1)-(-1-1)}$$

$$= \frac{2+2}{2+2} = \frac{4}{4} = 1$$

$$y = \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix} 3 & 3 & -1 \\ -1 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix}}{4}$$

$$= \frac{3(1-3)-3(-1-1)-(-3-1)}{4}$$

$$= \frac{-6+6+4}{4} = \frac{4}{4} = 1$$

$$z = \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & 3 \\ -1 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix}}{4}$$

$$= \frac{3(3-1)-(-3-1)+3(-1-1)}{4}$$

$$= \frac{6+4-6}{4} = \frac{4}{4} = 1$$

(c) Find the term independent of x in the expression of $(3x - y^2)^5$

$T_{r+1} = nc_r a^{n-r}\ b^r$

$= 5c_r(3x)^{5-r}(-y^2)^r$

We have to find

$T_4 = 5c_3(3x)^2(-y^2)^3$

$= 5c_3 . 3^2\ x^2\ (-1)^3\ y^6$

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$= -90x^2y^6$

(d) Prove that $\sin\theta + \sin3\theta + \sin5\theta + \sin7\theta = 4\cos\theta.\cos2\theta.\sin4\theta$

$\sin\theta + \sin3\theta + \sin5\theta + \sin7\theta$ $\left[\sin C + \sin D = 2.\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$= (\sin\theta + \sin7\theta) + (\sin3\theta + \sin5\theta)$

$= 2\sin4\theta.\cos3\theta + 2\sin4\theta.\cos\theta$

$= 2\sin4\theta(\cos3\theta + \cos\theta)$ $\left[\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$= 2\sin4\theta\ .\ 2(\cos2\theta\ .\ \cos\theta)$

$= 2\sin4\theta\ .\ \cos2\theta\ .\ \cos\theta$, hence the result.

(e) Prove that $\frac{\sin3\theta}{\sin\theta} + \frac{\cos3\theta}{\cos\theta} = 4\cos2\theta$

$\frac{\sin3\theta}{\sin\theta} + \frac{\cos3\theta}{\cos\theta}$

$= \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta} + \frac{4\cos^3\theta - 3\cos\theta}{\cos\theta}$

$= 3 - 4\sin^2\theta + 4\cos^2\theta - 3 = 4(\cos^2\theta - \sin^2\theta)$

$= 4\cos2\theta$

(f) Find the equation of the line passing through the point (2 , -1) and (-6 , 3). Also find the slope of the line.

Two points of a line is given by,

$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$

$(x_1, y_1) = (2, -1)$

$(x_2, y_2) = (-6, 3)$

$\frac{y--1}{3--1} = \frac{x-2}{-6-2}$

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$\frac{y+1}{4} = \frac{x-2}{-8}$

-2(y+1) = x – 2

-2y – 2 = x – 2

x + 2y = 0

Slope of the line x + 2y = 0 is m = $-\frac{a}{b} = -\frac{1}{2}$

(g) Find the value of k so that the following lines are concurrent.

5x + 2y – 4 = 0
2x + ky + 11 = 0
3x – 4y – 18 = 0

5x + 2y – 4 = 0
2x + ky + 11 = 0
3x – 4y – 18 = 0

Since the lines are concurrent

$\therefore \begin{vmatrix} 5 & 2 & -4 \\ 2 & k & 11 \\ 3 & -4 & -18 \end{vmatrix} = 0$

$5\begin{vmatrix} k & 11 \\ -4 & -18 \end{vmatrix} - 2\begin{vmatrix} 2 & 11 \\ 3 & -18 \end{vmatrix} - 4\begin{vmatrix} 2 & k \\ 3 & -4 \end{vmatrix} = 0$

5(-18k + 44) – 2(-36 – 33) – 4(-8 – 3k) = 0

-90k + 220 – 2 × -69 + 32 + 12k = 0

-78k + 220 + 138 + 32 = 0

390 = 78k

K = $\frac{390}{78} = 5$

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PART –C

(Maximummark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If A=$\begin{bmatrix}2 & 3\\0 & 1\end{bmatrix}$and I unit matrix of same order, then find $A^3 - 3A^2 + 2A + I$

$A^3 = A^2.A$

$A^3 = A^2.\ A = \begin{bmatrix}2 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\0 & 1\end{bmatrix}A$

$= \begin{bmatrix}4 & 9\\0 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\0 & 1\end{bmatrix} = \begin{bmatrix}8 & 21\\0 & 1\end{bmatrix}$

$A^3 - 3A^2 + 2A + I = \begin{bmatrix}8 & 21\\0 & 1\end{bmatrix} - 3\begin{bmatrix}4 & 9\\0 & 1\end{bmatrix} + 2\begin{bmatrix}2 & 3\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

(b) If A = $\begin{bmatrix}5 & 3\\2 & 2\end{bmatrix}$, B = $\begin{bmatrix}7 & 5\\4 & 3\end{bmatrix}$ show that $(AB)^{-1} = B^{-1}A^{-1}$

$AB = \begin{bmatrix}5 & 3\\2 & 2\end{bmatrix}\begin{bmatrix}5 & 3\\2 & 2\end{bmatrix}\begin{bmatrix}47 & 34\\22 & 16\end{bmatrix}$

$|AB| = \begin{vmatrix}47 & 34\\22 & 16\end{vmatrix} = 4$

Cofactor matrix = $\begin{bmatrix}16 & -22\\-34 & 47\end{bmatrix}$

Adj. (AB) = $\begin{bmatrix}16 & -34\\-22 & 47\end{bmatrix}$

Inverse of AB =$\dfrac{\text{Adj.AB}}{|AB|} = \dfrac{\begin{bmatrix}16 & -34\\-22 & 47\end{bmatrix}}{4} = \begin{bmatrix}4 & -\frac{17}{2}\\-\frac{11}{2} & \frac{47}{4}\end{bmatrix}$

A = $\begin{bmatrix}5 & 3\\2 & 2\end{bmatrix}$ $|A| = 4$

Cofactor matrix of A = $\begin{bmatrix}2 & -2\\-3 & 5\end{bmatrix}$

Adj. (A) = $\begin{bmatrix}2 & -3\\-2 & 5\end{bmatrix}$

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$A^{-1} = \frac{\text{Adj.A}}{|A|} = \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$

$B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$ $\qquad |B| = 1$

Cofactor matrix of B = $\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

Adj. (B) = $\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$

$B^{-1} = \frac{\text{Adj.B}}{|B|} = \frac{\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}}{1}$

$B^{-1}A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4} = \begin{bmatrix} 4 & -\frac{17}{2} \\ -\frac{11}{2} & \frac{47}{4} \end{bmatrix}$

$\therefore \ (AB)^{-1} = B^{-1}A^{-1}$

(c) Find x if $\begin{vmatrix} 2 & x & 3 \\ 4 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & 2 \\ 2 & 3 \end{vmatrix}$

$\begin{vmatrix} 2 & x & 3 \\ 4 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} - x\begin{vmatrix} 4 & 2 \\ 0 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix}$

$= 2(1 - 2) - x(4) + 3(4)$

$= -2 - 4x + 12 = 10 - 4x$

$\begin{vmatrix} x & 2 \\ 2 & 3 \end{vmatrix} = 3x - 4$

$\therefore 3x - 4 = 10 - 4x$

$\Longrightarrow 7x = 14$

$\Longrightarrow x = 2$

IV.

(a) If $A = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 6 & -5 \end{bmatrix}$, show that $(A + B)^T = A^T + B^T$

$A + B = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 6 & -5 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 10 & 4 \end{bmatrix}$

$(A + B)^T = \begin{bmatrix} 5 & 10 \\ 3 & 4 \end{bmatrix}$ —— (1)

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$A^T = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$

$B^T = \begin{bmatrix} 4 & 1 \\ 6 & -5 \end{bmatrix}^T = \begin{bmatrix} 4 & 6 \\ 1 & -5 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 5 & 10 \\ 3 & 4 \end{bmatrix}$ ——— (2)

From (1) & (2) it is clear that $(A + B)^T = A^T + B^T$

(b) Find k, if the following system of equation are consistent
x + y + 1 = 0, x + 2y + 1 = 0, 2x+3y + k = 0

If the system is consistent then,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & k \end{vmatrix} = 0$$

$$1\begin{vmatrix} 2 & 1 \\ 3 & k \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & k \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 0$$

1(2k – 3) – (k – 2) + (3 - 4) = 0

2k – 3 – k + 2 – 1 = 0

k – 2 = 0

k = 2

(c) Solve using inverse of the coefficient matrix
x + y + z = 1,
2x + 2y + 3z = 6,
x + 4y + 9z = 3

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

<u>Calculation for A^{-1}</u>

$m_{11} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6$

$m_{12} = \begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix} = 15$

$m_{13} = \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 6$

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$m_{21} = \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = 5$

$m_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8$

$m_{23} = \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = 3$

$m_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$

$m_{32} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$

$m_{33} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$

$|A| = -3$

Minor matrix = $\begin{bmatrix} 6 & \mathbf{15} & 6 \\ \mathbf{5} & 8 & \mathbf{3} \\ 1 & \mathbf{1} & 0 \end{bmatrix}$

Cofactor matrix = $\begin{bmatrix} 6 & -15 & 6 \\ -5 & 8 & -3 \\ 1 & -1 & 0 \end{bmatrix}$

Adjoint matrix = $\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$

So inverse matrix, $A^{-1} = \frac{\text{Adj.A}}{|A|} = \frac{\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}}{-3}$

$X = A^{-1}B = \frac{\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}}{-3} = \frac{\begin{bmatrix} -21 \\ 30 \\ -12 \end{bmatrix}}{-3}$

$x = -\frac{21}{-3} = 7$

$y = \frac{30}{-3}$-10

$z = -\frac{12}{-3} = 4$

V.

(a) If $20C_r = 20C_{r+2}$ find r

$nC_r = nC_s$ ===> r = s or r + s = n

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ie, r + r + 2 = 20

2r + 2 = 20

2r = 18, r = 9

(b) Find middle term in the expansion of $(x^2 + 3/x)^{20}$

$T_{r+1} = nc_r\, a^{n-r} b^r$, n= 20

n + 1 = 21, odd.

$\therefore \left(\frac{n+1}{2}\right)^{th}$ = 22/2 = 11^{th} term is the middle term

$ie, T_{11} = 20c_{10}\,(x^2)^{10}(3/x)^{10}$

$= 20c_{10}x^{20}3^{10}x^{-10}$

$= 20c_{10}x^{10}3^{10}$

$= 20c_{10}3^{10}x^{20}$

(c) Prove that $\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = 2 - \sqrt{3}$

$$\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{(\sqrt{3}-1)^2}{3-1} = \frac{3+1-2\sqrt{3}}{2} = 2 - \sqrt{3}$$

VI.

(a) Expand $(3x - \frac{y}{2})^4$ binomially

$(a + b)^n = a^n + nc_1\, a^{n-1}\, b + nc_2\, a^{n-2}\, b + \ldots\ldots. + nc_n b^n$

$(3x - \frac{y}{2})^4 = (3x)^4 - 4c_1\,(3x)^3\,(\frac{y}{2}) + 4c_2\,(3x)^2\,(\frac{y}{2})^2 - 4c_3\,(3x)^1\,(\frac{y}{2})^3 + 4c_4(\frac{y}{2})^4$

$= 81x^4 - 4 \times 27x^3(\frac{y}{2}) + 6 \times 9x^2 \times (\frac{y2}{4}) - 4 \times 3 \times x \times (\frac{y3}{8}) + (\frac{y4}{16})$

$= 81x^4 - 54x^3y + (\frac{27}{2})x^2y^2 - (\frac{3xy3}{2}) + \frac{y4}{16}$

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(b) Find the constant term in the expansion of $(\sqrt{x} + \frac{2}{x^2})^{10}$

$T_{r+1} = nc_r\, a^{n-r} b^r$

$T_{r+1} = 10c_r\, (\sqrt{x})^{10-r} (\frac{2}{x^2})^r$

$= 10c_r x^{\frac{10-r}{2}} 2^r x^{-2r}$

$= 10c_r x^{\frac{10-r}{2}}\, x^{-2r} 2^r$

$= 10c_r x^{\frac{10-r}{2} - 2r} 2^r$

$= 10c_r x^{\frac{10-5r}{2}} 2^r$

Then $\frac{10-5r}{2} = 0$ ===> $10 - 5r = 0$

===> $r = 2$

Therefore $T_3 = 10c_2 2^2$

$= 45 \times 4 = 180$ is the constant term.

(c) Prove that $\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta . \text{cosec}^2\theta$

$\sec^2\theta = \frac{1}{\cos^2\theta}$

$\text{cosec}^2\theta = \frac{1}{\sin^2\theta}$

$$\sec^2\theta + \text{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta\cos^2\theta}$$

$$= \sec^2\theta . \text{cosec}^2\theta$$

VII. (a)Prove that in ΔABC, $\sum a(\sin B - \sin C) = 0$

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inΔABC, we know that sinB = $\frac{b}{2R}$ (by sine rule)

andsinC = $\frac{c}{2R}$

$\sum a(\sin B - \sin C) = \sum a\left(\frac{b}{2R} - \frac{c}{2R}\right)$

$= \frac{1}{2R}\left(\sum a\,(b - c)\right)$

$= \frac{1}{2R}\left[a\,(b - c) + b\,(c - a) + c\,(a - b)\right]$

$= \frac{1}{2R}\left[ab - ac + bc - ba + ca - cb\right]$

$= \frac{1}{2R} \times 0 = 0$

(b)Prove that cos20.cos40.cos60.cos80 = $\frac{1}{16}$

We have cos60 = $\frac{1}{2}$

ie, $\frac{1}{2}$cos20.cos40.cos80 =

$= \frac{1}{2}\cos 20 . \frac{1}{2}\,[\cos 120 - \cos(-40)]$

$= {}^{1}\!/_{4} . \cos 20\ [-\frac{1}{2} + \cos 40]$

$= -\frac{1}{8} . \cos 20 + \frac{1}{4}\cos 20 . \cos 40$

$= -\frac{1}{8} . \cos 20 + \frac{1}{4} \times (\cos 60 + \cos 20)$

$= -\frac{1}{8} . \cos 20 + \frac{1}{8}\cos 60 + \frac{1}{8}\cos 20$

$= \frac{1}{8} . \cos 60 = \frac{1}{8} \times \frac{1}{8} = \frac{1}{16}$

(c)Show that sin120.cos330 + cos240.sin330 = 1

Sin120 = sin(1 x 90 + 30) = cos30 = $\frac{\sqrt{3}}{2}$

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Cos330 = cos(360 – 30) = cos(4 x 90 – 30) = cos30 = $\frac{\sqrt{3}}{2}$

Cos240 = cos(270 – 30) = cos(3 x 90 – 30) = -sin30 = $\frac{-1}{2}$

Sin330 = sin(360 – 30) = sin(4 x 90 – 30) = -sin30 = $\frac{-1}{2}$

sin120.cos330 + cos240.sin330

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{-1}{2} \times \frac{-1}{2} = \frac{3}{4} + \frac{1}{4} = 1$

VIII.

(a) Express 3cosθ + 4sinθ in the form of Rsin(θ + α) where α is acute.

$\sqrt{3}\cos x + \sin x = R.\sin(x + \alpha)$

$= R.\sin x.\cos\alpha + R\cos x.\sin\alpha$

Equating the similar terms on both sides,

$\sqrt{3}\cos x = R\sin\alpha.\cos\alpha$

$\sin x = R\sin x.\cos\alpha$

$\Longrightarrow \sqrt{3} = R\sin\alpha$ ——— (1)

$\Longrightarrow 1 = R\cos\alpha$ ——— (2)

Squaring and adding (1) & (2)

$3 + 1 = R^2 \sin^2\alpha + \cos^2\alpha$

$4 = R^2 \quad \Longrightarrow R = \pm 2$

(1) ÷ (2) $\quad \Longrightarrow \sqrt{3} = \frac{\sin\alpha}{\cos\alpha} \Longrightarrow \tan\alpha = \sqrt{3}$

$\Longrightarrow \alpha = \tan^{-1}(\sqrt{3})$

$\Longrightarrow \alpha = 60^o$

(b) Prove that sin(A + B).sin(A – B) = $\sin^2 A - \sin^2 B$

sin(A+B) = sinA.cosB + cosA.sinB

sin(A-B) = sinA.cosB - cosA.sinB

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$\sin(A+B).\sin(A-B) = (\sin A.\cos B + \cos A.\sin B)\ (\sin A.\cos B - \cos A.\sin B)$

$= \sin^2 A.\cos^2 B - \sin A.\cos A \sin B.\cos B$

$+ \sin A \cos A.\sin B \cos B - \cos^2 A.\sin^2 B$

$= \sin^2 A.\cos^2 B - \cos^2 A.\text{sins}^2 B$

$= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 A.\sin^2 B - \sin^2 B - \sin^2 A.\sin 2B$

$= \sin^2 A - \sin^2 B$

(c) In any ΔABC, show that (b + c)sinA/2 = $a.\cos(\frac{B-C}{2})$

LHS = $(a + b).\sin\frac{C}{2}$

$= (2R\sin A + 2R\sin B)\sin\frac{C}{2}$ $\left[\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= 2R(\sin A + \sin B)\sin\frac{C}{2}$

$= 2R.2.\sin\frac{A+B}{2}.\cos\frac{A-B}{2}.\sin\frac{C}{2}$

$= \cos\frac{A-B}{2}.4R.\sin\frac{A+B}{2}.\sin\frac{C}{2}$

$= \cos\frac{A-B}{2}.4R.\sin\left(90 - \frac{C}{2}\right).\sin\frac{C}{2}$

$= \cos\frac{A-B}{2}.2R.\left(2\cos\frac{C}{2}.\sin\frac{C}{2}\right)$

$= \cos\frac{A-B}{2}.2R.\sin C$

$= \cos\frac{A-B}{2}.c$ = RHS

IX.

(a) Solve ΔABC if a = 5cm, B = 30^o& c = 8cm

$\tan(\frac{A-B}{2}) = \frac{a-b}{a+b}\cot\frac{c}{2}$

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$$\frac{A-B}{2}=\tan^{-1}\left[\frac{a-b}{a+b}\cot\frac{c}{2}\right]$$

$$\frac{A-B}{2}=\tan^{-1}\left[\frac{5-8}{13}\cot\frac{30}{2}\right]$$

$$\frac{A-B}{2}=\tan^{-1}\left[\frac{-3}{13}\cot 15^{o}\right]$$

$$\frac{A-B}{2}=\tan^{-1}[-0.8612] = -40.736$$

A – B = -81.473 ——— (1)

A + B = 180 – 30 =150 ——— (2)

Solving (1) + (2)

A = 34.2635 = $34^{O}16'$ B = 150 - 34.2635 = $115^{O}44'$

Now we have to find 'C'

We have $$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{5}{\sin 34^{o}16'} = \frac{c}{\sin 30^{o}}$$

$$c = \frac{5}{0.5629996} \times \sin 30^{o} = 4.44\text{cm}$$

(b) Find the slope and intercept of the line 3x + 4y = 12

Slope of 3x + 4y = 12 is $-\frac{a}{b} = -\frac{3}{4}$

Intercept form of a line is $\frac{x}{a}+\frac{y}{b}= 1$

3x + 4y = 12

$$\frac{3x}{12}+\frac{4y}{12}= 1$$

$$\Longrightarrow \frac{x}{4}+\frac{y}{3}= 1$$

X intercept = 4

Y intercept = 3

(c) Find k so that the lines kx + 2y – 10 = 0, 2x – 4y + 15 = 0 are

(i). Perpendicular to each other.

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(ii). Parallel to each other.

(i). $m_1 \times m_2 = -1$

$-\frac{k}{2} x - \frac{2}{-4}$ = -1-1

$\frac{2k}{-8} =$ ===> 2k = 8 ===> k =4

(ii). $m_1 = m_2$

$-\frac{k}{2} = -\frac{2}{-4}$

$-\frac{k}{2} = -\frac{1}{-2}$

$k = -1$

X.

(a) Solve ΔABC using Napier's formula, given a = 87cm, b = 53cm and C = 70°

$\tan(\frac{A-B}{2}) = \frac{a-b}{a+b} \cot \frac{C}{2}$

$A - B = 2\tan^{-1}[\frac{a-b}{a+b} \cot \frac{C}{2}]$

$A - B = 2\tan^{-1}[\frac{87-53}{87+53} \cot 35]$

$= 2\tan^{-1}[\frac{34}{140} \cot 15^o]$

$= 2\tan^{-1}[0.3469] = 2 \times 19^o08' = 38^o16'$

$A + B = 180 - 70 = 110^o$

$2A = 148^o16'/2 = 74^o08'$

A = B = 110

$B = 110 - 74^o08' = 35^o52'$

Now we have to find 'c', we have

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{87}{\sin 74^o08'} = \frac{c}{\sin 70^o}$$

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$$===> c = \frac{\sin 70 \text{ x } 87}{\sin 74^{o} 08'} = \frac{0.9397 \text{ x } 87}{0.9619} = 84.99\text{cm}$$

(b) Find the equation to the line passing through the point of intersection of x – y + 1 = 0 and 2x – 3y +2 = 0 and perpendicular to the line x + y – 6 = 0

Given

x – y + 1 = 0

2x – 3y + 2 = 0

∴ x – y = -1

2x – 3y = -2

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\therefore x = \frac{\begin{vmatrix} -1 & -1 \\ -2 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}}$$

$$= \frac{3-2}{-3+2} = \frac{1}{-1} = -1$$

$$Y = \frac{\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}}$$

$$= \frac{-2+2}{-3+2} = \frac{0}{1} = 0$$

∴ Point of intersection = (-1, 0)

∴ Given the line is,

x + y – 6 = 0

a = 1, b = 1, c = -6

∴ Perpendicular line is,

bx - ay + k = 0

x – y + k = 0 ———— (1)

∴ (1) Passes through (-1, 0)

(1) ===> -1 – 0 + k = 0

K = 1

∴ (1) ===> x – y + 1 = 0

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(c) A line passes through (-6 , 3). The X-intrecept of the line is 3 times its Y-intercept. Find the equation of the line.

a = 3 b (given)

The equation of a line is $\frac{x}{a} + \frac{y}{b} = 1$

Ie, $\frac{x}{3b} + \frac{y}{b} = 1$……………….(1)

(1) Pass through (-6 , 3)

(1) Implies, $\frac{-6}{3b} + \frac{3}{b} = 1$

$\frac{-2}{b} + \frac{3}{b} = 1$

$\frac{1}{b} = 1$, b = 1

(1) Implies $\frac{x}{3} + \frac{y}{1} = 1$

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