

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/
TECHNOLIGY- OCTOBER, 2011

TECHNICAL MATHEMATICS- I
(Common – Except DCP and CABM)

[Time: 3 hours

(Maximum marks: 100)

Marks

PART –A
(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) If $\begin{vmatrix} 3x & 7 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix}$ find x.

$$\begin{vmatrix} 3x & 7 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix}$$

$$\implies 9x - 14 = 8 - 4$$

$$\implies 9x - 14 = 4$$

$$\implies 9x = 18$$

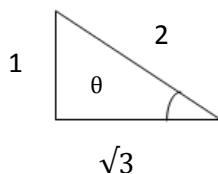
$$\implies x = 18/9 = 2$$

(b) If $n_{C_{20}} = n_{C_{23}}$ find n

$$n_{C_r} = n_{C_s} \implies r = s \quad \text{or} \quad r + s = n$$

$$\text{Then } n = r + s = 20 + 23 = 43$$

(c) If $\cos\theta = \sqrt{3}/2$ find $\sin\theta$ & $\tan\theta$



$$\sin\theta = 1/2 \quad \tan\theta = 1/\sqrt{3}$$

(d) If $\sin\theta = a$ find $\sin 3\theta$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\sin 3\theta = 3a - 4a^3$$

(e) Find the slope of the line joining the vertices (2, 6), (4, 0)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{4 - 2} = \frac{-6}{2} = -3$$

PART -B

Answer any five questions. Each question carries 6 marks

II.

(a) Solve using determinants:

$$x + y + z = 3$$

$$2x + 3y + z = -6$$

$$x + y - z = -3$$

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & 1 \\ -6 & 3 & 1 \\ -3 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{(-3-1)-(-2-1)+2-3}{-2} = \frac{-18}{-2} = 9$$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 2 & -6 & 1 \\ 1 & -3 & -1 \end{vmatrix}}{-2} = \frac{18}{-2} = -9$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & -6 \\ 1 & 1 & -3 \end{vmatrix}}{-2} \\ = \frac{-6}{-2} = 3$$

(b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ evaluate AB & BA

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1+2+6 & 3+4+9 \\ 8 & -4+5+12 & 12+10+18 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 16 \\ 8 & 13 & 40 \\ 2 & -1 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4+3 & 4-5 & 6-6 \\ 4+2 & 5 & 6 \\ 8+3 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 6 & 5 & 6 \\ 11 & 10 & 12 \end{bmatrix}$$

(c) Find the middle term of $(x^2 + 1/x)^{12}$

$$T_{r+1} = nC_r a^{n-r} b^r, n=12$$

$$T_{r+1} = 12C_r (x^2)^{12-r} (1/x)^r$$

$$n+1 = 13, \text{ odd.}$$

$$\therefore \left(\frac{n+1}{2}\right)^{\text{th}} = \frac{12}{2} + 1 = 6 + 1 = 7^{\text{th}} \text{ term is the middle term}$$

$$\text{ie, } T_7 = 12C_6 (x^2)^6 (1/x)^6$$

$$= 12C_{10} x^{12-6}$$

$$= 12C_6 x^6$$

$$= 924x^6$$

(d) Prove that $\sin\theta + \sin3\theta + \sin5\theta + \sin7\theta = 4 \cos\theta \cdot \cos2\theta \cdot \sin4\theta$

$$\sin\theta + \sin3\theta + \sin5\theta + \sin7\theta \quad \left[\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \right]$$

$$\begin{aligned}
&= (\sin\theta + \sin 7\theta) + (\sin 3\theta + \sin 5\theta) \\
&= 2\sin 4\theta \cdot \cos 3\theta + 2\sin 4\theta \cdot \cos \theta \\
&= 2\sin 4\theta (\cos 3\theta + \cos \theta) \quad \left[\cos C + \cos D = 2 \cos \frac{(C+D)}{2} \cdot \cos \frac{(C-D)}{2} \right] \\
&= 2\sin 4\theta \cdot 2(\cos 2\theta \cdot \cos \theta) \\
&= 2\sin 4\theta \cdot \cos 2\theta \cdot \cos \theta, \text{ hence the result.}
\end{aligned}$$

(e) Find the equation of the line parallel and perpendicular to $2x - 3y + 10 = 0$ and passing through $(1, 1)$.

$$2x - 3y + 10 = 0$$

$$a = 2, b = -3, c = 10$$

Case I

Equation of a line parallel is $ax + by + k = 0$

$$2x - 3y + k = 0 \quad \text{--- (1)}$$

(1) Passes through $(1, 1)$

$$(1) \implies 2 \times 1 - 3 \times 1 + k = 0$$

$$2 - 3 + k = 0$$

$$-1 + k = 0$$

$$K = 1$$

$$\text{So, (1) } \implies 2x - 3y + 1 = 0$$

Case II

Equation of a line perpendicular is, $bx - ay + k = 0$

$$-3x - 2y + k = 0 \quad \text{--- (2)}$$

(2) Passing through $(1, 1)$

$$(2) \implies -3 \times 1 - 2 \times 1 + k = 0$$

$$-3 - 2 + k = 0$$

$$-5 + k = 0$$

$$K = 5$$

$$\text{So, (1) } \implies -3x - 2y + 5 = 0$$

(f) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$

$$\begin{aligned}
 \cos 3A &= \cos(2A + A) \\
 &= \cos 2A \cdot \cos A - \sin 2A \cdot \sin A \\
 &= (2\cos^2 A - 1)\cos A - 2\sin A \cdot \cos A \cdot \sin A \\
 &= 2\cos^3 A - \cos A - 2\sin^2 A \cdot \cos A \\
 &= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A \\
 &= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A \\
 &= 4\cos^3 A - 3\cos A
 \end{aligned}$$

(g) The straight line through (4, 3) makes intercepts of 4a and 3a on the X axis and Y axis respectively. Find a

Equation of a line in intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{4a} + \frac{y}{3a} = 1 \quad \text{--- (1)}$$

(1) Passing through (4, 3) then $\frac{x}{4a} + \frac{3}{3a} = 1$

$$\implies \frac{1}{a} + \frac{1}{a} = 1$$

$$\implies a = 2$$

Then (1) becomes,

$$\frac{x}{8} + \frac{y}{6} = 1$$

PART -C
(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Find x if $\begin{vmatrix} 2 & 3 & 5 \\ 2 & x & 5 \\ 3 & -1 & 2 \end{vmatrix} = 0$

$$\begin{vmatrix} 2 & 3 & 5 \\ 2 & x & 5 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$$\implies 2(2x + 5) - 3(4 - 15) + 5(-2 - 3x) = 0$$

$$\implies 4x + 10 - 12 + 45 + -10 - 15x = 0$$

$$\implies -11x = -33$$

$$x = -33/-11 = 3$$

(b) Express the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

$$\frac{A+A^T}{2} = \frac{\begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}}{2}$$

$$= \frac{\begin{bmatrix} 2 & 6 & 8 \\ 6 & 4 & 4 \\ 8 & 4 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

————— (1) a symmetric matrix

$$\frac{A-A^T}{2} = \frac{\begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}}{2}$$

$$= \frac{\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 2 \\ -2 & -2 & 0 \end{bmatrix}}{2} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

————— (2) a skew symmetric matrix

Adding (1) & (2)

$$\begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = A$$

Hence the result.

(c) Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

Minors

$$m_{11} = \begin{vmatrix} 5 & 0 \\ 4 & 3 \end{vmatrix} = 15$$

$$m_{12} = \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} = 0$$

$$m_{13} = \begin{vmatrix} 0 & 5 \\ 2 & 4 \end{vmatrix} = -10$$

$$m_{21} = \begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} = 6$$

$$m_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 3-6=-3$$

$$m_{23} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0$$

$$m_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 0 \end{vmatrix} = -15$$

$$m_{32} = \begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix} = 0$$

$$m_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} = 5$$

$$\text{Minor matrix of } A = \begin{bmatrix} 15 & 0 & -10 \\ -6 & -3 & 0 \\ -15 & 0 & 5 \end{bmatrix}$$

$$\text{Cofactor matrix of } A = \begin{bmatrix} 15 & 0 & -10 \\ 6 & -3 & 0 \\ -15 & 0 & 5 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{vmatrix} = 1 \begin{vmatrix} 5 & 0 \\ 4 & 3 \end{vmatrix} - 2 \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} + 3 \begin{vmatrix} 0 & 5 \\ 2 & 4 \end{vmatrix}$$

$$= 15 \times 2 \times 0 + 3 \times -10 = 15 - 30 = -15$$

$$\text{So inverse matrix} = \frac{\text{Adj.}A}{|A|} = \frac{\begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}}{-15}$$

IV.

(a) Find the value of 'p'

$$2x + 3y + 9 = 0$$

$$4x + py + 13 = 0$$

$$px - 2y - 25 = 0$$

Since the system is consistent,

$$\text{The eliminant, } \begin{vmatrix} 2 & 3 & 9 \\ 4 & p & 13 \\ p & -2 & -25 \end{vmatrix} = 0$$

$$2(-25p + 26) - 3(-100 - 13p) + 9(-8 - p^2) = 0$$

$$-50p + 52 + 300 + 39p - 72 - 9p^2 = 0$$

$$-9p^2 - 11p + 280 = 0$$

$$P = \frac{-11 \pm \sqrt{11^2 - 4 \times 9 \times -280}}{2 \times 9} = \frac{-11 \pm \sqrt{10201}}{18} = \frac{-11 \pm 101}{18}$$

$$P = \frac{-112}{18} \text{ or } p = \frac{90}{18} = 5$$

(b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ show that $A^2 - 4A - 5I = 0$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) If $A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$ show that $(AB)^{-1} = B^{-1}A^{-1}$

$$AB = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 47 & 34 \\ 22 & 16 \end{bmatrix}$$

To find $(AB)^{-1}$

Minors

$$m_{11} = 16$$

$$m_{12} = 22$$

$$m_{21} = 34$$

$$m_{22} = 47$$

$$\therefore \text{Minor matrix} = \begin{bmatrix} 16 & 22 \\ 34 & 47 \end{bmatrix}$$

$$\text{Cofactor of } AB = \begin{bmatrix} 16 & -22 \\ -34 & 47 \end{bmatrix}$$

$$\text{Adj } AB = \begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}$$

$$\begin{aligned} \therefore |AB| &= \begin{vmatrix} 47 & 34 \\ 22 & 16 \end{vmatrix} \\ &= 752 - 748 \\ &= 4 \end{aligned}$$

$$\begin{aligned} (AB)^{-1} &= \frac{\text{adj } AB}{|AB|} \\ &= \frac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4} \end{aligned}$$

To find B^{-1}

$$B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$$

Minors

$$m_{11} = 3$$

$$m_{12} = 4$$

$$m_{21} = 5$$

$$m_{22} = 7$$

$$\therefore \text{Minor matrix} = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$$

$$\text{Cofactor of B} = \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$$

$$\text{AdjB} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$$

$$\therefore |B| = \begin{vmatrix} 7 & 5 \\ 4 & 3 \end{vmatrix}$$

$$= 1$$

$$B^{-1} = \frac{\text{adj B}}{|B|}$$

$$= \frac{\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}}{1}$$

To find A⁻¹

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$$

Minors

$$m_{11} = 2$$

$$m_{12} = 2$$

$$m_{21} = 3$$

$$m_{22} = 5$$

$$\therefore \text{Minor matrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$$

$$\text{Cofactor of A} = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\text{AdjA} = \begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 5 & 3 \\ 2 & 2 \end{vmatrix}$$

$$= 10 - 6$$

$$= 4$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$$

$$B^{-1} \times A^{-1} = \frac{\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}}{1} \times \frac{\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}}{4}$$

$$B^{-1} \times A^{-1} = \frac{\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}}{4}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

V.

(a) Expand $(x + 1/\sqrt{x})^5$ binomially

$$(a + b)^n = a^n + nc_1 a^{n-1} b + nc_2 a^{n-2} b^2 + \dots + nc_n b^n$$

$$\begin{aligned} (x + 1/\sqrt{x})^5 &= x^5 + 5c_1 x^4(1/\sqrt{x}) + 5c_2 x^3(1/\sqrt{x})^2 + 5c_3 x^2(1/\sqrt{x})^3 + 5c_4 x(1/\sqrt{x})^4 + (1/\sqrt{x})^5 \\ &= x^5 + 5x^4(1/\sqrt{x}) + 10x^3(1/x) + 10x^2(1/x\sqrt{x}) + 5x(1/x^2) + (1/x^2\sqrt{x}) \\ &= x^5 + 5x^4 x^{-1/2} + 10x^2 + 10x \cdot x^{-1/2} + 5x^{-1} + x^{-5/2} \\ &= x^5 + 5x^{7/2} + 10x^2 + 10x^{1/2} + 5x^{-1} + x^{-5/2} \end{aligned}$$

(b) Find the 10th term in the expansion of $(x^2 + 1/x)^{20}$

$$T_{r+1} = nc_r a^{n-r} b^r, \quad n = 20$$

$$T_{r+1} = 20c_r (x^2)^{20-r} (1/x)^r$$

$$= 20c_r x^{40-2r} x^{-2r}$$

$$= 20c_r x^{40-4r}$$

$$\text{ie, } T_{10} = 20c_9 x^{40-36} = 20c_9 x^4$$

(c) Prove that $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$

$$\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)\sin A}$$

$$\begin{aligned}
&= \frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{(1 + \cos A)\sin A} \\
&= \frac{1 + 1 + 2\cos A}{(1 + \cos A)\sin A} = \frac{2 + 2\cos A}{(1 + \cos A)\sin A} \\
&= \frac{2(1 + \cos A)}{(1 + \cos A)\sin A} = 2\operatorname{cosec} A
\end{aligned}$$

VI.

(a) Find the middle term of $(2x + 3/x)^9$

$$T_{r+1} = nC_r a^{n-r} b^r, \quad n = 9$$

$$T_{r+1} = 9C_r (2x)^{9-r} (3/x)^r$$

$$n + 1 = 10, \text{ even.}$$

\therefore 5th term and 6th term are the middle term.

$$\begin{aligned}
ie, T_5 &= 9C_4 (2x)^5 (3/x)^4 \\
&= 9C_4 2^5 x^5 3^4 x^{-4} \\
&= 9C_4 2^5 x^3 3^4 \\
&= 9C_4 2^5 3^4 x
\end{aligned}$$

$$\begin{aligned}
ie, T_6 &= 9C_5 (2x)^4 (3/x)^5 \\
&= 9C_5 2^4 x^4 3^5 x^{-5} \\
&= 9C_5 2^4 x^{-1} 3^5 \\
&= 9C_5 2^4 3^5 x^{-1}
\end{aligned}$$

(b) Find the coefficient of x^4 in the expansion of $(x^4 - 1/x^3)^{15}$

$$T_{r+1} = nC_r a^{n-r} b^r, \quad n = 15$$

$$\begin{aligned}
T_{r+1} &= 15C_r (x^4)^{15-r} (-1/x^3)^r \\
&= 15C_r x^{60-4r} (-1)^r x^{-3r} \\
&= 15C_r x^{60-7r} (-1)^r
\end{aligned}$$

$$\text{Now } 60 - 7r = 4$$

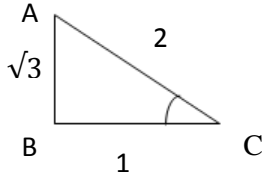
$$-7r = -56$$

$$r = 56/7 = 8$$

$$T_9 = 15C_8 x^4 (-1)^8$$

∴ Coefficient of x^4 is $15c_8$

(c) If $\sin A = 1/2$, A lies in first quadrant. Find all t- functions.



$$AB = \sqrt{2^2 - 1^2} = \sqrt{3}$$

$$\sin A = 1/2$$

$$\cos A = \sqrt{3}/2$$

$$\sec A = 2/\sqrt{3}$$

$$\tan A = 1/\sqrt{3}$$

$$\cot A = \sqrt{3}$$

$$\operatorname{cosec} A = 2$$

VII.

(a) If $\tan A = 18/17$, $\tan B = 1/35$, prove that $A - B = 45^\circ$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{18}{17} - \frac{1}{35}}{1 + \frac{18}{17} \times \frac{1}{35}}$$

$$= \frac{\frac{630 - 17}{595}}{\frac{595 + 18}{595}}$$

$$= \frac{613}{613} = 1$$

(b) Prove that $\frac{\cos(90+A) \cdot \sec(360+A) \cdot \tan(180-A)}{\sec(A-720) \cdot \sin(540+A) \cdot \cot(A-90)} = 1$

$$\cos(90 + A) = -\sin A$$

$$\sec(360 + A) = 1/\cos(360 + A) = 1/\cos A = \sec A$$

$$\tan(180 - A) = -\tan A$$

$$\sec(A - 720) = \sec -(720 - A) = \sec A$$

$$\sin(540 + A) = \sin(360 + 180 + A) = \sin(180 + A) = -\sin A$$

$$\cot(A - 90) = \cot(90 - A) = -\tan A$$

Substituting the above values

$$\frac{\cos(90+A).\sec(360+A).\tan(10-A)}{\sec(A-720).\sin(540+A).\cot(A-90)} = \frac{-\sin A.\sec A.-\tan A}{\sec A.-\sec A.-\tan A} = 1$$

(c) Prove that in ΔABC , $(a + b)\sin C/2 = c.\cos(\frac{A-B}{2})$.

$$\text{LHS} = (a + b).\sin C/2$$

$$\begin{aligned} &= (2R\sin A + 2R\sin B)\sin C/2 && \left[\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \right] \\ &= 2R(\sin A + \sin B)\sin C/2 \\ &= 2R.2.\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right).\sin C/2 \\ &= \cos\left(\frac{A-B}{2}\right).4R.\sin\left(\frac{A+B}{2}\right).\sin C/2 \\ &= \cos\left(\frac{A-B}{2}\right).4R.\sin\left(90 - \frac{C}{2}\right).\sin C/2 \\ &= \cos\left(\frac{A-B}{2}\right).2R.(2\cos C/2.\sin C/2) && [\sin A = 2 \sin A/2.\cos A/2] \\ &= \cos\left(\frac{A-B}{2}\right).2R.\sin C \\ &= \cos\left(\frac{A-B}{2}\right).c = \text{RHS} \end{aligned}$$

VIII.

(a) Express $\sqrt{3}\cos x + \sin x$ in the form of $R\sin(x + \alpha)$ where α is acute.

$$\begin{aligned} \sqrt{3}\cos x + \sin x &= R.\sin(x + \alpha) \\ &= R.\sin x.\cos \alpha + R\cos x.\sin \alpha \end{aligned}$$

Equating the similar terms on both sides,

$$\sqrt{3}\cos x = R\sin \alpha.\cos \alpha$$

$$\sin x = R\sin x.\cos \alpha$$

$$\implies \sqrt{3} = R\sin \alpha \quad \text{--- (1)}$$

$$\implies 1 = R\cos \alpha \quad \text{--- (2)}$$

Squaring and adding (1) & (2)

$$3 + 1 = R^2 \sin^2 \alpha + \cos^2 \alpha$$

$$4 = R^2 \implies R = \pm 2$$

$$(1) \div (2) \implies \sqrt{3} = \frac{\sin \alpha}{\cos \alpha} \implies \tan \alpha = \sqrt{3}$$

$$\implies \alpha = \tan^{-1}(\sqrt{3})$$

$$\implies \alpha = 60^\circ$$

(b) Show that $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = 1/16$

We have $\cos 60^\circ = 1/2$

$$\text{ie, } \frac{1}{2} \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ =$$

$$= \frac{1}{2} \cdot \cos 20^\circ \cdot \frac{1}{2} [\cos 120^\circ - \cos(-40^\circ)] \quad [\cos A \cdot \cos B = \frac{1}{2} [\cos(A+B) - \cos(A-B)]]$$

$$= \frac{1}{4} \cdot \cos 20^\circ [-\frac{1}{2} + \cos 40^\circ]$$

$$= -\frac{1}{8} \cdot \cos 20^\circ + \frac{1}{4} \cos 20^\circ \cdot \cos 40^\circ$$

$$= -\frac{1}{8} \cdot \cos 20^\circ + \frac{1}{4} \times (\cos 60^\circ + \cos 20^\circ)$$

$$= -\frac{1}{8} \cdot \cos 20^\circ + \frac{1}{8} \cos 60^\circ + \frac{1}{8} \cos 20^\circ$$

$$= \frac{1}{8} \cdot \cos 60^\circ = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$$

(c) In a ΔABC , $A = 30^\circ, C = 45^\circ, a = 2\text{cm}$ find c

$$A = 30^\circ, C = 45^\circ \quad B = 180^\circ - (A + C)$$

$$= 180^\circ - 75^\circ$$

$$= 105^\circ$$

Using sine rule we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\implies \frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\implies \frac{2}{\sin 30^\circ} = \frac{c}{\sin 45^\circ}$$

$$\implies c = \frac{2}{1/2} \times \frac{1}{\sqrt{2}}$$

$$c = 4 \times \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828$$

IX.

(a) Solve $\triangle ABC$ if $a = 24.5\text{cm}$, $b = 18.6\text{cm}$ & $c = 26.4\text{cm}$

$$\begin{aligned}\text{We have } \tan \frac{A}{2} &= \frac{\sqrt{((s-b)(s-c))}}{s(s-a)} \text{ where } s = \frac{a+b+c}{2} = \frac{24.5+18.6+26.4}{2} = 34.75 \\ &= \frac{\sqrt{((34.75-18.6)(34.75-26.4))}}{34.75(34.75-24.5)} \\ &= 0.6153\end{aligned}$$

$$\tan \frac{A}{2} = 0.6153$$

$$\implies A/2 = \tan^{-1}(0.6153) = 31^{\circ}36'$$

$$A = 2 \times 31^{\circ}36' = 63^{\circ}12'$$

$$\begin{aligned}\tan B/2 &= \frac{\sqrt{((s-a)(s-c))}}{s(s-b)} = \frac{\sqrt{((34.75-24.5)(34.75-26.4))}}{34.75(34.75-18.6)} \\ &= 0.3905\end{aligned}$$

(b) The x - intercept of a line is 3 times its y - intercept. The line passes through $(-2, 3)$. Find its equation.

Intercept form of a line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{3b} + \frac{y}{b} = 1$$

The equation passes through $(-2, 3)$

$$\frac{-2}{3b} + \frac{3}{b} = 1$$

$$-2b + 9b = 3b^2$$

$$7b = 3b^2$$

$$\implies b = 7/3$$

\therefore Equation of the line is

$$\frac{x}{7} + \frac{y}{7/3} = 1$$

(c) Find the value of k so that the following lines are concurrent.

$$5x + 2y - 4 = 0$$

$$2x + ky + 11 = 0$$

$$3x - 4y - 18 = 0$$

Eliminant = 0

$$\begin{vmatrix} 5 & 2 & -4 \\ 2 & k & 11 \\ 3 & -4 & 18 \end{vmatrix} = 0$$

$$5 \begin{vmatrix} k & 11 \\ -4 & -18 \end{vmatrix} - 2 \begin{vmatrix} 2 & 11 \\ 3 & -18 \end{vmatrix} - 4 \begin{vmatrix} 2 & k \\ 3 & -4 \end{vmatrix} = 0$$

$$5(-18k + 44) - 2(-36 - 33) - 4(-8 - 3k) = 0$$

$$-90k + 220 + 72 + 66 + 32 + 12k = 0$$

$$-78k + 390 = 0$$

$$-78k = -390$$

$$K = \frac{-390}{-78} = 5$$

X.

(a) Solve ΔABC $a = 4$, $b = 5$ and $C = 50^\circ$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{c}{2}$$

$$A - B = 2 \tan^{-1} \left[\frac{a-b}{a+b} \cot \frac{c}{2} \right]$$

$$A - B = 2 \tan^{-1} \left[\frac{4-5}{4+5} \cot 50/2 \right]$$

$$= 2 \tan^{-1} \left[\frac{-1}{9} \cot 25^\circ \right]$$

$$= 2 \tan^{-1} [-0.111 \times 2.145]$$

$$A - B = -26.7849$$

$$A + B = 180 - 50 = 130$$

Solving (1) (2)

$$A - B = -26.7849$$

$$A + B = 130$$

$$2A = 103.215$$

$$A = 51.6075 = 51^{\circ}36'$$

$$B = 130 - A = 130 - 51^{\circ}36' = 78^{\circ}24'$$

Now we have to find 'C', we have

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ \frac{4}{\sin 51^{\circ}36'} &= \frac{c}{\sin 50^{\circ}} \\ \implies c &= \frac{\sin 50^{\circ} \times 4}{\sin 51^{\circ}36'} = 3.909\end{aligned}$$

- (b) Find the equation to the line passing through the point of intersection of $x - y + 1 = 0$ and $2x + 3y + 2 = 0$ and perpendicular to the line $x + y - 6 = 0$

Given $x - y + 1 = 0$
 $2x - 3y + 2 = 0$

$$\therefore x - y = -1$$

$$2x - 3y = -2$$

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\begin{aligned}\therefore x &= \frac{\begin{vmatrix} -1 & -1 \\ -2 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}} \\ &= \frac{3-2}{-3+2} = \frac{1}{-1} = -1\end{aligned}$$

$$\begin{aligned}Y &= \frac{\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}} \\ &= \frac{-2+2}{-3+2} = \frac{0}{-1} = 0\end{aligned}$$

\therefore Point of intersection = (-1, 0)

\therefore Given line is

$$x + y - 6 = 0$$

$$a = 1, b = 1, c = -6$$

∴ Perpendicular line is

$$bx - ay + k = 0$$

$$x - y + k = 0$$

①

∴ ① Pass through $(-1, 0)$

$$① \implies -1 - 0 + k = 0$$

$$K = 1$$

$$\therefore ① \implies x - y + 1 = 0$$

(c) Find the foot of the perpendicular from the origin to the line $3x - 2y - 13 = 0$

$$3x - 2y - 13 = 0$$

Slope of the line perpendicular to the line $3x - 2y = 13$ is $-2/3$

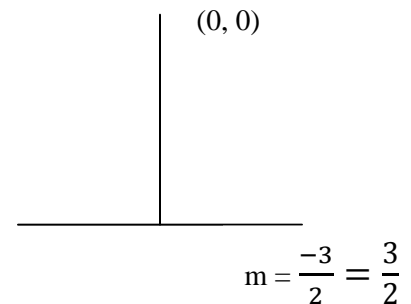
Equation of the line perpendicular to $3x - 2y = 13$ is

$$y - 0 = -2/3 (x - 0)$$

$$y = -2/3 x$$

$$3y = -2x$$

$$2x + 3y = 0$$



Foot of the perpendicular is obtained by solutions

$$3x - 2y = 13 \quad \text{①}$$

$$2x + 3y = 0 \quad \text{②}$$

$$6x - 4y = 26$$

$$6x + 9y = 0$$

$$\hline -13y = 26$$

$$y = \frac{26}{-13} = -2$$

$$3x - 2 \times -2 = 13$$

$$3x = 13 - 4$$

$$x = 9/3 = 3$$

∴ Foot perpendicularis (3, -2)

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