TED (10)-1015 Reg. No. …………………………

(REVISION-2010) Signature …….……………………

SECOND SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- OCTOBER, 2012

TECHNICAL MATHEMATICS- II

(Common – Except DCP and CABM)

[*Time:* 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) Evaluate $\lim_{x\to\infty} \frac{x^3-3x+5}{2x^3-4x-6}$

We have $\lim_{x\to\infty} \frac{x^3-3x+5}{2x^3-4x-6} = \lim_{x\to\infty} \frac{x^3(1-\frac{3}{x^2}+\frac{5}{x^3})}{x^3\left(2-\frac{4}{x^2}-\frac{6}{x^3}\right)}$

$$= \frac{(1-\frac{3}{\infty^2}+\frac{5}{\infty^3})}{\left(2-\frac{4}{\infty^2}-\frac{6}{\infty^3}\right)}$$

$$= \frac{1}{2}$$

(b) Find 'k', if $f(x) = \begin{cases} kx^2, & x \neq 2 \\ 3, & x = 2 \end{cases}$ is continuous at x= 2

F(2) = 3

$\lim_{x\to 2} f(x) = \lim_{x\to 2} kx^2$

$= k \times 2^2$

$= 4k$

$\therefore \lim_{x\to 2} f(x) = f(2)$

$\therefore 4k = 3$

$$k = \frac{3}{4}$$

(c) If the displacement of a particle at a time 't' is given by $S = t^2 - 4t + 3$. Find the velocity at t = 4 seconds

$$S = t^2 - 4t + 3$$

Velocity V = $\frac{ds}{dt}$

= 2t – 4

= 2 x 4 – 4

= 8

(d) Find $\int \sec x \, dx$

$\int \sec x \, dx = \int \frac{\sec x(\sec x+\tan x)}{(\sec x+\tan x)} dx$

$= \int \frac{\sec^2 x+\sec x \tan x}{(\sec x+\tan x)} dx$

Put secx + tanx = u

$\frac{du}{dx} =$ secx tanx + $\sec^2$x

du = ($\sec^2$x+secx tanx)dx

$= \int \frac{du}{u}$

$= \int \frac{1}{u} du$

= log u

= log(secx + tanx)+ c

(e) Solve $\frac{dy}{dx} + 3y = 0$

Apply variable – separable form $\frac{dy}{dx} = -3y$

$$\frac{dy}{y} = -3dx$$

Integrating on both sides

$$\int \frac{dy}{y} = \int -3\ dx$$

$\log y$ = -3x + c

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Find the differential coefficient of 'cosecx' with respect to 'x' using 1st principle.

$$f(x) = cosecx$$

$$f(x+\Delta x) = cosecx(x+\Delta x)$$

$$f(x+\Delta x) - f(x) = cosecx(x+\Delta x) - cosecx$$

$$= \frac{1}{sin(x+\Delta x)} - \frac{1}{sinx}$$

$$= \frac{sinx - sin(x+\Delta x)}{sinx \text{ x } sin(x+\Delta x)}$$ sinC- sinD=2cos $\left(\frac{C+D}{2}\right)$.sin $\left(\frac{C-D}{2}\right)$

$$= \frac{2\cos\left(\frac{x+x+\Delta x}{2}\right) \text{ x } \sin\left(\frac{x-x+\Delta x}{2}\right)}{sinx\ \sin(x+\Delta x)}$$

$$= \frac{2\cos\left(x+\frac{\Delta x}{2}\right) \text{ x } \sin\left(\frac{-\Delta x}{2}\right)}{sinx\ \sin(x+\Delta x)}$$

$$= \frac{-2\cos\left(x+\frac{\Delta x}{2}\right) \text{ x } \sin\left(\frac{\Delta x}{2}\right)}{sinx\ \sin(x+\Delta x)}$$

$$\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{-2cos\left(x+\frac{\Delta x}{2}\right)\sin(\frac{\Delta x}{2})}{sinx\ \sin(x+\Delta x)\frac{\Delta x}{2}\text{x } 2}$$

$$= \frac{-\cos(x+\frac{0}{2})}{sinx\ \sin(x+0)}$$

$$= \frac{-\cos x}{sinx \sin x}$$

$$\therefore \frac{dy}{dx} = -cotx\ cosecx$$

(b) For what value of ‘x’ is the tangent to the curve $y = 2x^3 - 9x^2 + 12x - 3$ parallel to the x – axis

$$\frac{dy}{dx} = 0$$

$$ie 6x^2 - 18x + 12 = 0$$

$$ie 6(x^2 - 3x + 2) = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

x = 1, 2

(c) A particle moves such that the displacement from a point ‘o’ is always given by $S = 5cosnt + 4sinnt$, where ‘n’ is a constant. Prove that the acceleration varies as the displacement.

$$s = 5cosnt + 4sinnt$$

Acceleration, a = $\frac{d^2s}{dt^2}$

Velocity, v = $\frac{ds}{dt}$

$= -5n\ sin\ nt + 4n\ cos\ nt$

a = $-5n^2 cos\ nt - 4n^2 sin\ nt$

$= -n^2[5cosnt + 4sinnt]$

$= -n^2 s$

a = ks, where k = $-n^2$

∴ Acceleration ∝ displacements

(d) $\int_0^{\frac{\pi}{2}} sin5x\; cosx\; dx$

$$\int_0^{\frac{\pi}{2}} sin5x\; cosx\; dx = \frac{1}{2}[\int_0^{\frac{\pi}{2}}(sin5x + 2x) + \sin(5x - 2x)]\, dx$$

$$= \frac{1}{2}[\int_0^{\frac{\pi}{2}}(sin7x + \sin 3x)]\, dx$$

$$= \frac{1}{2}\left[-\frac{cos7x}{7} - \frac{cos3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$=- \frac{1}{2}\left[\frac{cos7x}{7} + \frac{cos3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \; -\frac{1}{2}\left[\frac{cos7x}{7} + \frac{cos3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \; -\frac{1}{2}\left[\frac{cos7\frac{\pi}{2}}{7} + \frac{cos3\frac{\pi}{2}}{3}\right] + \frac{1}{2}[cos0 + cos0]$$

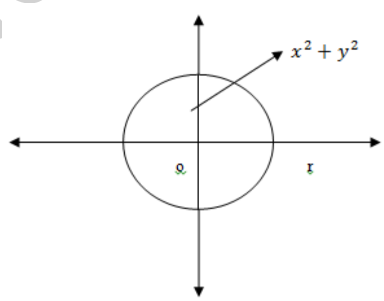
$$= -\frac{1}{2}\left[\frac{0}{7} + \frac{0}{3}\right] + \frac{1}{2}[1 + 1]$$

$$= -\frac{1}{2}[0] + \frac{1}{2}[2]$$

$$= 1$$

(e) Obtain the area of the quadrant of a circle of radius ‘r’ limits, using integration.

We know the equation of a circle is



$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2} \qquad\qquad x = 0, x = r$$

Area in the 1st quadrant, A = $\int_a^b y\, dx$

$$= \int_0^r \sqrt{r^2 - x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{r^2 - x^2} + \frac{r^2}{2}\sin^{-1}\left(\frac{x}{r}\right)\right]_0^r$$

$$= \frac{r}{2}\sqrt{r^2 - r^2} + \frac{r^2}{2}\sin^{-1}\left(\frac{r}{r}\right)$$

$$- \frac{0}{2}\sqrt{r^2 - 0^2} - \frac{r^2}{2}\sin^{-1}\left(\frac{0}{r}\right)$$

$$= \frac{r}{2} \times 0 + \frac{r^2}{2}\sin^{-1}(1) - 0 - \frac{r^2}{2} \times 0$$

$$= \frac{r^2}{2} \times \frac{\pi}{2} = \frac{\pi r^2}{4}$$

(f) Find $\int_1^e logx$ dx

$$\int_1^e logx\,dx = (logx \times x)_1^e - \int_1^e \left(x \times \frac{1}{x}\right) dx$$

$$= (logx \times x)_1^e - \int_1^e 1\,dx$$

$$= (x\,logx)_1^e - (x)_1^e$$

$$= (e\,loge - 1log1) - (e - 1)$$

$$= (e \times 1 - 1 \times 0) - e + 1$$

$$= e - e + 1$$

$$= 1$$

(g) Solve $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

$$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$$

Dividing $(1 + x^2)$ on both sides, we get

$$\frac{dy}{dx} + \frac{1}{(1 + x2)}y = \frac{e^{\tan^{-1}x}}{(1 + x2)}$$

$$\frac{dy}{dx} + Py = Q$$

$\therefore P = \frac{1}{(1 + x2)}$

$Q = \frac{e^{\tan^{-1} x}}{(1 + x2)}$

$\therefore$ I.F = $e^{\int p\, dx} = e^{\int \frac{1}{(1 + x2)} dx}$

$= e^{\tan^{-1} x}$

Let u = $e^{\tan^{-1} x}$

$\frac{du}{dx} = e^{\tan^{-1} x} \frac{1}{(1 + x2)}$

du = $e^{\tan^{-1} x} \frac{1}{(1 + x2)} dx$

$\therefore$ Solution is Y x IF = $\int Q$ x IF dx

$$Y \text{ x } e^{\tan^{-1} x} = \int \frac{e^{\tan^{-1} x}}{(1 + x2)} \text{ x } e^{\tan^{-1} x}\, dx$$

$\therefore$ Solution is Y x $e^{\tan^{-1} x} = \int u\, du$

$$= \frac{u^2}{2}$$

$$Y \text{ x } e^{\tan^{-1} x} = \frac{e^{2\tan^{-1} x}}{2} + c$$

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) Evaluate $\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16}$

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16} = \frac{na^{n-1}}{ma^{m-1}}$$

$$= \frac{3 \text{ x} 4^{3-1}}{2 \text{ x} 4^{2-1}}$$

$$= \frac{3 \text{ x } 16}{2 \text{ x } 4}$$

$=\frac{3 \text{ x } 16}{8}$
$= 3 \text{ x } 2 = 6$

(b) If $y = x^2 cosx$, show that $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (x^2 + 6)y = 0$

$$y = x^2 cosx$$

$\frac{dy}{dx} = x^2 \text{ x} - sinx + cosx \text{ x } 2x$

$\frac{d^2y}{dx^2} = -x^2 \ cosx + sinx \text{ x} - 2x + cosx \text{ x } 2 - 2xsinx$

$= -x^2cosx - 2xsinx + 2cosx - 2xsinx$

$= -x^2cosx - 4xsinx + 2cosx$

$$\therefore x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + x^2y + 6y$$

$=x^2(-x^2cosx - 4xsinx + 2cosx) -$

$4x(-x^2cosx - 4xsinx + 2cosx) + x^2 \text{x } x^2cosx + 6x^2 \text{x } x^2cosx$

$= -x^4cosx - 4x^3sinx + 2x^2cosx +$

$4x^3sinx - 8x^2cosx + x^4cosx + 6x^2cosx$

$= 0$

(c) Find

i. $\frac{d}{dx}\log\left(x + \sqrt{1 + x^2}\right)$

ii. $\frac{d}{dx}\frac{e^x sinx}{1+logx}$

i. $\frac{d}{dx}\log\left(x + \sqrt{1 + x^2}\right)$

$=\frac{1}{x+\sqrt{1+x^2}}\ (1+\frac{1}{2\sqrt{1+x^2}}$ x 2x)

$=\frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}$

ii. $\frac{d}{dx}[\frac{e^x sinx}{1+logx}]$

$=\frac{(1+logx)[e^x cosx+sinx e^x]-e^x sinx\ \text{x}\ \frac{1}{x}}{(1+logx)^2}$

IV.

(a) Find the differential coefficient with respect to x

i. Log(logx)

ii. $\frac{e^x-1}{e^x+1}$

i. Y = Log(logx)

$\frac{dy}{dx}=\frac{1}{logx}$ x $\frac{1}{x}$

ii. Y = $\frac{e^x-1}{e^x+1}$

$\frac{dy}{dx}=\frac{(e^x+1)e^x-(e^x-1)e^x}{(e^x+1)^2}$

(b) Find $\frac{dy}{dx}$, if $x^2+y^2+2gx+2fy+C$

$x^2+y^2+2gx+2fy+C=0$

$2x+2y\frac{dy}{dx}+2g+2f\frac{dy}{dx}=0$

$\frac{dy}{dx}(2y+2f)=-2x-2g$

$\frac{dy}{dx}=\frac{-2x-2g}{2y+2f}$

$$= \frac{-2(x+g)}{2(y+f)}$$

$$= \frac{-(x+g)}{(y+f)}$$

(c) Find $\frac{dy}{dx}$, if x = $a\cos^3 t$, y = $b\sin^3 t$

if $x = a\cos^3 t$

$$\frac{dx}{dt} = -3a\cos^2 t \sin t$$

$$y = b\sin^3 t$$

$$\frac{dy}{dt} = 3b\sin^2 t \cos t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{-3a\cos^2 t \sin t}{3b\sin^2 t \cos t}$$

$$= \frac{-a}{b} \cot t$$

V.

(a) Find the equation of a tangent at normal to the curve $x^2 + y^2 = 25$ at the point (3, -4)

$$x^2 + y^2 = 25$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\therefore m = \frac{-3}{-4} = \frac{3}{4}$$

∴ Equation of the tangent is $y - y_1 = m(x - x_1)$

$$y + 4 = \frac{3}{4}(x - 3)$$

$$4y + 16 = 3x - 9$$

$$3x - 4y - 25 = 0$$

Equation of the normal is $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y + 4 = -\frac{4}{3}(x - 3)$$

$$3y + 12 = -4x + 12$$

$$4x + 3y = 0$$

(b) Prove that a rectangle of fixed perimeter has its maximum area when it became a square

Area of a rectangle $A = l \times b$

Perimeter $P = 2(l + b)$

$$\frac{P}{2} = l + b$$

$$b = \frac{P}{2} - l$$

$$\therefore A = l \times \left(\frac{P}{2} - l\right)$$

$$= \frac{lP}{2} - l^2$$

$$\frac{dA}{dl} = \frac{P}{2} - 2l$$

$$\frac{d^2A}{dl^2} = -2 < 0$$

$\therefore$ Area is maximum when $\frac{dA}{dl} = 0$

$$ie, \frac{P}{2} - 2l = 0$$

$$\frac{P}{2} = 2l$$

$$\frac{P}{4} = l$$

$$l = \frac{P}{4}$$

$$b = \frac{P}{2} - l$$

$b = \frac{P}{2} - \frac{p}{4}$

$b = \frac{P}{4}$

$\therefore l = b = \frac{P}{4}$

∴it becomes a square

(c) Find the values of ‘x’ for which $x^2 - 3x + 4$ is

i. Increasing
ii. Decreasing

$y = x^2 - 3x + 4$

$\frac{dy}{dx} = 2x - 3$

i. $\frac{dy}{dx} > 0$

$2x - 3 > 0$

$2x > 3$

$x > \frac{3}{2}$

ii. $\frac{dy}{dx} < 0$

$2x - 3 < 0$

$2x < 3$

$x < \frac{3}{2}$

VI.

(a) A spherical bladder of radius 3’’ has air pumped into it. If the radius increases at a uniform rate of 1’’ per minute. Find the rate at which the volume is increasing at the end of 3 minutes

$\frac{dr}{dt} = 1$

Volume of the sphere, V = $\frac{4}{3}\pi r^3$

$$\frac{dr}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = 4\pi r^2 \text{ x } 1$$

$$\frac{dr}{dt} = 4\pi r^2$$

But given radius r = 3

∴ Required radius r = 3 + 3 x 1

= 6

$$\frac{dv}{dt} = 4\pi 6^2$$

$=4\pi$ x 36

$=144\pi$ inch^3/minutes

(b) Find the equation of the tangent to the curve $y = \sqrt{25 - x^2}$ at (4, 3)

$y = \sqrt{25 - x^2}$ (4, 3)

$$\frac{dy}{dx} = \frac{1}{2\sqrt{25-x^2}}\,{}^{-}2x$$

$$= \frac{-x}{\sqrt{25-x^2}} = \frac{-4}{\sqrt{25-4^2}}$$

$$= \frac{-4}{\sqrt{25-16}} = \frac{-4}{\sqrt{9}}$$

$$= \frac{-4}{3}$$

Equation of the tangent is $y - y_1 = m(x - x_1)$

$y - 3 = \frac{-4}{3}(x - 4)$

$3y - 9 = -4x + 16$

$4x + 3y - 25 = 0$

Equation of the normal is $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - 3 = \frac{3}{4}(x - 4)$$

$$4y - 12 = 3x - 12$$

$$3x - 4y = 0$$

(c) Find the maximum value of the cone whose slant height is ‘l’ cm

Volume of cone $V = \frac{1}{3}\pi r^2 h$

We have x, $l^2 = r^2 + h^2$

$$\therefore r^2 = l^2 - h^2$$

$\therefore V = \frac{1}{3}\pi(l^2 - h^2)h$

$V = \frac{1}{3}\pi l^2 h - \frac{1}{3}\pi h^3$

$\therefore \frac{dv}{dh} = \frac{1}{3}\pi l^2 - \frac{1}{3}\, 3\pi h^2$

$= \frac{1}{3}\pi l^2 - \pi h^2$

$\frac{d^2v}{dh^2} = -2\pi h < 0$

h is +ve

$\therefore$ V is maximum when $\frac{dv}{dh} = 0$

$ie \frac{1}{3}\pi l^2 - \pi h^2 = 0$

$$\frac{1}{3}\pi l^2 = \pi h^2$$

$l^2 = 3h^2$

$$l = \sqrt{3}\, h$$

$\therefore$ Volume $V = \frac{1}{3}\pi l^2 h - \frac{1}{3}\pi h^3$

$= \frac{1}{3}\pi(\sqrt{3}\, h)^2 h - \frac{1}{3}\pi h^3$

$= \frac{1}{3}\pi 3h^3 - \frac{1}{3}\pi h^3$

$= \pi h^3 - \frac{1}{3}\pi h^3$

$= \frac{2}{3}\pi h^3$

VII.

(a) Find $\int \frac{2+3sinx}{\cos^2 x}$dx

$\int \frac{2+3sinx}{\cos^2 x}$dx

$= \int \frac{2}{\cos^2 x}dx + \int \frac{3sinx}{\cos^2 x}\,dx$

$= 2\int \sec^2 x\, dx + 3\int secxtanx\, dx$

$= 2tanx + 3secx + C$

(b) Find $\int_0^{\frac{\pi}{2}} \cos^2 x\ dx$

$$\int_0^{\frac{\pi}{2}} \cos^2 x\ dx$$

$= \int_0^{\frac{\pi}{2}} (\frac{1+cos2x}{2})\, dx$

$= \frac{1}{2}\left[x + \frac{sin2x}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{sin2\frac{\pi}{2}}{2} - 0 - sin0\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} + 0\right]$

$= \frac{\pi}{4}$

(c) Find $\int xsin(x^2)dx$

Let u = x^2

$\int xsin(x^2)dx = \int sinu \times \frac{du}{2}$

$\frac{du}{dx}$ = 2x

$\frac{du}{2}$ = x dx

$= \frac{1}{2}$x –cosu

$= \frac{-\cos(x^2)}{2} + C$

(d) Find $\int \frac{2x^4}{1+x^{10}}$dx

$\int \frac{2x^4}{1+x^{10}}dx = \int \frac{2x^4}{1+(x^5)^2}dx$

$= \int \frac{2 \text{ x } \frac{du}{5}}{1+u^2}$

$= \frac{2}{5} \int \frac{1}{1+u^2}du$

$= \frac{2}{5}\tan^{-1}u$

$= \frac{2}{5}\tan^{-1}(x^5) + C$

Let u = x^5

$\frac{du}{dx} = 5x^4$

$\frac{du}{5} = x^4\, dx$

VIII.

(a) Evaluate $\int x^2 e^x$dx

$\int x^2 e^x dx = x^2 e^x - \int e^x \text{x } 2x \text{ dx}$

$= x^2 e^x - 2 \int x e^x dx$

$= x^2 e^x - 2[xe^x - \int e^x\, dx]$

$= x^2 e^x - 2xe^x + 2e^x + C]$

(b) Evaluate $\int \sin^3 x$dx

$\int \sin^3 x dx = \int \left(\frac{3sinx - sin3x}{4}\right)dx$

$= \frac{1}{4}[-3cosx + \frac{cos3x}{3}]$

$= \frac{-3}{4}cosx + \frac{cos3x}{12} + C$

(c) Evaluate $\int_0^{\frac{\pi}{2}} cos4x\ cosx$ dx

$$\int_0^{\frac{\pi}{2}} cos4x\ cosx\ dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}\ (cos5x + \ cos3x)\ dx$$

$$= \frac{1}{2}\left[\frac{sin5x}{5} + \frac{sin3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left[\frac{sin5\frac{\pi}{2}}{5} + \frac{sin3\frac{\pi}{2}}{3}\right] - 0 - \ 0\right]$$

$$= \frac{1}{2}\left[\frac{1}{5} + \frac{-1}{3}\right]$$

$$= \frac{1}{2}\left[\frac{3-5}{15}\right]$$

$$= \frac{1}{2}\left[\frac{-2}{15}\right]$$

$$= \frac{-1}{15}$$

(d) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$

$$\int_0^1 \frac{1}{1+x^2}dx = [\tan^{-1}x]_0^1$$

$$= \tan^{-1}1 - \tan^{-1}0$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$

IX.

(a) Find the area enclosed between one arch of the curve $2x + y = 1$ and the curve $y = x^2 - 6x + 4$

$$2x + y = 1$$

$$y = x^2 - 6x + 4$$

$$y = 1 - 2x$$

$$\therefore 1 - 2x = x^2 - 6x + 4$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1, 3$$

∴ Area = $\int_a^b f(x)g(x)dx$

= $\int_a^b (1-2x) - (x^2 - 6x + 4)dx$

= $\int_1^3 (-x^2 + 4x - 3)dx$

= $\left[\frac{-x^3}{3} + \frac{4x^2}{2} - 3x\right]_1^3$

= $\left[\frac{-x^3}{3} + 2x^2 - 3x\right]_1^3$

= $-\left(\frac{3^3}{3} - \frac{1^3}{3}\right) + 2(9-1) - 3(3-1)$

= $-\left(9 - \frac{1}{3}\right) + 2$ x $8 - 3$ x 2

= $-\frac{26}{3} + 16 - 6$ =-26/3 +10

= $-\frac{26+30}{3}$

= $\frac{4}{3}$

∴ Area = $\frac{4}{3}$ square unit

(b) Solve $\frac{dy}{dx} = \frac{xy^2+x}{yx^2+y}$

$\frac{dy}{dx} = \frac{xy^2+x}{yx^2+y}$

$\frac{dy}{dx} = \frac{x(y^2+1)}{y(x^2+1)}$

$\frac{ydy}{(y^2+1)} = \frac{x}{(x^2+1)}dx$

$\int \frac{y}{(y^2+1)}$dy $= \int \frac{x}{(x^2+1)}dx$

u = $x^2 + 1$

$\frac{du}{dx}$=2x

$\frac{du}{2}$ = x dx

$= \int \frac{1}{u} \frac{du}{2}$

$= \frac{1}{2} \log u$

$= \frac{1}{2} \log (x^2 + 1)$

∴ Solution is,

$\frac{1}{2}\log (y^2 + 1) = \frac{1}{2} \log (x^2 + 1)$

$\log (y^2 + 1) = \log (x^2 + 1) + C$

(c) Find the volume generated by the rotation of the area bounded by the curve $y = 2x^2 + 1$, the y – axis and the lines y = 3 and y = 9 about the y – axis.

$y = 2x^2 + 1$ y = 3, y = 9

$$\therefore x^2 = \frac{y-1}{2}$$

∴ Volume V = $\pi \int_a^b x^2 dy$

$$V = \pi \int_3^9 \frac{y-1}{2} dy$$

$$V = \frac{\pi}{2} \int_3^9 y - 1 dy$$

$$V = \frac{\pi}{2} \left[\frac{y^2}{2} - y\right]_3^9$$

$$V = \frac{\pi}{2} \left[\left(\frac{9^2}{2} - \frac{3^2}{2}\right) - (9 - 3)\right]$$

$$V = \frac{\pi}{2} \left[\left(\frac{81}{2} - \frac{9}{2}\right) - 6\right]$$

$$V = \frac{\pi}{2} (36 - 6)$$

$V = \frac{30}{2}\ \pi$

$= 15\ \pi$ cubic units

X.

(a) Find the volume generated when the portion of the parabola $y^2 = 4x$ between x = 0, x = 4 revolves about the x – axis

$$V = \pi \int_a^b y^2 dx$$

$$V = \pi \int_0^4 4x\ dx$$

$= \pi \left[\frac{4x^2}{2}\right]_0^4$

$= \pi [2x]_0^4$

$= \pi$ (2 x 16)

$= 32\pi$ cubic units

(b) solve $\frac{dy}{dx} + ycotx$ = cosecx

$\frac{dy}{dx} + ycotx$ = cosecx

$\frac{dy}{dx} + Py$ = Q

P = cotx

Q = cosecx

∴ I.F = $e^{\int p\ dx}$

$= e^{\int cotx\ dx}$

$= e^{\log sinx}$

= sinx

Solution is y x IF = $\int (Q$ x IF$)\ dx$

Ysinx = $\int cosecx$ x $sinx\ dx$

Ysinx = $\int \frac{1}{sinx}$ x $sinx\ dx$

Ysinx = x + C

(c) find the integrity factor of $\frac{dy}{dx} + 3y = e^{2x}$

$\frac{dy}{dx} + 3y = e^{2x}$

$\frac{dy}{dx} + Py = Q$

P = 3

Q = e^{2x}

∴ IF = $e^{\int p\ dx}$

= $e^{\int 3\ dx}$

= e^{3x}