TED (10)-1002 Reg. No. ………………………….

(REVISION-2010) Signature ……..…………………..

FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLIGY- MARCH, 2014

TECHNICAL MATHEMATICS- I

(Common – Except DCP and CABM)

[*Time*: 3 hours

(Maximum marks: 100)

Marks

PART –A

(Maximum marks: 10)

(Answer all questions. Each question carries 2 marks)

I.

(a) If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$ find$(A + B)^T$

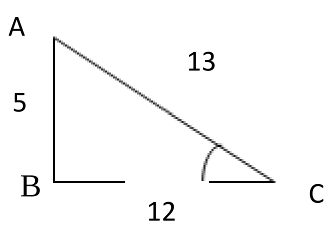
$$(A + B)^T = \left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}\right)^T$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$$

(b) Evaluate $\begin{vmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{vmatrix}$

$$\begin{vmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{vmatrix} = \sec^2\theta - \tan^2\theta = 1$$

(c) If sinθ= 5/13 find cosθ and tanθ



$\text{Sin}\theta = \frac{5}{13}$

$\text{Cos}\theta = \frac{12}{13}$

$\text{Tan}\theta = \frac{5}{12}$

(d) Prove that sin2θ = 2sinθ.cosθ

$\text{Sin}2\theta = \sin(\theta + \theta) = \sin\theta.\cos\theta + \cos\theta.\sin\theta$

$2\sin\theta\cos\theta$

(e) Find the slope of the line joining the points (3,3) and (1,2)

$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-3}{1-3} = \frac{-1}{-2} = \frac{1}{2}$

PART –B

Answer any five questions. Each question carries 6 marks

II.

(a) Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$|A| = 4$

<u>Minors</u>

$m_{11} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$m_{12} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$

$m_{13} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$

$m_{21} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$

$m_{22} = \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} = 4$

$m_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$

$m_{31} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$

$m_{32} = \begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix} = 2$

$m_{33} = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 4$

Minor matrix = $\begin{bmatrix} 0 & \mathbf{-2} & -2 \\ \mathbf{2} & 4 & \mathbf{2} \\ 2 & \mathbf{2} & 4 \end{bmatrix}$

Cofactor matrix of A = $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$

Adjoint matrix of A = $\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj A}}{|A|} = \frac{\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}}{4}$

(b) Solve using determinants:

2a – 3b + c = 1

A + 4b + 2c = 3

4a – b + 3c = 11

AX = B

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 11 \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} -1 & -3 & 1 \\ 3 & 4 & -2 \\ 11 & -1 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ 4 & -1 & 3 \end{vmatrix}}$$

$$= \frac{-(12-2)+3(9+22)+(-3-44)}{2(12-2)+3(3+8)+1(-1-16)}$$

$$= \frac{36}{36} = 1$$

$$y= \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 4 & 11 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ 4 & -1 & 3 \end{vmatrix}}$$

$$= \frac{2(9+22)+(3+8)+(11-12)}{36}$$

$$= \frac{72}{36} = 2$$

$$z= \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 2 & -3 & -1 \\ 1 & 4 & 3 \\ 4 & -1 & 11 \end{vmatrix}}{36}$$

$$= \frac{2(44+3)+3(11-12)-(-1-16)}{16}$$

$$= \frac{108}{36} = 3$$

(c) Find the term independent of x in the expression of $(x^3 - {}^{3}/_{x^2})^5$

$T_{r+1} = nc_r a^{n-r}\ b^r$

$= 5c_r(x^3)^{5-r}(-3/x^2)^r$

$= 5c_6 x^{15-3r}(-3^r)\ x^{-2r}$

$= 15c_6 x^{15-5r}(-3^r)$

$15 - 5r = 0$ ===>r=3

∴ Term independent of x is $T_4 = 5c_3 x^0(-3)^3$

$=5c_3(-3)^3$

(d) Prove that $\frac{1+\sin A-\cos A}{1+\sin A+\cos A} = \tan\frac{A}{2}$

$$\text{L.H.S} = \frac{1+\sin A-\cos A}{1+\sin A+\cos A}$$

$$= \frac{(1-\cos A)+\sin A}{(1+\cos A)+\sin A}$$

$$= \frac{2\sin^2\left(\frac{A}{2}\right)+2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{2\cos^2\left(\frac{A}{2}\right)+2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{A}{2}\right)\left(\sin\left(\frac{A}{2}\right)+\cos\left(\frac{A}{2}\right)\right)}{2\cos\left(\frac{A}{2}\right)\left(\sin\left(\frac{A}{2}\right)+\cos\left(\frac{A}{2}\right)\right)}$$

$$= \tan\frac{A}{2}$$

(e) In a ΔABC, $R(a^2 + b^2 + c^2) = abc\,(\cot A + \cot B + \cot C)$

$$\text{R.H.S} = abc\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right)$$

$$= abc\frac{\cos A}{\sin A} + abc\frac{\cos B}{\sin B} + abc\frac{\cos C}{\sin C}$$

$$= \frac{a}{\sin A}\,bc.\cos A + \frac{b}{\sin B}\,ac.\cos B + \frac{c}{\sin C}\,ab.\cos C$$

$$= 2R.bc.\cos A + 2R.ac.\cos B + 2R.ab.\cos C$$

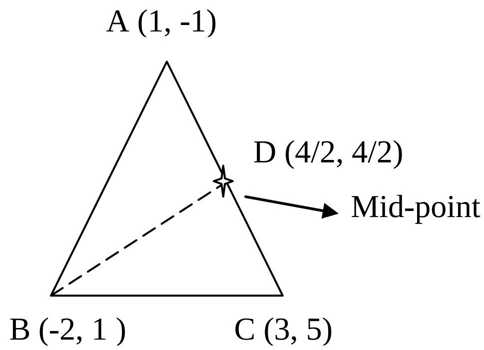
$$= 2R\left[bc\,.\frac{b^2+c^2-a^2}{2bc} + ac.\frac{a^2+c^2-b^2}{2ac} + ab.\frac{a^2+b^2-c^2}{2ab}\right]$$

$$= 2R\left[\frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2}\right]$$

$$= 2R\left[\frac{a^2+b^2+c^2}{2}\right]$$

$$= R\,(a^2 + b^2 + c^2) = \text{L.H.S}$$

(f) If A(1, -1) , B(-2 , 1) and C(3, 5) are the vertices of a triangle , then find the equation ofmedian through B



Slope of AB = $\frac{y2-y1}{x2-x1}$

$= \frac{1--1}{-2-1} = \frac{2}{3}$, ($m_1$)

Slope of AC = $\frac{5--1}{3-1} = \frac{6}{2}$ = 3, (m_2)

Slop of BC = $\frac{5--1}{3--2} = \frac{4}{5}$, ($m_3$)

Equation of BD

$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$

$\frac{y-y1}{2-1} = \frac{x--2}{2--2}$

$\frac{y-1}{1} = \frac{x+2}{4}$

4(y – 1) = x + 2

4y – 4 = x + 2

x – 4y = -6

x – 4y + 6 = 0

(g) The x – intercept of a line is 3 times in the y – intercept. If it passes through (3,2), find the equation.

Intercept form of a line is

$\frac{x}{a} + \frac{y}{b}$= 1ginen a = 3b

$\frac{x}{3b} + \frac{y}{b}$= 1 ——— (1)

Given equation (1) passes through (3,2) we have

$\frac{3}{3b} + \frac{2}{b}$= 1

===>$\frac{1}{b} + \frac{2}{b}$= 1 ===>$\frac{3}{b}$= 1 ===> b = 3

∴ Equation (1) becomes,

$\frac{x}{9} + \frac{y}{3}$= 1

===> 3x + 9y = 27

===> x + 3y = 9

PART –C

(Maximum mark: 60)

Answer four full questions. Each question carries 15 marks.

III.

(a) If A(θ) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ show that A(θ).A(θ′) = A(θ + θ′)

A (θ) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

A (θ′) = $\begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix}$

A(θ).A(θ′) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix}$

= $\begin{bmatrix} \cos\theta.\cos\theta' - \sin\theta.\sin\theta' & -(\cos\theta.\sin\theta' + \sin\theta.\cos\theta') \\ \sin\theta.\cos\theta' + \cos\theta.\sin\theta' & -\sin\theta.\sin\theta' + \cos\theta'.\cos\theta' \end{bmatrix}$

= $\begin{bmatrix} \cos(\theta + \theta') & -\sin(\theta + \theta') \\ \sin(\theta + \theta') & \cos(\theta + \theta') \end{bmatrix}$

= A(θ + θ′)

(b) Find a, b, c if $\begin{bmatrix} a + 3 & 3a - 2b \\ 3a - c & a + b + c \end{bmatrix}$ = $\begin{bmatrix} 2 & -7 + 2b \\ b + 4 & 2a \end{bmatrix}$

$\begin{bmatrix} a + 3 & 3a - 2b \\ 3a - c & a + b + c \end{bmatrix}$ = $\begin{bmatrix} 2 & -7 + 2b \\ b + 4 & 8a \end{bmatrix}$

a + 3 = 2 ===> a = -1

3a – 2b = -7 + 2b ===> 3 x -1 – 2b = -7 + 2b

$===> -3 – 2b = -7 + 2b$

$===> 4 = 4b$

$===> b = 1$

$3a – c = b + 4$

$3 \times -1 – c = 1 + 4$

$-3 – c = 5$

$c = -8$

(c) If $A=\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix}$ prove that $A^3 -3A^2 + 3A – I = 0$

$A^3 = A^2.A$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & -2 \\ 6 & 0 & 1 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & -2 \\ 6 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -15 & 1 & -3 \\ 9 & 0 & 1 \end{bmatrix}$$

$$3A^2 = \begin{bmatrix} 3 & 0 & 0 \\ -21 & 3 & -6 \\ 18 & 0 & 3 \end{bmatrix} \quad , \quad 3A = \begin{bmatrix} 3 & 0 & 0 \\ -6 & 3 & -3 \\ 9 & 0 & 3 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 -3A^2 + 3A – I = \begin{bmatrix} 1 & 0 & 0 \\ -15 & 1 & -3 \\ 9 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ -21 & 3 & -6 \\ 18 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ -6 & 3 & -3 \\ 9 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

IV.

(a) Find A and B, if $A – B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $A + B = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

Given $A - B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ ——— (1)

$A + B = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$ ——— (2)

Adding (1) and (2) we get

$2A = \begin{bmatrix} 4 & 6 \\ 4 & 4 \end{bmatrix}$

$A = \dfrac{\begin{bmatrix} 4 & 6 \\ 4 & 4 \end{bmatrix}}{2} = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$

Then $B = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} - A$

$= \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}$

(b) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix}$ show that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric

$A + A^T = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 8 \\ -2 & 2 & 8 \\ 8 & 8 & 14 \end{bmatrix}$

$(A + A^T)^T = \begin{bmatrix} 2 & -2 & 8 \\ -2 & 2 & 8 \\ 8 & 8 & 14 \end{bmatrix}^T = \begin{bmatrix} 2 & -2 & 8 \\ -2 & 2 & 8 \\ 8 & 8 & 14 \end{bmatrix}$

Clearly $A + A^T$ is symmetric

$A - A^T = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ 5 & 6 & 7 \end{bmatrix}^T = \begin{bmatrix} 0 & -2 & -2 \\ 2 & 0 & -4 \\ 2 & 4 & 0 \end{bmatrix}$

$(A - A^T)^T = \begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 4 \\ -2 & -4 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 4 \\ -2 & -4 & 0 \end{bmatrix} = -(A - A^T)$

Clearly $A - A^T$ is skew symmetric.

(c) Solve using inverse of the coefficient matrix

$x + y + z = 1,$

$2x + 2y + 3z = 6,$

x + 4y + 9z = 3

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

<u>Calculation for A^{-1}</u>

$m_{11} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6$

$m_{12} = \begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix} = 15$

$m_{13} = \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 6$

$m_{21} = \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = 5$

$m_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8$

$m_{23} = \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = 3$

$m_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$

$m_{32} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$

$m_{33} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$

$|A| = -3$

Minor matrix $= \begin{bmatrix} 6 & \mathbf{15} & 6 \\ \mathbf{5} & 8 & \mathbf{3} \\ 1 & \mathbf{1} & 0 \end{bmatrix}$

Cofactor matrix $= \begin{bmatrix} 6 & -15 & 6 \\ -5 & 8 & -3 \\ 1 & -1 & 0 \end{bmatrix}$

Adjoint matrix $= \begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$

So inverse matrix, $A^{-1} = \frac{\text{Adj.A}}{|A|} = \frac{\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}}{-3}$

$X = A^{-1}B = \frac{\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}}{-3} = \frac{\begin{bmatrix} -21 \\ 30 \\ -12 \end{bmatrix}}{2}$

$x = -\frac{21}{-3} = 7$

$y = \frac{30}{-3} = -10$

$z = -\frac{12}{-3} = 4$

V.

(a) Prove that $nC_r = nC_{n-r}$

$nC_{n-r} = \frac{n!}{(n-(n-r))!(n-r)!}$

$= \frac{n!}{(n-n+r)!(n-r)!} = \frac{n!}{r!(n-r)!}$

$= \frac{n!}{(n-r)!r!} = nC_r$

(b) Find the 9th term in the expansion of $(x^2 - 1/x)^{18}$

$T_{r+1} = (-1)^r nc_r\, a^{n-r} b^r$, n= 18

$T_9 = (-1)^8 18c_8\, (x^2)^{10} (1/x)^8$

$= 18c_8 x^{20} \frac{1}{x^8}$

$= 18c_8 x^{12}$

(c) Prove that in ΔABC, $\tan(\frac{B-C}{2}) = (\frac{b-c}{b+c}).\cot A/2$

Consider $({}^{b-c}/_{b+c})\cot A/2 = \frac{2R\sin B - 2R\sin C}{2R\sin B + 2R\sin C}.\cot A/2$

$= \frac{2R(\sin B - \sin C)}{2R(\sin B + \sin C)}.\cot A/2$

$$= \frac{(\sin B - \sin C)}{(\sin B + \sin C)} . \cot A/2$$

$$= \frac{2\cos\left(\frac{B+C}{2}\right) . \sin\left(\frac{B-C}{2}\right)}{2\sin\left(\frac{B+C}{2}\right) . \cos\left(\frac{B-C}{2}\right)} . \cot A/2$$

$$= \cot\left(\frac{B+C}{2}\right) . \tan\left(\frac{B-C}{2}\right) . \cot\left(\frac{A}{2}\right)$$

$$= \tan\left(\frac{B-C}{2}\right) . \cot\left(90 - \frac{A}{2}\right) . \cot\left(\frac{A}{2}\right)$$

$$= \tan\left(\frac{B-C}{2}\right) . \tan\frac{A}{2} . \cot\frac{A}{2}$$

$$= \tan\left(\frac{B-C}{2}\right)$$

VI.

(a) Find the coefficient of x^5 in the expansion of $(3x+4/x)^{11}$

$T_{r+1} = nc_r\, a^{n-r} b^r$, n = 11

$T_{r+1} = 11c_r\, (3x)^{11-r}\, (4/x)^r$

$= 11c_r 3^{11-r}\, x^{11-r}\, 4^r x^{-r}$

$= 11c_r 3^{11-r}\, x^{11-2r}\, 4^r$

Now $11 - 2r = 5$

$-2r = 5 - 11$

$r = 3$

$T_4 = 11c_3 3^8 4^3 = 69284160$

(b) Find the constant term in the expansion of $(x - 2/x^2)^{15}$

$T_{r+1} = (-1)^r nc_r\, a^{n-r} b^r$

$T_{r+1} = (-1)^r 15c_r\, (x)^{15-r}\, (2/x^2)^r$

$= (-1)^r 15c_r x^{15-r}\, (2)^2\, x^{-2r}$

$= (-1)^r 15c_r x^{15-3r}\, (2)^2$

$15 - 3r = 0$

$r = 5$

$T_5 = (-1)^5 15c_5 (2)^5$

$= -96096$

(c) Prove that $\frac{\sin2A+\sin4A}{\cos2A+\cos4A} = \tan3A$

$\frac{\sin2A+\sin4A}{\cos2A+\cos4A} = 2\sin(\frac{2A+4A)}{2}\cos(\frac{2A-4A}{2})/ 2\cos(\frac{2A+4A}{2})\text{Cos}(\frac{2A-4A}{2})$

$= \frac{\sin3A}{\cos3A} = \tan3A$

VII.

(a) If A & B are acute angles, where $\tan A = \frac{1}{11}$ and $\tan B = \frac{5}{6}$. Show that $A + B = 45^0$

$\tan A = \frac{1}{11}$

$\tan B = \frac{5}{6}$

$\tan(A + B) = \frac{\tan A+\tan B}{1-\tan A.\tan B}$

$= \frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}.\frac{5}{6}}$

$= \frac{\frac{6+55}{66}}{1-\frac{5}{66}}$

$= \frac{61}{66} \text{ X } \frac{66}{61} = 1$

$===> \tan(A + B) = 1$

$===> A + B = 45^0$

(b) Prove that cos3A + cos5A + cos9A + cos17A = 4cos4A.cos6A.cos7A

cos3A + cos5A + cos9A + cos17A

= (cos3A + cos5A) + (cos9A + cos17A)

= 2cos4A.cosA + 2cos13A.cos4A

$= 2\cos 4A\ (\cos A + \cos 13A)$ [cos(-A) = cosA]

$= 2\cos 4A.2\cos 7A.\cos 6A$

$= 4\cos 4A.\cos 6A.\cos 7A$

(c) Find the value of tan75° and hence show that tan75° + cot75° = 4

$\text{Tan}75^o = \tan(45 + 30)^o$

$= \tan 45 + \tan 30 / 1 - \tan 45.\tan 30$

$$= \frac{1 + \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3}$$

$$\text{Cot}75 = \frac{1}{\tan 75} = \frac{1}{2+\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}} \text{ x } \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

$\therefore \tan 75^o + \cot 75^o = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

VIII.

(a) Prove that in ΔABC, $\sum a(\sin B - \sin C) = 0$

In ΔABC, we know that $\sin B = \frac{b}{2R}$ (by sine rule)

And $\sin C = \frac{c}{2R}$

$$\sum a(\sin B - \sin C) = \sum a\left(\frac{b}{2R} - \frac{c}{2R}\right)$$

$$= \frac{1}{2R}\left(\sum a\ (b - c)\right)$$

$$= \frac{1}{2R}\left[a\ (b - c) + b\ (c - a) + c\ (a - b)\right]$$

$$= \frac{1}{2R}\left[\ ab - ac + bc - ba + ca - cb\right]$$

$$= \frac{1}{2R} \text{x } 0 = 0$$

(b) Solve ΔABC if a = 2, b = 3 & c = 4

A = $\cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \cos^{-1}\left(\frac{3^2+4^2-2^2}{2\times 3\times 4}\right)$

$= \cos^{-1}\left(\frac{25-4}{24}\right) = \cos^{-1}\left(\frac{21}{24}\right)$

$= \cos^{-1}(0.875)$

$= (28.96)^{o} = 28^{o}57'$

B = $\cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)$

$= \cos^{-1}\left(\frac{2^2+4^2-3}{2\times 2\times 4}\right)$

$= \cos^{-1}\left(\frac{4+16-9}{16}\right) = \cos^{-1}\left(\frac{11}{16}\right)$

$= \cos^{-1}(0.6875) = 46^{o}34'$

C = $180^{o} - (A + B) = 180^{o} - (28^{o}57' + 46^{o}34') = 104^{o}29'$

(c) Derive expression for cos3A

(i) Cos3A = cos(2A + A)

= cos2A.cosA – sin2A.sinA

= $(2\cos^2 A - 1)\cos A - 2\sin A.\cos A.\sin A$

= $2\cos^3 A - \cos A - 2\sin^2 A.\cos A$

= $2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

= $2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

= $4\cos^3 A - 3\cos A$

IX.

(a) Prove that sinA + sin(120 +A) + sin(240 + A) = 0

sinA + sin(120 +A) + sin(240 + A)

$= \sin A + 2\sin\left(\frac{120+A+240+A}{2}\right)\cos\left(\frac{120+A-240-A}{2}\right)$

$= \sin A + 2\sin\left(\frac{360+2A}{2}\right)\cos\left(\frac{-120}{2}\right)$ $[Cos(-A) = cosA]$

= sinA +2sin(180 + A).cos60

= sinA + 2sin(180 + A) . $\frac{1}{2}$

= sinA +2 sin(2×90 + A)

= sinA - 2sinA ×1/2

= sinA -sinA

= 0

(b) Write down the equation of the line having x intercept = 5 and passing through the point (3,-2)

Intercept form of a line is

$\frac{x}{a} + \frac{y}{b} = 1$

Given $\frac{x}{5} + \frac{y}{b} = 1$ ———— (1)

Equation (1) passes through (3, -2)

Then $\frac{3}{5} + \frac{-2}{b} = 1$

===> 3b – 10 = 5b

===> 2b= -10

===> b = -5

The equation of the line is

$\frac{x}{5} + \frac{y}{-5} = 1$ or x – y = 5

(c) Find the equation to the line passing through the point of intersection of 2x – y – 3 = 0 and

x – 2y = 1 = 0 and perpendicular to the line x – y = 5

Given 2x – y – 3 = 0 ———— (1)

x – 2y = 1 = 0 ———— (2)

Solving (1) (2) we get

$$x = \frac{\begin{vmatrix} 3 & -1 \\ -1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}} = \frac{-6-1}{-4+1} = -\frac{-7}{-3} = 7/3$$

$$y = \frac{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}}{-3} = \frac{-2-3}{-3} = -\frac{5}{-3} = \frac{5}{3}$$ (By Cramer's rule)

∴ Point of intersection is $\left(\frac{7}{3}, \frac{5}{3}\right)$

∴ Given line is x – y = 5

ie, x – y – 5 = 0

∴ Equation of perpendicular line is,

bx – ay + k = 0 (Here a = 1, b = -1)

ie, -x – y + k = 0 ——— (1)

But (1) passes through $\left(\frac{7}{3}, \frac{5}{3}\right)$

∴ (1) ===> $-\frac{7}{3} - \frac{5}{3} + k = 0$

$-\frac{12}{3} + k = 0$

-4 + k = 0

K = 4

∴ (1) ==> -x – y + 4 = 0

X.

(a) Using Napier's formula, find the values of the angles A + B in ΔABC , if a = 5cm, b = 8cm and C = 30°

$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$

$\frac{A-B}{2} = \tan^{-1}\left[\frac{a-b}{a+b}\cot\frac{C}{2}\right]$

$\frac{A-B}{2} = \tan^{-1}\left[\frac{5-8}{13}\cot\frac{30}{2}\right]$

$\frac{A-B}{2} = \tan^{-1}\left[\frac{-3}{13}\cot 15^{\circ}\right]$

$\frac{A-B}{2}$ = $\tan^{-1}[-0.8612]$ = -40.736

A – B = -81.473 ——— (1)

A + B = 180 – 30 =150 ——— (2)

Solving (1) + (2)

A = 34.2635 = $34^{O}16'$ B = 150 - 34.2635 = $115^{O}44'$

Now we have to find 'C'

We have $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{5}{\sin 34^o16'} = \frac{c}{\sin 30^o}$$

$$c = \frac{5}{0.5629996} \times \sin 30^o = 4.44\text{cm}$$

(b) Find the equation to the line passing through the points (2, -1) and (-6, 3). Also find the slope of the line.

Two points form of a line is given by

$$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$$

$$\frac{y--1}{3--1} = \frac{x-2}{-6-2}$$

$$\frac{y+1}{4} = \frac{x-2}{-8}$$

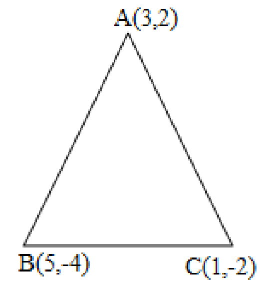
===> $-\frac{8}{4}(y+1) = \frac{x-2}{-8}$

===>-2y – 2 = x – 2

===> x + 2y = 0

Slope of x + 2y = 0 is $-\frac{a}{b} = -\frac{1}{2}$

(c) Find the angle of a triangle having vertices (3 , 2), (5 , -4) and (1 , -2)



Slope of AB = $m_1 = \frac{6}{-2} = -3$

Slope of AC = $m_2 = \frac{4}{2} = 2$

Slope of BC = $m_3 = \frac{2}{-4} = -\frac{1}{2}$

Angle between AB & AC = $\theta = \tan^{-1}\left|\frac{m1-m2}{1+m1m2}\right|$

$= \tan^{-1}\left|\frac{-3-2}{1+-6}\right| = \tan^{-1}\left|\frac{-5}{-5}\right|$

$= \tan^{-1}1 = \frac{\pi}{4} = 45^o$

Angle between AC & BC = $\theta = \tan^{-1}\left|\frac{m2-m3}{1+m2m3}\right|$

$= \tan^{-1}\left|\frac{2+\frac{1}{2}}{1+2\text{ x}\frac{1}{2}}\right| = \tan^{-1}\left|\frac{\frac{5}{2}}{0}\right|$

$= \tan^{-1}(\infty) = \frac{\pi}{2} = 90^o$

Angle between AB & AC = 180 – (90 + 45) = 45^o