TED (15) – 1002

(REVISION — 2015)

Reg. No. ....

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FIRST SEMESTER DIPLOMA EXAMINATION IN ENGINEERING/ TECHNOLOGY — MARCH, 2016

ENGINEERING MATHEMATICS – I

(Common to all branches except DCP and CABM)

[*Time* : 3 hours

(Maximum marks : 100)

PART— A

(Maximum marks : 10)

Marks

I Answer all questions. Each question carries 2 marks.

1. Evaluate $\sin\frac{\pi}{2} + \operatorname{cosec}\frac{\pi}{6} + \cot\frac{\pi}{4}$
2. In Δ ABC, show that abc = $4\Delta R$, where Δ is the area and R is the circum radius of the triangle.
3. Calculate $\lim\limits_{x \to \infty} \frac{7-x}{3x+1}$
4. Find the derivative of $x^2 \sin x$.
5. Find the range of values of x for which $y = 2x^2 - 8x + 1$ is increasing. (5×2=10)

PART— B

(Maximum marks : 30)

II Answer *any five* of the following questions. Each question carries 6 marks.

1. Prove that $\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A+1} = 2\sec^2 A$.
2. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 meters away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.
3. Prove that $\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8} = 0$.
4. Solve Δ ABC, given that a = 8cm, b = 5cm, $\angle C = 30°$.
5. If $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$ show that $\frac{dy}{dx} = \tan t$.

Marks

6. If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2y^{11} + xy^1 + y = 0$.

7. Find the equation of tangent and normal to the curve $x^2 + y^2 = 25$ at $(3, -4)$. Find also the points on this curve at which the tangent is parallel to the x-axis.

(5×6=30)

PART — C

(Maximum marks : 60)

(Answer *one* full question from each unit. Each full question carries 15 marks.)

UNIT—I

III (a) Prove that $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$. 4

(b) Simplify $\dfrac{\sin(90^\circ+\theta)\ \sec(-\theta)\ \cot(180^\circ-\theta)}{\cos(270^\circ+\theta)\ \operatorname{cosec}(180^\circ+\theta)\ \tan(90^\circ-\theta)}$. 4

(c) Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$. 4

(d) If A and B are acute angles, $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ show that $A+B = \frac{\pi}{4}$. 3

OR

IV (a) Prove that $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \ \operatorname{cosec}^2 x$. 4

(b) If $\sin A = \frac{-3}{5}$, $\sin B = \frac{12}{13}$, A lies in 3rd quadrant, B lies in second quadrant, find $\cos(A+B)$ and $\sin(A-B)$. 4

(c) Prove that $\dfrac{\cos A - \sin A}{\cos A + \sin A} = \tan(45^\circ - A)$ 4

(d) If $\theta = 30^\circ$, verify that $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$ 3

UNIT—II

V (a) Prove that $\dfrac{1+\cos 2A}{\sin 2A} = \cot A$ and deduce the value of $\cot 15^\circ$. 5

(b) Prove that $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$. 5

(c) Solve ΔABC, given that $a = 4$cm, $b = 5$cm, $c = 7$cm. 5

OR

VI (a) If $\sin A = \frac{3}{5}$, A is acute, find $\sin 2A$, $\cos 2A$, $\sin 3A$ and $\cos 3A$. 5

(b) Show that $\cos 55^\circ + \cos 65^\circ + \cos 175^\circ = 0$. 5

(c) Show that $a(b^2+c^2)\cos A + b(c^2+a^2)\cos B + C(a^2+b^2)\cos C = 3abc$. 5

Marks

UNIT—III

VII (a) Evaluate (i) $\lim_{x \to 1} \left(\frac{x^2+4x-5}{x^2+x-2} \right)$.

(ii) $\lim_{x \to 0} \left(\frac{1-\cos 2x}{x^2} \right)$. (3+3 = 6)

(b) Using quotient rule show that $\frac{d}{dx}(\sec x) = \sec x \tan x$. 4

(c) Show that $\frac{d}{dx}[\log(x+\sqrt{1+x^2})] = \frac{1}{\sqrt{1+x^2}}$. 5

OR

VIII (a) Evaluate (i) $\lim_{x \to 2} \left(\frac{x^4-16}{x^5-32} \right)$

(ii) $\lim_{\theta \to 0} \left(\frac{\sin 3\theta \ \cos \theta}{\theta} \right)$ (3+3 = 6)

(b) Find the second derivative of $y = \sin^2 x$. 4

(c) Find the derivative of sin x by the method of first principles. 5

UNIT—IV

IX (a) Find the turning values of $y = x^3 - 3x^2 - 9x + 5$. 5

(b) A particle is projected vertically upwards and its height 'h' and time 't' are connected by, $h = 80t - 16t^2$. Find the greatest height attained and acceleration at that time. 5

(c) Find the maximum area of a rectangle whose perimeter is 100m. 5

OR

X (a) The distance 'S' meters travelled by a particle is given by $S = ae^{nt} + be^{-nt}$, where 't' represents the time. Show that the acceleration varies as the distance. 5

(b) Show that the maximum value of the function, $M = 2x^3 - 9x^2 + 12x$ is 5. 5

(c) A balloon is spherical in shape. Gas is escaping from it at the rate of 10cc/sec. How fast is the surface area shrinking when the radius is 15cm. 5